

Properties of Fluids, Fluid Statics, and Fluid Dynamics



1.1 PROPERTIES OF FLUIDS

Fundamental properties of fluids like air, water, oil, gases, etc. must be known for the analysis of flow. Some of the common properties of fluids are discussed below.

1.1.1 Fluid Density

It is defined as the mass per unit volume of fluid denoted by symbol ρ . Density of water at normal temperature and pressure (NTP) is 1000 kg/m^3 .

1.1.2 Unit Weight

It is the weight of the fluid per unit volume at standard temperature and pressure, also called as specific weight. In MKS unit, it is expressed as N/m^3 . It is usually denoted by the symbol γ and expressed as the product of density and acceleration due to gravity, i.e.

$$\gamma = \rho \cdot g \quad (1.1)$$

Unit weight of water $\gamma_w = 1000 \text{ kgf/m}^3$ or 9800 N/m^3 (since $g = 9.8 \text{ m/sec}^2$ and $\text{N} = 1 \text{ kg (mass)} \times 1 \text{ m/s}^2$)

1.1.3 Viscosity

It is the property of fluid due to which shear stress is induced by the fluid in motion. In an ideal fluid, there is no shear stress and hence viscosity is zero. This is termed *inviscid flow* which is hypothetical since all fluids have viscosity. In all real fluid flows, there will be some resistance, low or high, depending on the viscosity governed by the molecular composition of the fluid. As per Newton's law of motion, shear stress is expressed as

$$\tau = \mu \cdot du/dy \quad (1.2)$$

where, μ is the coefficient of dynamic viscosity and du/dy is the velocity gradient in the flow direction, u is the flow velocity and y is the flow depth normal to u . Referring to (Fig. 1.1), when a fluid moves above a flat plate, the velocity of flow (u) in the x -direction changes along the depth (y), varying from zero at the static bottom surface to a maximum value u_{max} at the top of the boundary layer. The velocity gradient is maximum at the bottom where shear stress is also maximum. In an ideal flow (hypothetical), there is no viscosity ($\mu = 0$) and hence there is no shear stress ($\tau = 0$). The unit of dynamic viscosity is

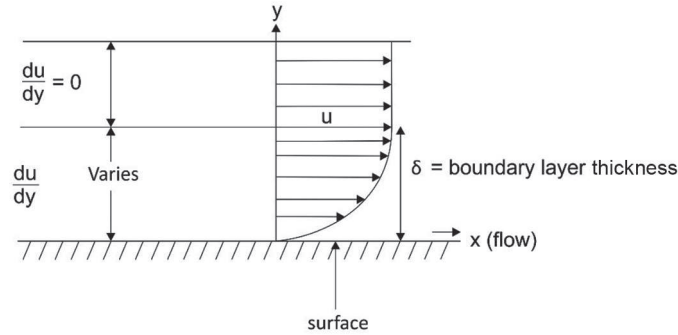


Fig. 1.1: Velocity distribution over a flat surface

$\text{N}\cdot\text{m}/\text{s}^2$ or $\text{kg}\cdot\text{m}/\text{s}$. Viscosity changes with temperature as for liquids, viscosity decreases with a rise in temperature. For gases, viscosity increases with a rise in temperature.

Kinematic viscosity is expressed as

$$\nu = \mu / \rho \quad (1.3)$$

Kinematic viscosity has dimension $\text{M}^0\text{L}^2\text{T}^{-1}$ and the unit is m^2/s , called stoke. One centistoke is one-hundredth of a stoke. At NTP, water has a kinematic viscosity of 10^{-6} stokes or 10^{-2} centistokes.

1.1.4 Specific Gravity

It is the relative density of a fluid with respect to a standard fluid. For solids like iron and liquids like oil, etc. the specific gravity (SG) is measured with respect to the density of water at 4°C . For gases, specific gravity is measured with respect to hydrogen. The specific gravity of oil is less than 1 as oil is lighter than water. The SG of mercury, on the other hand is 13.6 which means it is 13.6 times heavier than water. Specific gravity is a *non-dimensional quantity*.

1.1.5 Surface Tension

It is the molecular property of liquid due to which stress is developed when it is in contact with any surface. It is usually denoted by the symbol σ . It has a dimension of N/m in the SI unit. The capillary rise of a liquid like water in a glass tube (Fig. 1.2) is due to surface tension. If d is the diameter of a capillary tube and γ_w is the unit weight of water in the tube, then the weight of water (W) above the water surface held due to the surface tension is given by

$$W = \gamma_w \times \frac{\pi}{4} (d)^2 h \quad (1.4)$$

where h is the capillary rise. Force acting vertically (F_v) on the circular meniscus of the perimeter (πd) due to surface tension (σ) in the vertical direction is given by

$$F_v = \sigma \pi d \cos \theta \quad (1.5)$$

where, θ is the angle between the liquid and the tube wall. Under equilibrium conditions, $W = F_v$.

Equating Eqs (1.4) and (1.5), we have

$$= \gamma_w d \frac{h}{4} \cos \theta \quad (1.6)$$

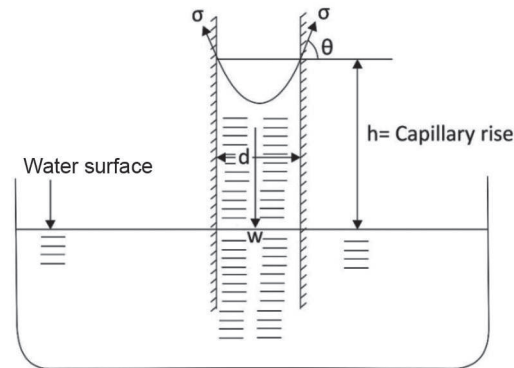


Fig. 1.2: Illustrating capillary rise of water in a glass tube

when $\theta = 0^\circ$ as in the case of water in the capillary tube, $\sigma = \gamma_w d \frac{h}{4}$.

When land is irrigated, water infiltrates into the soil and is retained in soil pores due to the surface tension effect. Without surface tension, all the water would have percolated into the groundwater table and the plants would have withered away. It is the capillary water held in the soil pores against gravity that supplies moisture (through roots) to meet the consumptive requirement of crops for growth and food production.

1.1.6 Bulk Modulus of Elasticity

It governs the compressibility of the fluid. It is denoted by the symbol K which is the ratio between stress and strain to which the fluid is subjected. Strain is defined as the change in volume to the initial volume of the fluid not under any stress. Strain being non-dimensional, bulk modulus of elasticity has the same dimension as stress, i.e. $M L^{-1} T^{-2}$ (N/m^2).

The K -value of water is about $10^5 N/m^2$ which is 100 times more compressible than steel having a K -value of about $10^7 N/m^2$.

The physical properties of some of the common fluids at 20° under atmosphere pressure are given in Table 1.1.

1.2 UNITS AND DIMENSIONS

Physical quantities, such as mass, velocity, acceleration, etc. are usually expressed in terms of standardized units. Earlier, FPS (foot-pound-second) and MKS (meter-kilogram-second) units were popular. But now-a-days SI (Système International d'Unités) units are used. In this book, SI units have been followed. Table 1.2 gives six of the primary units and some derived units. All other units can be derived from these fundamental primary units.

1.2.1 Dimensional Homogeneity

It is extremely important to check not only the numerical equations but its dimensional homogeneity also. This is more important when there are some coefficients in the equation. For example, the discharge equation for free flow over a weir is given by

$$Q = C_d L \cdot H^{3/2} \quad (1.7)$$

Table 1.1: Some fluid properties at 20°C under atmospheric pressure

<i>Name of liquid</i>	<i>Density (ρ) (kg/m³)</i>	<i>Dynamic viscosity (μ) (kg. m/sec)</i>	<i>Kinematic viscosity (m²/sec)</i>
Water	9.98×10^2	1.00×10^{-3}	1×10^{-6}
Octane	7.02×10^2	5.42×10^{-4}	7.72×10^{-7}
Ethyl alcohol	7.89×10^2	1.20×10^{-3}	1.52×10^{-6}
Methyl alcohol	7.92×10^2	5.84×10^{-4}	7.37×10^{-7}
Benzene	8.79×10^2	6.52×10^{-4}	7.42×10^{-7}
Ethylene glycol	10.10×10^2	1.99×10^{-2}	1.79×10^{-5}
Glycerin	12.6×10^2	1.49×10^0	1.14×10^{-7}
Mercury	135.55×10^2	1.55×10^{-3}	1.14×10^{-7}
<i>Name of Perfect Gases</i>			
Air	1.204	1.82×10^{-5}	1.51×10^{-5}
Hydrogen	8.832×10^{-2}	8.83×10^{-6}	1.51×10^{-5}
Helium	1.66×10^{-1}	1.95×10^{-5}	1.17×10^{-4}
Water vapor	7.498×10^{-1}	9.57×10^{-6}	1.28×10^{-5}
Carbon monoxide	1.165	1.76×10^{-5}	1.51×10^{-5}
Nitrogen	1.165	1.76×10^{-5}	1.51×10^{-5}
Oxygen	1.330	2.03×10^{-5}	1.53×10^{-5}
Argon	1.660	2.25×10^{-5}	1.36×10^{-5}
Carbon dioxide	1.830	1.47×10^{-5}	8.03×10^{-6}

where dimension of discharge (Q) = $v/t = L^3T^{-1}$ (LHS), on RHS, $LH^{3/2}$ has a dimension of $L^{5/2}$. Therefore, C_d has the dimension of $L^3T^{-1}/L^{5/2} = L^{1/2}T^{-1}$. C_d in Eq. (1.7) can be expressed as a function of \sqrt{g} having a dimension of $L^{1/2}T^{-1}$.

In fact, if the discharge equation (Eq. (1.7)) is derived from the fundamentals of flow (to be discussed later) over a rectangular weir, the discharge equation can be expressed as

$$Q = \frac{2}{3} C \sqrt{g} L H^{3/2} \quad (1.8)$$

where C is a dimensionless coefficient that accounts for the loss in head over the weir, etc. Here Q is the flow rate, L is the weir length, and H is the head above the crest of the weir.

Comparing Eqs (1.7) and (1.8), $C_d = \frac{2}{3} C \sqrt{g}$, i.e. C_d in Eq. (1.7) has the same dimension as \sqrt{g} in Eq. (1.8). If the discharge (Q) is to be found in m^3/s , then all the quantities L , H , and g must be expressed in meter units. If L and H are in m, it follows that g must be expressed as 9.8 m/s.

Table 1.2: Primary and derived units in SI system

Primary Quantity	Symbol of unit	SI unit	Symbol of unit
Length	L	Metre	m
Mass	M	Kilogram	kg
Time	T	Second	s
Electric current	I	Ampere	A
Luminous intensity	J	Candela	Cd
Temperature	t	Kelvin	K
<i>Derived units</i>	<i>Dimensional quantity</i>		
Velocity	LT^{-1}	Metre per second	m/s
Acceleration	LT^{-2}	Metre per second square	m/s^2
Force	MLT^{-2}	Newton	N
Pressure	$ML^{-1}T^{-2}$	Newton per square meter	N/m^2
Energy	ML^2T^{-2}	Newton meter	Nm
Power	ML^2T^{-3}	Newton meter per second	Nm/s^{-1}
Density	ML^{-3}	Mass per unit volume	kg/m^3
Unit weight	$ML^{-2}T^{-2}$	Force per unit volume	N/m^3
Specific gravity	$M^0L^0T^0$	Dimensionless	
Dynamic viscosity (μ)	$ML^{-1}T^{-1}$	Newton-second per square meter	Ns/m^2
Surface tension	MT^{-2}	Force per unit length	N/m

1.3 FLUID STATICS

It deals with the hydrostatics of fluid at rest. Many civil engineering works like dams, barrages, gates, locks, etc. are subjected to water pressure and all these structures are subjected to hydrostatic thrust. The buoyancy of floating bodies like a ship, uplifts pressure, the center of pressure, gauging of pressure, etc. are to be determined by engineers in finding stability. All these subjects are covered under fluid statics.

1.3.1 Hydraulic Pressure/Thrust

Pressure at any point in a static fluid is the same in all directions as per Pascal's law. As shown in Fig. 1.3, the pressure at point A at a depth of h below the surface is given by the weight of the column of fluid above a unit area of 1 m^2 , i.e. by the weight

$$W = (l \times h) \gamma_w \quad (1.9)$$

The pressure exerted at a depth h below the fluid surface $p = \gamma h$, where γ is the unit weight of the fluid. The pressure is normal in the unit area. If it would have been inclined to the area, it would have a tangential component which is not possible since fluid at rest cannot have any shear stress. Since the pressure is the same in all directions,

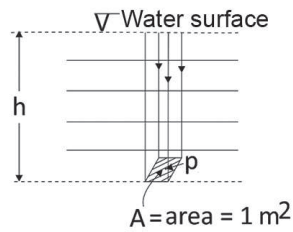


Fig. 1.3: Showing water pressure (p) at a point A lying at depth h below water surface

it is obvious that the lateral pressure at point A from left to right or right to left is also the same, i.e. γh . Hydrostatic thrust is the product of pressure and the area of the surface normal to it.

Example 1.1: A dam has both upstream and downstream water as shown in Fig. 1.4. Calculate the hydrostatic pressure at the base of the dam and the lateral thrust experienced per 10 m length down the on left and right sides and the uplift force at the base and show the pressure variation at the base AB assuming the linear variation of pressure. Take the unit weight of water as 9.8 kN/m^3 .

Solution:

$$h_1 = \text{Upstream depth of water} = 100 - 70 = 30 \text{ m}$$

$$h_2 = \text{Downstream depth of water} = 80 - 70 = 10 \text{ m}$$

Lateral pressure at the base =

$$p_1 \text{ upstream at A} = \gamma_w h_1 = 9.8 \times 30 = 294 \text{ kN/m}^2$$

$$p_2 \text{ downstream at B} = \gamma_w h_2 = 9.8 \times 10 = 98 \text{ kN/m}^2$$

Since lateral pressure at A and B will be the same in all directions, it is obvious that the uplift pressure at A and B will be the same as the lateral pressure at the respective points found above. The pressure distribution on the base (AB) and on the vertical plane through A \times B is drawn in Fig. 1.5.

The hydrostatic pressure variation on the vertical plane through A and B will be linear since pressure due to water is linearly proportional to depth ' h ' at any point. The pressure variation along in base AB, however, will not be linear as indicated. It depends on the rate of head loss when water seeps (or creeps) through the base of the dam.

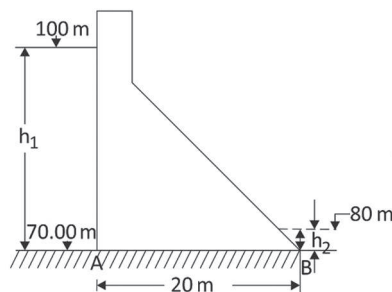


Fig. 1.4: Showing section of a dam

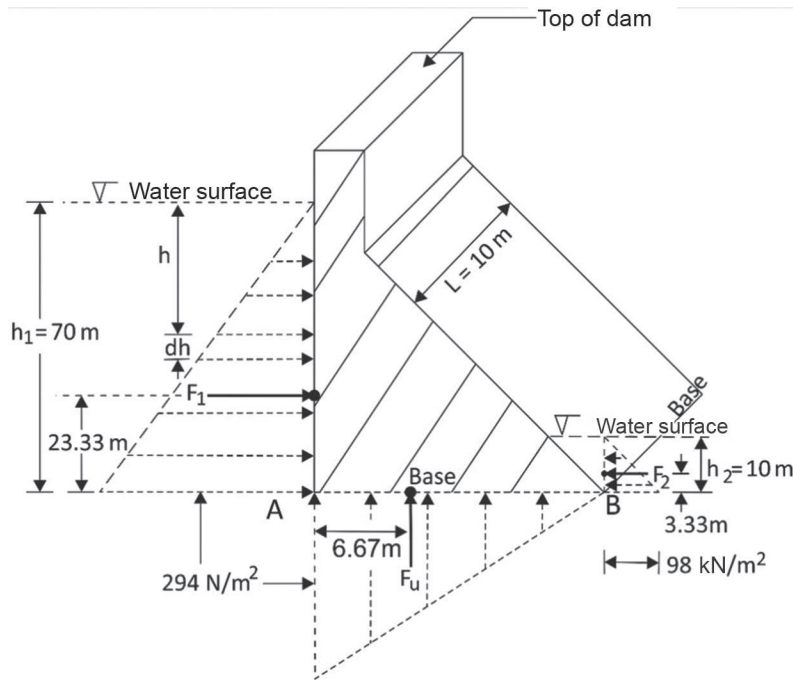


Fig. 1.5: Showing uplift pressure diagram and CP with rectangle and triangle

Thrust on a length of 10 m of the dam

Consider a point 'x' at a depth 'h' below the height surface, water pressure at p_x

$$p_x = \gamma_w h$$

and this pressure will be the same at all points along the length (L) of the dam every point of which is 'h' below the water surface.

The elementary thrust dF on a strip area ($dh \cdot L$), normal to the pressure is

$$dF = p(dh \cdot L) = \gamma_w h L dh$$

The total water thrust on the u/s face of the dam (F_1) is given by

$$\begin{aligned} F_1 &= \int_0^{h_1} \gamma_w \cdot L \cdot h \cdot dh \\ &= \gamma_w \cdot L \int_0^{h_1} h \cdot dh \\ &= \frac{\gamma_w \cdot L \cdot h_1^2}{2} \end{aligned}$$

Taking $h_1 = 30$ m, $L = 10$ m, $\gamma_w = 9.8$

$$F_1 = 9.8 \times 10 \times 30 \times \frac{30}{2} = 98 \times 30 \times 15 = 44100 \text{ kN}$$

Similarly, horizontal thrust on the d/s face of the dam

$$\begin{aligned} F_2 &= \frac{\gamma_w \cdot L \cdot h_2^2}{2} \\ &= 9.8 \times 10 \times \frac{10 \times 10}{2} = 98 \times 50 = 4900 \text{ kN} \end{aligned}$$

It may be seen that the thrust per unit length ($L = 1$) of the dam on the u/s face is given by $\frac{\gamma_w \cdot h_1^2}{2}$ is the area of the pressure triangle upstream, i.e. $\frac{1}{2} \times 294 \times 30 = 4410$ kN.

For a length of 10 m, the thrust is 4410 kN, i.e. same as above.

Uplift force on the base per meter length (U)

Given, the area of the trapezium

$$F_U = \frac{1}{2} (294 + 98) \times 20$$

$$= 48020 \text{ kN (same as above)}$$

Total uplift for 10 m length = $48020 \times 10 = 480200$ kN

Pressure and thrust on an inclined submerged surface

Consider an elementary area δA at a depth h below the water surface (Fig. 1.6), elementary thrust δF is given by

$$\delta F = p \cdot \delta A \text{ (Fig. 1.6)} = \gamma_w \cdot h \cdot dA$$

$$= \gamma_w \cdot l \sin \theta \cdot dA$$

or $F = \int_0^A \gamma_w \cdot h \cdot dA$ (1.10)

Similarly,

$$h = l \cdot \sin \theta \quad (1.11)$$

$$F = \int_0^A \gamma_w \cdot l \cdot \sin \theta \cdot dh \quad (1.12)$$

$$= \gamma_w \sin \theta \cdot \int_A l dA \quad (1.13)$$

By definition (in Fig. 1.6),

$$\bar{l} = LG = \frac{\int l dA}{A} \quad (1.14)$$

From Eqs (1.13) and (1.14)

$$F = (\gamma_w \cdot \sin \theta \cdot A \cdot \bar{l}) \quad (1.15)$$

But $\bar{l} \sin \theta = \bar{h}$, i.e. depth of CG of the area below the center surface

$$F = \gamma_w \cdot \bar{h} \cdot A \quad (1.16)$$

Equation (1.16) states that hydrostatics thrust on any surface of area A is given by the pressure at the center of gravity (G) of the area (i.e. $\gamma_w \cdot \bar{h}$) multiplied by the total area of the surface. Since the pressure acts normal to the surface, the thrust (F) which is the integration of pressure on the area (A) must also act normal to the surface as shown in Fig. 1.6.

Location where the thrust acts on the surface

In stability and other hydrostatic analyses, it is important not only to the thrust (F) on area A but one must also consider the exact location of the thrust on the surface. The location of the line of action of thrust is known as the center of pressure.

Moment of the elementary force dF about 'O' is given by

$$dM = (dF \cdot l) = \gamma_w \cdot \sin \theta \cdot \int l dA \cdot l = \gamma_w \cdot \sin \theta \int l^2 \cdot dA \quad (1.17)$$

Moment of all the forces acting on the surface is given by

$$M = \gamma_w \cdot \sin \theta \int_A l^2 dA \quad (1.18)$$

This moment must be the same as the moment of the thrust F about 'O', i.e. $F \times L_C$

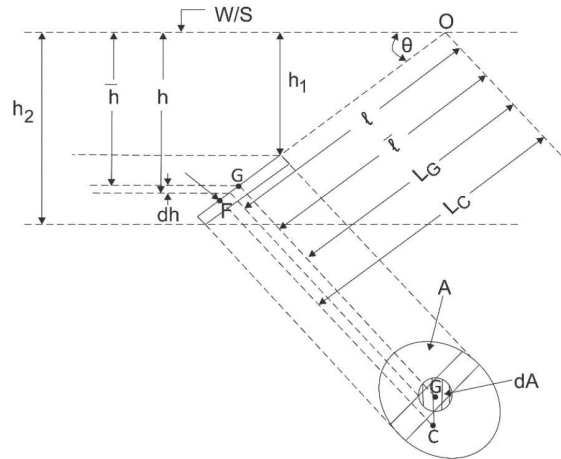


Fig. 1.6: Pressure on a submerged circular surface

$$\text{or } F.L_C = \gamma_w \cdot \sin \theta \int_A l^2 dA$$

$$\text{or } A\bar{L}_C = \int_A l^2 \cdot dA$$

$$\text{or } L_C = \frac{\int_A l^2 \cdot dA}{A\bar{I}}$$

The entity $\int_A l^2 \cdot dA$, i.e. second moment of area about d also known as moment of inertia, i.e. I .

$$I_C = I/A\bar{I}$$

$$(L_C - L_G) = (I/A\bar{I}) - \bar{I} = \bar{I}/(A\bar{l}^2) - l \tag{1.19}$$

Since the quantity I is always greater than $A\bar{l}^2$, the quantity $I/A\bar{l}^2$ is always greater than 1 and hence center pressure (C) will always be at a point below the center of gravity (G) of the area.

Example 1.2: A sluice gate 4 m × 2 m is inclined at an angle of 30° as shown in Fig. 1.7. If the top edge is 2 m below the water surface, find the hydrostatic thrust on the gate, the distance of the center of pressure (C), and also find the thrust on the sill on which the gate rests as its base.

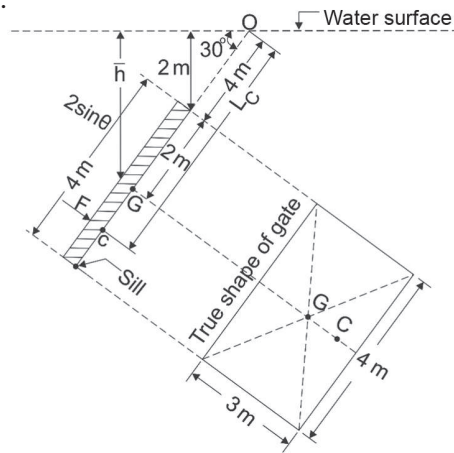


Fig. 1.7: Thrust on an inclined sluice gate

Solution:

Since the gate is rectangular, its CG is 2 m before the top edge along the inclined face

Distance of CG below the water surface is given by

$$\begin{aligned}\bar{h} &= 2 + 2 \sin \theta \\ &= 2 + 2 \sin 30^\circ = 3 \text{ m}\end{aligned}$$

Pressure at CG

$$\begin{aligned}\bar{p} &= \gamma_w \bar{h} = 9.8 \times 3 \text{ kN/m}^2 \\ &= 29.4 \text{ kN/m}^2\end{aligned}$$

Hydrostatics thrust on the gate, F is given by

$$\begin{aligned} &= \bar{p} \cdot A = 29.4 \times 3 \times 4 \\ &= 352.8 \text{ kN}\end{aligned}$$

Centre of pressure C , lying at a distance l_C below the origin is given by

$$l_C = \frac{I}{A\bar{l}}$$

\bar{l} = distance of CG below O is given by

$$2 \operatorname{cosec} \theta + 2 = 4 + 2 = 6 \text{ m}$$

$$\begin{aligned}l_C &= I/6A \\ &= I/(6 \times 3 \times 4) = I/72\end{aligned}$$

I = Second moment of the area about O

$$= I_0 + A\bar{l}^2$$

$$I_0 = \frac{1}{12} \cdot b \cdot d^3 = \frac{1}{12} \times 3 \times 4^3 = 16$$

and

$$A \cdot \bar{l}^2 = 12 \times 6^2 = 432$$

$$I_0 + A\bar{l}^2 = 16 + 432 = 448$$

$$l_C = \frac{448}{6 \times 12} = 6.22 \text{ m}$$

$L_C - L_G = 6.22 - 6 = 0.22$, i.e. center of pressure lies at 0.22 m below the center of gravity (CG) along the inclined face.

Pressure on curved surface

Since hydrostatic pressure at any point B (Fig. 1.8) on a curved surface, ABC is normal to the surface, the pressure p_B at B is given by

$$p = \gamma_w \cdot y$$

Thrust on the curved length ds is given by

$$dF_B = \gamma_w \cdot y (ds \cdot l) \quad (1.20)$$

per unit length of the surface normal to the plane (i.e. the paper).

Resolving dF_B in x and y directions as shown in Fig. 1.8.

$dFBx = dFB \sin \theta$ and $dFBy = dFB \cos \theta$, where θ is the angle between the tangent ds and horizontal axis x (Fig. 1.8).

From Eq. (1.15)

$$dF_{Bx} = \gamma_w \cdot y \cdot ds \sin \theta \text{ and } dF_{By} = \gamma_w \cdot y \cdot ds \cos \theta$$

$$\text{but } ds \sin \theta = dy \text{ and } ds \cos \theta = dx$$

$$dF_{Bx} = \gamma_w \cdot y \cdot dy \text{ and } dF_{By} = \gamma_w \cdot y_B \cdot dx$$

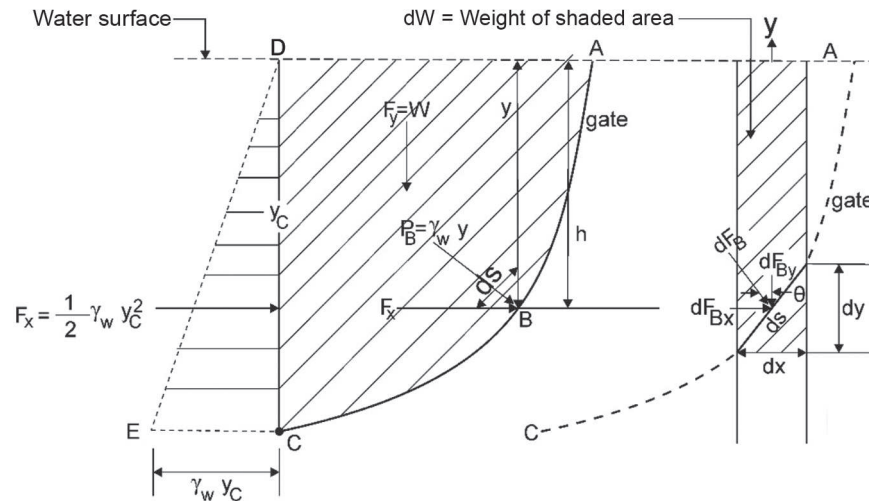


Fig. 1.8: Showing thrust on a curved surface

Thrust on the entire curved surface ABC

$$F = \int_0^{y_c} \gamma_w \cdot y \cdot dy \cdot l + \int_A^C \gamma_w \cdot y \cdot dx \cdot l = \bar{F}_x + \bar{F}_y = I + II$$

The first integral (I) $I = \bar{F}_x = \frac{1}{2} \gamma_w y_c^2$, i.e. area of the pressure triangle ADE (Fig. 1.8)

The second integral (II) $\bar{F}_y = \int_A^C \gamma_w \cdot y \cdot dx \cdot l = \int_A^C \gamma_w d_w = W\omega$ (Fig. 1.8)

Since $(\gamma_w \cdot y \cdot dx \cdot l)$ is the height of the fluid column above dx per unit length of the surface and integration of such elementary fluid column along ABC is negligible. The weight of the third area ABCDA, i.e. the shaded area is shown in Fig. 1.8.

The total thrust (F) per unit length of the curved surface is given by $\bar{F} = \bar{F}_x + \bar{F}_y$ and the resultant thrust on the curved surface is $\sqrt{F_x^2 + F_y^2}$

Thus the resultant thrust must be normal to the curved surface.

Example 1.3: In example 1.1 fluid, the thrust on the d/s face of the dam of the s/s face slope is 1:1 for a length of 10 m of the dam.

Solution: Drawing a vertical through the toe of the dam (T), it meets the railway (TWL) at S (Fig. 1.9).

$$F_x = \text{horizontal thrust} = \frac{1}{2} \times 98 \times 10 \times 10 = 49 \times 10^3 \text{ kN}$$

$$= (\text{Area of pressure triangle} \times 10)$$

$$F_y = \text{weight of water RST to a length of 10 m} = \frac{1}{2} \times RS \times ST \times \gamma_w \times 10$$

$$= \frac{1}{2} \times 10 \times 10 \times 9.8 \times 10 \text{ kN}$$

$$= 4.9 \times 10^3 \text{ kN}$$

The total thrust on the shaded surface = (\bar{F} is the resultant of F_x and F_y)

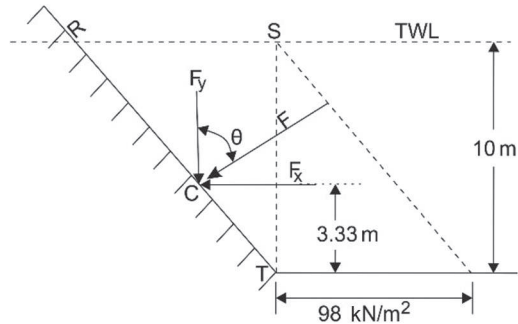


Fig. 1.9: Showing total thrust on inclined d/s face of dam (Example 1.1)

$$\begin{aligned}
 F &= \sqrt{(4.9 \times 10^3)^2 + (4.9 \times 10^3)^2} \\
 &= 4.9 \times 10^3 \sqrt{2} \\
 &= 6.9286 \times 10^3 \text{ kN} \\
 &= 6928 \text{ kN}
 \end{aligned}$$

Alternatively,

$$\begin{aligned}
 A &= \text{Area of the surface RT for 10 m length} \\
 &= 14.14 \times 10 \\
 &= 141.4 \text{ m}^2
 \end{aligned}$$

CG of the area (G) lies 7.07 m along the slope or 5 m below tailwater

$$\begin{aligned}
 F &= \text{Pressure at CG at the area} \times A \\
 &= \gamma_w \times 5 \times 141.4 \\
 &= 9.8 \times 5 \times 141.4 = 6928 \text{ kN}
 \end{aligned}$$

[Same as before]

From the force parallelogram law

$$\begin{aligned}
 F_x / F_y &= \tan \theta \\
 \theta &= \tan^{-1} \frac{F_x}{F_y} = \tan^{-1} \frac{4.9 \times 10^3}{4.9 \times 10^3} = 1 \\
 &= 45^\circ
 \end{aligned}$$

Thus, the resultant thrust (F) is normal to the surface RT at a distance $\frac{10}{3} = 3.33$ m above the base or 9.43 from R or $3.33 \sqrt{2} = 4.71$ along the sloping surface.

Alternatively

$$A = 14.14 \times 10 = 141.4 \text{ m}^2$$

$$\begin{aligned}
 I_{(\text{about R})} &= I_0 + A \bar{r}^2 = \frac{1}{12} \times 10 \times 14.14^3 + 141.4 \times 70.7^2 \\
 &= 2355.9 + 7067.8 = 9423.7 \text{ m}^4
 \end{aligned}$$

$$L_{\text{C (from R along sloping face)}} = \frac{I}{A \bar{r}} = \frac{9423.7}{141.4 \times 7.07} = 9.43 \text{ m}$$

Example 1.4: Calculate the total thrust on the downstream face of the dam of 10 m length if it is curved as shown in Fig. 1.10.

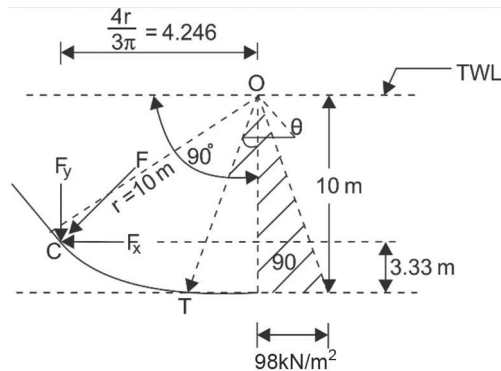


Fig. 1.10: Showing thrust on the curved d/s surface of the dam

Solution:

$$F_x = \frac{1}{2} \times 10^2 \times 10 \times 10 = 49 \times 10^2 \text{ kN acting at a height 3.33 m above the base}$$

$$F_y = \text{Weight of water at the quadrant of the circle} = \frac{\pi}{4} \times 10^2 \times 10 \times 9.8$$

$$= 7693 \text{ kN at a distance } \left(\frac{4}{3\pi}\right) \times r \text{ from centre 'O', i.e. 4.246 m}$$

Total thrust on the curved surface

$$= \sqrt{F_x^2 + F_y^2} = \sqrt{(4900)^2 + (7693)^2}$$

$$= 9120.9 \text{ kN}$$

Thus, the resultant thrust will be radial and pass through the center can be proved by taking the moment of forces about 'O'. Σ of moment must be zero (since the moment of thrust taken anywhere on the line of action must be zero)

$$\Sigma M_0 = F_x (10 - 3.33) - F_y \left(\frac{4r}{3\pi}\right)$$

$$= 4900 \times 6.67 - 7693 \times \frac{4 \times 10}{3 \times 3.14}$$

$$= 32683 - 32683$$

$$= 0$$

Point of application (R) of the trust along the curved surface

$$\tan \theta = \frac{F_x}{F_y} = \frac{4900}{7693} = 0.6369$$

$$\theta = 32.49^\circ$$

$$TR = R \cdot \theta^c = 10 \times \frac{32.49}{180} \times \pi$$

$$= 5.667 \text{ m}$$

1.4 FLOTATION/BUOYANCY

Figure 1.11 shows a floating cube weight (W) of which is in equilibrium with upward thrust (F_u) on the body given by

$$F_u = p \cdot A = \gamma_w \cdot y \cdot A = \gamma_w \cdot Y \cdot (ab) \quad (1.21)$$

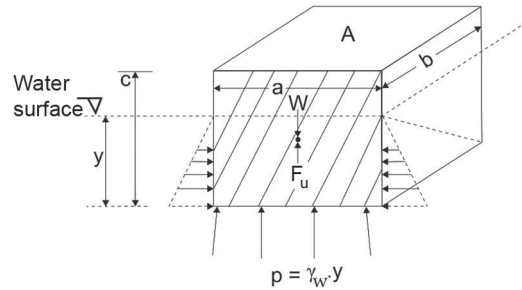


Fig. 1.11: Showing a floating cube

where p is the pressure at the bottom surface and a, b, c are the length, breadth, and height of the cube respectively.

The weight of the cube is given by

$$W = (abc) \cdot \gamma_s \quad (1.22)$$

where γ_s is the unit weight of the solid. For equilibrium condition

$$\gamma_w \cdot Y \cdot (ab) = \gamma_s (abc)$$

or

$$\frac{y}{c} = \frac{\gamma_s}{\gamma_w} = S_s$$

where S_s is the specific gravity of the solid material of the cube.

If $S_s = 1$, i.e. if the unit weight of the material is just equal to the unit weight of water, $y = c$, i.e. the body will be fully submerged up to its top surface.

If $S_s < 1$, then $y < c$, i.e. the body will be partly underwater and partly above the water surface. If $S_s > 1$, the body will sink since the weight will be more than the upward thrust to which the weight is subjected, i.e. how much it will be submerged will be given by the specific gravity of the material. For example, if the wooden cube has specific gravity 0.5, then $\frac{y}{c} = \frac{1}{2}$ or $y = \frac{c}{2}$, i.e. 50% of its height will be below the water and 50% above the water surface.

Example 1.5: A circular cylinder has a height of 1 m and the radius of the circle is 0.5 m, if the specific gravity of the cylinder is 0.7, determine the % depth it will be underwater (Fig. 1.12).

Solution:

Let the depth of the cylinder below the water surface be y .

Then upward thrust on the bottom circular face is given by

$$\begin{aligned} F_u &= \gamma_w \cdot y \cdot A \\ &= \gamma_w \cdot y \cdot \pi (0.5)^2 \end{aligned}$$

The weight of the body is given by

$$W = S_s \cdot \gamma_w \cdot \pi (0.5)^2 c$$

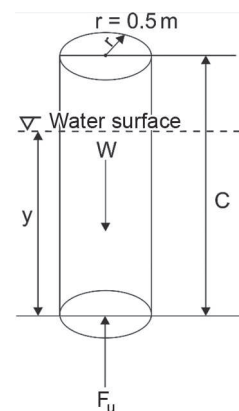


Fig. 1.12: Showing a circular cylinder floating in water

Condition for floating,

$$W = F_u$$

$$y/c = S\delta = 0.7$$

i.e. 70 cm below the water surface and 30% above the water surface.

1.4.1 Buoyancy

When a body is submerged, it is subjected to buoyant force given by the weight of water displaced by the given volume of the body underwater. This is known as the *Archimedes principle*. The center of buoyancy is the same location as the buoyancy where the buoyant force acts upward. It is given by the CG of the body, to upward.

1.4.2 Stability of Floating Bodies

Floating bodies, e.g. 1–2–3–4 in Fig. 1.13, are stable so long as it does not influenced by small distributors onces like water force in case of a strip floating in a section.

When the body is at rest, the CG and the center of buoyancy (same as center of pressure force (C) are in the same vertical 5–6 and the body is stable. When a small disturbance (in the form of a moment) is nitrogen (C), the body tilts and occupies the brew position 1'–2'–3'–4' as shown by the closed line in Fig. 1.13. The center of buoyancy (i.e. the buoyancy force F_u now offered by the water body displayed, i.e. 7–8–3'–4' passed through the center of C' indicated in Fig. 1.13. The vertical component of the force meets the line of the axis at rest (5–6) at a point M is known as the *meter center for equilibrium* must be the same as a comet having moment 1 M in a diversity opposite to M, i.e. body in opposite direction trying to have vertical stability.

If the water center lies above the CG (G) of the body. It is stable. If it coincides with G, it is neutrally stable and when M lies below G, the body is unreadable, since the moment due to the compound will be in the same as that of the disturbance moment M (Fig. 1.13).

It can be proven that the metacenter height MN of a floating body is given by $MB = I/V$, where I is the moment of inertia of the body about the axis of rotation, and V is the volume of water displayed by the floating body.

Example 1.6: A wooden block having a length of 3 m, width of 2 m and 1.5 m height float in water. Find the metacentric height from the center of gravity of the block, if it rotates about its shorter side, i.e. along the 2 m edge as shown in Fig. 1.14. The specific gravity of wood is 0.8. Is the block stable?

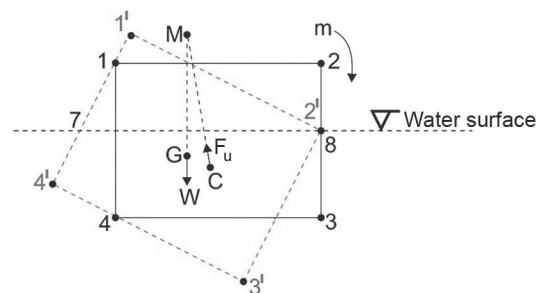


Fig. 1.13: Illustrating stability of a floating body

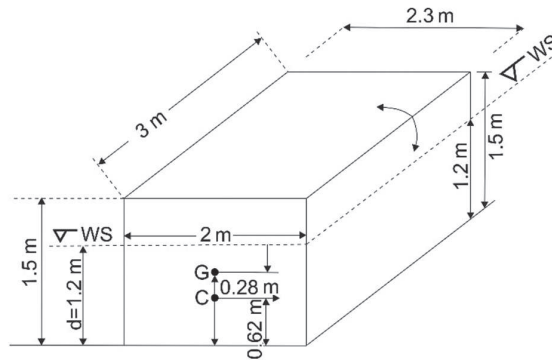


Fig. 1.14: Showing a wooden block floating in water

Solution:

For the floating objects with CG at 0.75 m below the top, the weight of a displaced volume of water must be equal to the weight of the body as per Archimedes' principle. Therefore, if draft $d = 3 \times 2 \times 1.5 \times \gamma_w = S_s = 0.3 \times 2 \times d \times \gamma_w$

$$\text{or } D = \text{draft} = \frac{1.5 \times S_s \times \gamma_w}{\gamma_w} = 1.5 \times S_s = 1.5 \times 0.88 = 1.2 \text{ m}$$

Height of the center of buoyancy (C) along depth $= d/2 = 0.6 \text{ m}$

$$I = 3 \times 2^3 / 12 = 2 \text{ m}^4$$

$$V = 3 \times 2 \times 1.2 = 7.2 \text{ m}^3$$

$$MB = \frac{I}{V} = \frac{2}{7.2} = 0.28 \text{ m}$$

$$MG = 1.5 - (0.6 + 0.28) = 0.62 \text{ m}$$

Since the metacenter M lies 0.62 m below the CG of the block (G), the block is unstable.

Manometric principle of pressure measurement in fluid devices

Manometers are devices by which fluid pressure of any point can be measured by balancing the pressure against a column of liquid (same or different) under static equilibrium conditions. Consider a pipe shown in Fig. 1.15 carrying liquid under

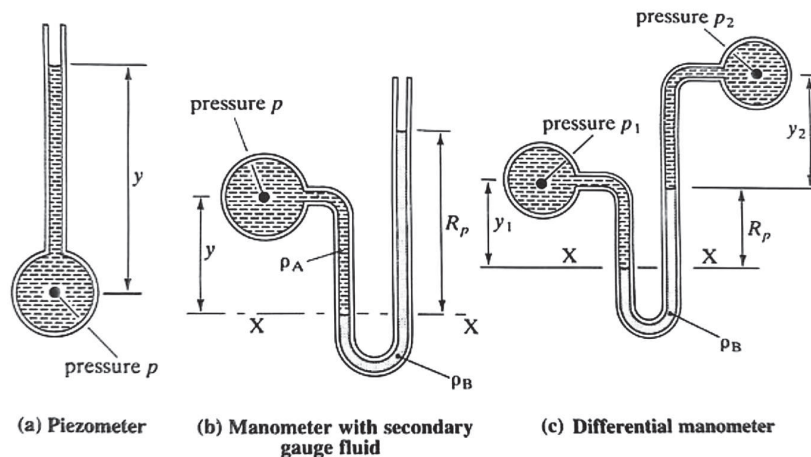


Fig. 1.15: Illustrating pizeometric head (h) in a pipe carrying liquid under pressure

pressure. If the pipe is pierced through and a vertical transparent tube is connected to it, the liquid will rise in the tube up to a height ' h ' measured above the centerline of the pipe such that pressure ' p ' at any point in the pipe will be balanced by the pressure exerted by the weight of liquid of height h as shown in Fig. 1.15. It may be noted that pressure at all points in the pipe at any section is the same, the only representative pressure head will be given by h measured from the centerline of the pipe.

$$p = \gamma h.$$

where ' p ' is pressure in N/m^2 , ' γ ' is the unit weight of liquid in N/m^3 , and ' h ' is the pressure head in metre. The device for measuring pressure is termed piezometric device and the tube ' D ' is commonly called a piezometer. For every high-pressure head (h), the length of the piezometric tube will be too high. Denser liquid like mercury is commonly used for balancing the pressure head in the pipe illustrated in Fig. 1.16.

Let the pressure in the pipe be ' p ' and assume that the liquid flowing in the pipe has a unit weight ' γ ' and the unit weight of mercury $\gamma_g = S_s \gamma$ (where S_s is the specific gravity of mercury (i.e. 13.6)). Since the pressure at points 1 and 2 (Fig. 1.16) is the same under equilibrium conditions, then

$$p + \gamma h_1 = \gamma_g h_2 = S_s \gamma h_2$$

$$\text{or} \quad p = S_s \gamma h_2 - \gamma h_1$$

$$\text{or} \quad p/\gamma = (S_s h_2 - h_1)$$

Thus, the pressure head (p/γ) in the pipe is given by the difference between $S_s h_2$ and h_1 . Values of h_2 and h_1 can be measured and on knowing the specific gravity of the index liquid (usually mercury), the pressure head in the pipe p/γ can be determined. It is obvious that the use of heavier index liquid reduces the height of the manometer. For example, if a pressure of $98 \text{ kN}/\text{m}^2$ in the pipe carrying water is to be measured by a simple manometer, the height of the piezometer above the pipe centerline will be $98/9.8 = 10 \text{ m}$. With the use of a differential manometer with mercury as index liquid, the height (h_2) will be

$$\frac{98}{9.8} = 13.6 \times h_2 - h_1$$

$$\text{Assuming } h_1 = 0.5 \text{ m, } h_2 = \frac{10 + .5}{13.6} = \frac{10.05}{13.6} = 0.77 \text{ m.}$$

$$(h_2 - h_1) = 0.77 - 0.5 = 0.27 \text{ m,} \\ \text{i.e. } 27 \text{ cm}$$

Therefore, the minimum required height of the manometer above the pipe center lies is 27 cm only.

1.4.3 Differential Pressure Measurement

a. High pressure differential

Differential pressure between two pipes or between two points in a conduit can be measured by the use of a manometer. As shown in Fig. 1.17, the difference in pressure heads in a venturi meter (flow meter) between points 1 and 2 can be found by using a

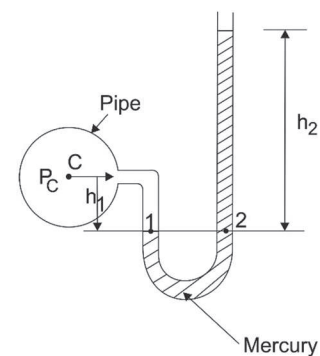


Fig. 1.16: Showing a mercury manometer

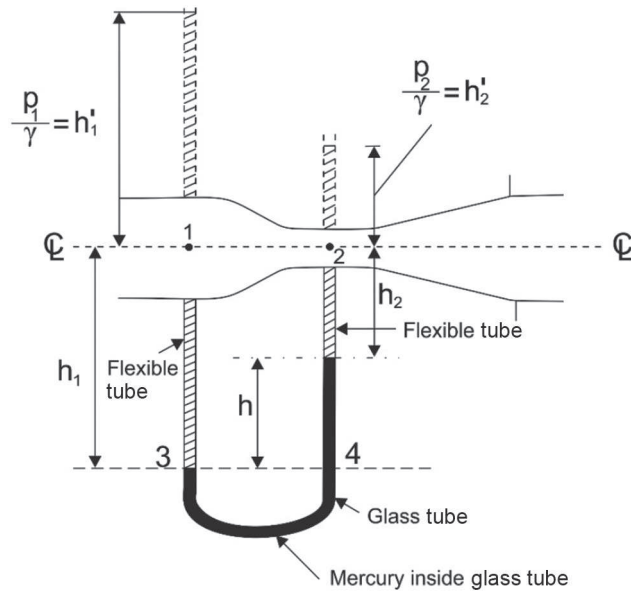


Fig. 1.17: Differential mercury manometer

mercury manometer. Referring to Fig. 1.17, if p_1 and p_2 are the pressure at points 1 and 2 respectively, under stable conditions, pressure at 3 and 4 must be the same.

$$p_1 + \gamma h_1 = p_2 + \gamma h_2 + \gamma S_s h$$

or

$$p_1 - p_2 = \gamma(h_2 - h_1) + \gamma S_s h$$

$$= -\gamma h + \gamma S_s h$$

or

$$(p_1 - p_2) / \gamma = h (S_s - 1)$$

$$= 12.6 h$$

where S_s is the specific gravity of the index liquid (mercury). Assuming that the differential pressure head of the flowing liquid ($p_1/\gamma - p_2/\gamma$) is 3 m.

$$H = 3/12.6 = 0.238 \text{ m} = 23.8 \text{ cm}$$

As a result, a small size manometer of about 30 cm or so will suffice for measuring the pressure head difference. If the same pressure differential is to be measured by open stand type piezometer, then

$$(p_1/\gamma - p_2/\gamma) = (h_{1(w)}' - h_{2(w)}') = 12.6 \times 0.238 = 2.998 = 3 \text{ m.}$$

Supposing $p_2/\gamma = 1 \text{ m}$, then $p_1/\gamma = 4 \text{ m}$. thus a very long piezometer is to be connected at point 1. Reading the watermark at such a height physically would pose very difficult problem.

b. Low pressure differential

Sometimes, a very low-pressure difference is to be measured by the use of a manometer as illustrated in Fig. 1.18. For example, in the measurement of low flow velocity by using Prandtl type pitot tube and static pressure at point 2 (Fig. 1.18) the pressure difference between stagnation point 1 and the static pressure at point 2 is low (Fig. 1.18)

Low flow velocity (a) Pitot tube (b) differential manometer is very low. The velocity of head flow of 0.5 m/s in a pipe carrying water, the pressure difference

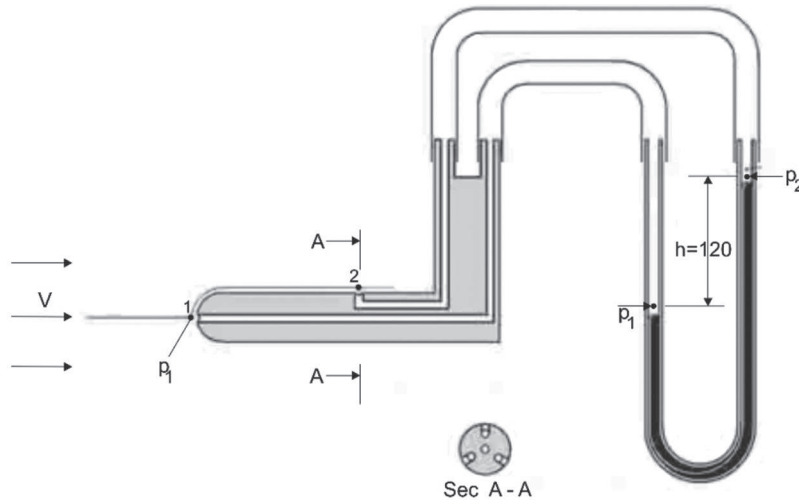


Fig. 1.18: Use of Prandtl type pitot tube for measuring velocity

$(p_1/\gamma - p_2/\gamma)$ will be only $0.5^2/2g = 0.25/19.6 = 0.0127 \text{ m} = 12.7 \text{ mm}$ only even a 1 mm physical error in reacting the manometer scale will result in 7 % error in velocity flow.

If an inverted manometer with oil is considered as an index liquid, the differential pressure head between points 1 and 2 will be much higher. For example, if an oil of specific gravity 0.9 is used, the head h shall be

$$(p_1/\gamma - p_2/\gamma) = 0.5^2/2g = h/(1 - S)$$

$$\text{or} \quad h = 0.0127/(1 - 0.9) \text{ m} = 0.127 \text{ m} = 127 \text{ mm}$$

Hence, a 1 mm error in reading the scale will result in 0.8% error only in the velocity measurement.

1.5 FLUID DYNAMICS

Engineering flows are mostly complex and it is extremely difficult to find a precise solution. A great deal can be learned from the equations derived from hydrodynamics of such flows. Solutions are easy as the fluid is assumed to be ideal having no viscosity (inviscid fluid) and incompressible. Fluid dynamics for compressible and viscous flow is very complex and usually taught at a higher level. However, a brief mention of them has been made at the end. Fluid dynamics basically deals with the laws of motion of the fluid flow.

1.5.1 Flow Visualization

It is a powerful technique to understand the motion of the flowing water.

Streamlines

A streamline is a line that is tangential to the velocity vector at every point of its path of motion at any given instant as illustrated in Fig. 1.19, where the velocity vectors V_1, V_2, V_3, V_4, V_5 are tangential at the points 1, 2, 3, 4, 5 on the streamline, respectively.

Stream tube

Figure 1.20 illustrates a stream tube consisting of a number of streamlines. Since there is no motion at the right angle to the streamline, volume of fluid entering section A of the tube must be the same as the volume of fluid leaving the stream tube at section B.

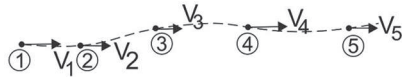


Fig. 1.19: Illustrating a typical streamline

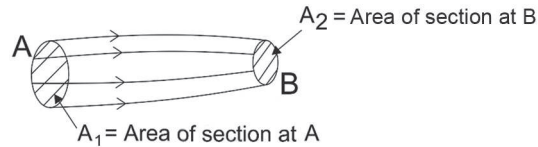


Fig. 1.20: Showing a stream tube

Streaklines

To identify streamlines in a flowing fluid, a colored dye is commonly injected and the photographed path of the dye is assumed to be the streamline. It is really a streakline since the characteristics of the dye may be different from that of the fluid and its photograph at a given time during which fluid flows may change during the photographic time interval. So, the photograph gives an average path traced by the dye in a given time interval which is really a streak line and not a streamline.

One-, two-, and three-dimensional flows

In a real fluid flow within a given boundary, the velocity vector may not always be passed to the boundary. Referring to Fig. 1.21, if the flow U_x is parallel to the x-direction only without any component in the y- or z-direction, then the flow is one-dimensional (Fig. 1.21a). In two-dimensional flow (Fig. 1.21b), in a diffuser, velocity vector (V) has components in both x- and y-directions, i.e. U_x and V_y respectively. Flow within a hydraulic jump (Fig. 1.21c), there are all the three components (u_x , v_y , and w_z) of the velocity vector and hence it is three-dimensional.

The fundamental equations of fluids dynamics

Fundamental equation of laws of physics:

- i. Conservation of mass
- ii. Conservation of energy and
- iii. Conservation of momentum are applicable to fluid flows also.

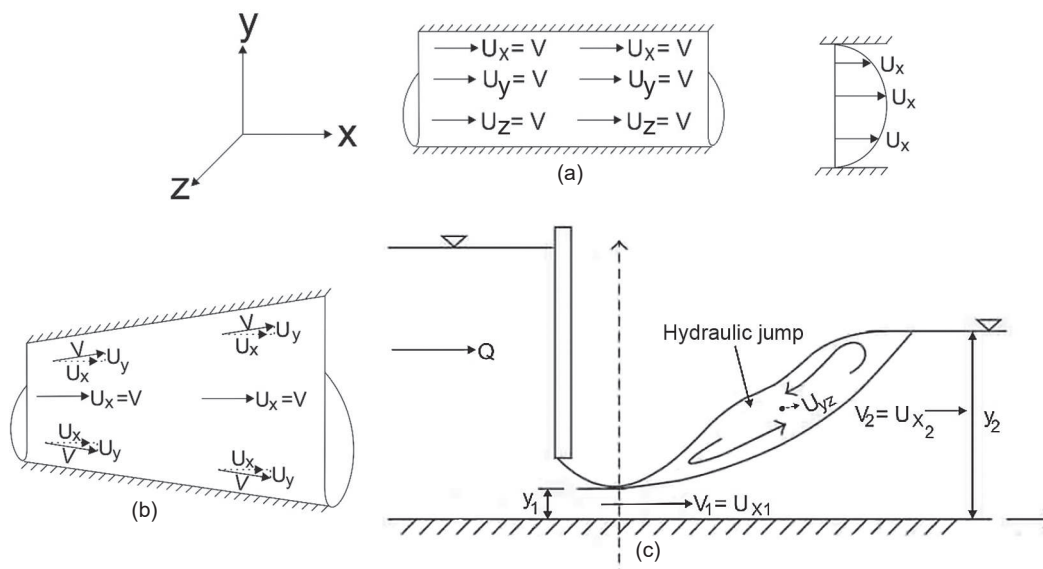


Fig. 1.21: Illustrating one-, two- and three-dimensional flows

1.5.2 Law of Conservation of Mass

Referring to Fig. 1.20, the mass of fluid energy in the stream tube at A and leaving the tube at B must be the same.

$$\text{Amount entering at A} = \int_{A_1}^A A \rho u_1 dA_1 \quad (1.23)$$

$$\text{Amount leaving at B} = \int_{A_2}^A A \rho u_2 dA_2 \quad (1.24)$$

From Eqs (1.23) and (1.24)

$$u_1 dA_1 = u_2 dA_2$$

Assuming ρ and u_1, u_2 as constants

$$u_1 \int_{A_2}^A A \rho dA_1 = u_2 \int_{A_2}^A A \rho dA_2$$

$$u_1 A_1 = u_2 A_2$$

or,

$$A_1 V_1 = A_2 V_2 = A_3 V_3 \quad (1.25)$$

where $u_1 = V_1$ and $u_2 = V_2, u_3 = V_3$; V_1, V_2 and V_3 are mean flow velocities at sections 1, 2 and 3 respectively.

Equation (1.25) is also known as the *continuity equation*.

1.5.3 Laws of Conservation of Energy

Refer to Fig. 1.22, the volume of fluid in a length L of the tube = $A_1 L$

Mass of the fluid = $\rho A_1 L = m$

If u_1 is the velocity of flow, then KE of the fluid is given by

$$\text{KE} = \frac{1}{2} m u_1^2 = \frac{1}{2} (\rho A_1 L) u_1^2 \quad (1.26)$$

$$\text{Pressure energy} = (P_1 A_1) \times L \quad (1.27)$$

$$\text{Potential energy (PE)} = mg \cdot Z_1 = (P A_1 L) \cdot g \cdot Z_1 \quad (1.28)$$

The total energy (TE) of the fluid element at an elevation Z_1 above the datum is given by

$$\text{TE} = \text{KE} + \text{pressure energy} + \text{potential energy}$$

$$= \frac{1}{2} \rho A_1 L u_1^2 + P_1 A_1 \cdot L + P A_1 L \cdot g \cdot Z_1$$

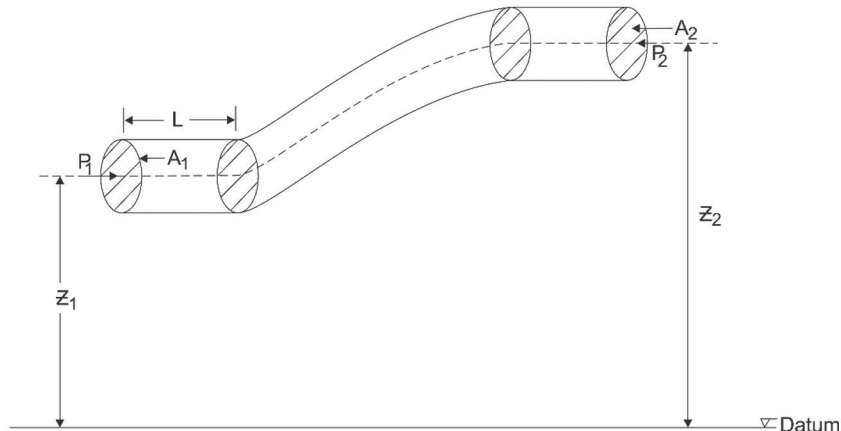


Fig. 1.22: Illustrating conservation of energy

TE per unit weight of flow, $H_1 = TE/\rho g A_1 L$

$$H_1 = u_1^2/2g + P_1/\gamma + Z_1 = P_1/\gamma + Z_1 + u_1^2/g \quad (1.29)$$

where $\gamma = \rho g$

In a similar manner, it can be shown that the total energy per unit weight of flow for the fluid element at section Z_2 (Fig. 1.23) is given by

$$H_2 = P_2/\gamma + Z_2 + u_2^2/2g \quad (1.30)$$

Since, energy can neither be created nor destroyed, the law of conservation of energy states that

$$H_1 = H_2 \quad (1.31a)$$

$$\text{or} \quad P_1/\gamma + Z_1 + u_1^2/2g = P_2/\gamma + Z_2 + u_2^2/2g \quad (1.31b)$$

Equation (1.31) is also called the Bernoulli equation in the name of Daniel Bernoulli (1700–1782). It may be noted that in Eq. (1.31), all the terms are in length (m) unit and as such, the total head H is also in length (m) unit.

1.5.4 Laws of Conservation of Momentum (Euler's Equation)

Considering the fluid element of Fig. 1.24, the net force acting in the direction that is given by

$$P_1 A - (P_1 + dP_1) \cdot A = -dP \cdot A \quad (1.32)$$

Since the fluid is considered as ideal and the pressure along the workface remains constant, the force on the periphery of fluid element = 0

$$\text{Net force along flow due to weight} = -W \cos \theta = -W; \frac{d\delta}{\delta L}$$

Total net force in the direction of motion

$$F = -Ad_1 P_1 - W \cdot d_2/dl$$

where, W = weight of the fluid element

$$= P g A \cdot dh$$

or

$$F = -AdP_1 - P g A \cdot d_2$$

$$= -AdP - P g A \frac{d_2}{\delta L} \quad (1.33)$$

As per Newton's second law of motion

Net force = rate of change of momentum, i.e.

$$-AdP_1 - P g A dZ = m(du/dt) = m \times \frac{du}{dt} \times \frac{dL}{dt} = mu \frac{du}{dt} \quad (1.34)$$

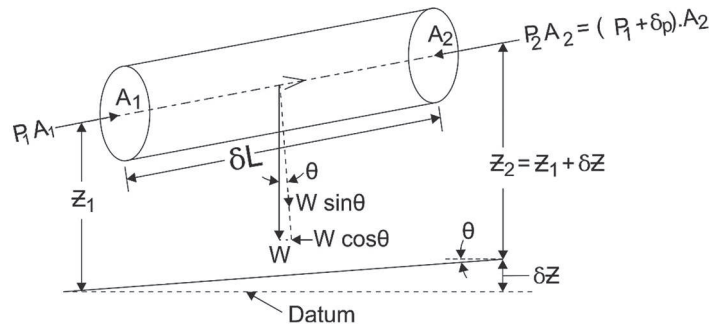


Fig. 1.23: Laws of conservation of momentum

$$\text{or} \quad \rho u \frac{du}{dL} + \rho A dp_1 + \rho g A dZ = 0$$

$$\text{or} \quad \rho A dL \cdot u \frac{du}{dL} + \rho A dp + \rho g A \frac{dZ}{dL} = 0 \quad (1.35)$$

Multiply Eq. (1.35) by PA ,

$$dL \cdot \frac{du}{dL} + \frac{1}{\rho} dp + g \frac{dZ}{dL} = 0$$

$$\text{or} \quad \frac{1}{\rho} \frac{dp}{dL} + u \frac{du}{dL} + g \frac{dZ}{dL} = 0 \quad (1.36)$$

Equation (1.36) is called Euler's equation of motion in the name of Swiss mathematician Leonhard Euler (1707–85).

$$\frac{1}{\rho} \frac{dP}{dL} + \frac{u du}{dL} + \frac{g dZ}{dL} = \text{constant}$$

$$\text{or} \quad \frac{P}{\rho} + \frac{u^2}{2} + dZ = \text{constant}$$

$$\text{or} \quad \frac{P}{\rho g} + \frac{u^2}{2g} + Z = \text{constant}$$

$$\text{or} \quad \frac{P}{\rho} + \frac{u^2}{2} + Z = \text{constant} \quad (1.37)$$

$$\text{i.e.} \quad p_1/\rho + u_1^2/2 + Z_1 = p_2/\rho + u_2^2/2 + Z_2 = 1.1 \quad (1.38)$$

(It is same in Bernoulli's equation)

It may be noted that Bernoulli's and Euler's equations, assumed to be componentwise and are individually correct.

1.5.5 Energy and Momentum Coefficients

In both momentum and energy equations, it was assumed that the distribution of velocity is uniform across the section. However, in a large section, still the energy and momentum equation is valid we apply the same correction factors known popular as KE correction factor (α), and momentum correction factor (β), are defined as

$$\alpha = \frac{\int u^3 dA}{V^3 A} \text{ and } \beta = \frac{\int u^2 dA}{V^2 A}$$

where V is the mean velocity of flow, u is the path velocity, A is the area of cross-section.

With the above correction factors, the energy and momentum (or Bernoulli's) equation may be written as follows:

$$H = Z + \alpha \frac{v^2}{2g} + \frac{p}{\rho g} \quad (1.39)$$

and net force in this direction

$$F_x = \beta \rho Q (v_2 - v_1) \quad (1.40)$$

1.5.6 Bernoulli's Equation (Application)

For solving problems in one-dimensional flow using Bernoulli's equation is extremely popular. A few examples of the application of Bernoulli's equation are illustrated (Fig. 1.24) through examples.

Example 1.7: Consider point 1 at the top of the water surface in the vent and 2 at the existing end of the syphon. Ensure during the exist of syphon as the datum, as per Eq. (3.16)

Solution:
$$\frac{P_1}{\gamma} + \frac{u_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{u_2^2}{2g} + Z_2$$

Since points 1 and 2 are in air, the pressure $P_1/\gamma = 0^2$. $P_2/\gamma = Pn/\gamma$, considering atmospheric pressure head Pa/γ is the sum at 1 and 2.

$$\frac{u_1^2}{2g} + Z_1 = \frac{u_2^2}{2g} + Z_2$$

But $u_1 = 0$, being on a large vessel with a negotiable velocity at the surface.

$$\frac{u_2^2}{2g} = Z_1 - Z_2 = 137 - 0 = 1.22 \text{ m}$$

$$R_2 = \sqrt{2g} \times 1.37 = 5.18 \text{ m/s}$$

$$Q = \frac{\pi}{4} .d^2 \times u_2 = \frac{\pi}{4} \times 0.15^2 \times 5.18 = 0.092 \text{ m}^3/\text{s}$$

Pressure head at A

Again applying Bernoulli's equation below points 1 and A, we have

$$\frac{P_1}{\gamma} + \frac{u_1^2}{2g} + Z_1 = \frac{P_A}{\gamma} + \frac{u_2^2}{2g} + Z_2$$

Taking the atmospheric pressure head as zero,

$$0 + 0 + Z_1 = \frac{P_A}{\gamma} + \frac{u_2^2}{2g} + Z_2$$

From the continuity equation,

$$Q = u_A \times \frac{\pi}{A} \times 0.2^2 = 0.092$$

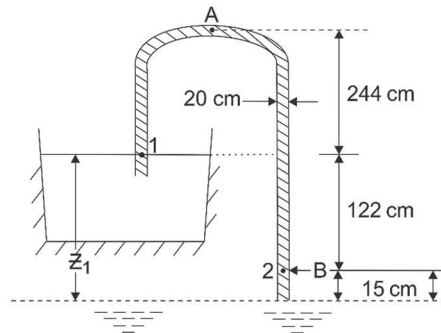


Fig. 1.24

$$u_A = 0.092 \times \frac{\pi}{4} \times 0.2^2 = 0.93 \text{ m/s}$$

Substituting the value of u_A above, we have

$$Z_1 = \frac{P_A}{\gamma} + \frac{2.93^2}{2g} = -2.44 - \frac{2.93^2}{2g} = -2.88 \text{ m}$$

Thus, the pressure head at A is sub-atmospheric, i.e. 2.88 m below the atmospheric pressure head

Pressure head at B

Applying Bernoulli's equation between points A + B with respect to point 2 we have:

$$\frac{P_A}{\gamma} + \frac{u_A^2}{2g + Z_A} = \frac{P_B}{\gamma} + \frac{u_B^2}{2g + Z_B}$$

$$\text{or} \quad -2.88 + \frac{2.93^2}{2g} + (1.22 + 2.44 + 0.15) + \frac{P_B}{\gamma} + \frac{u_B^2}{2g + 0.15}$$

(since the pipe diameter is the same at A and B from the continuity equation $u_B = u_A = 2.93 \text{ m/s}$)

$$-2.88 + 3.81 = \frac{P_B}{\gamma} + 0.15$$

$$\frac{P_B}{\gamma} = 0.93 - 0.15 = 0.78 \text{ m}$$

Modified Bernoulli's equation with loss in head

Example 1.8: Consider two tanks as shown in Fig. 1.25. If the water levels in the tanks remain constant, find the flow in the rough pipe of diameter $d = 10 \text{ cm}$.

Assume, head loss is given by $h_L = (fL/d) \cdot V^2/2g$

Take a length of pipe as 500 m, and $f = 0.01$

Solution: Applying Bernoulli's equation between points 1 or 2

$$\frac{P_1}{\gamma} + u_1^2/2g + Z_1 = \frac{P_2}{\gamma} + u_2^2/2g + Z_2 + h_L$$

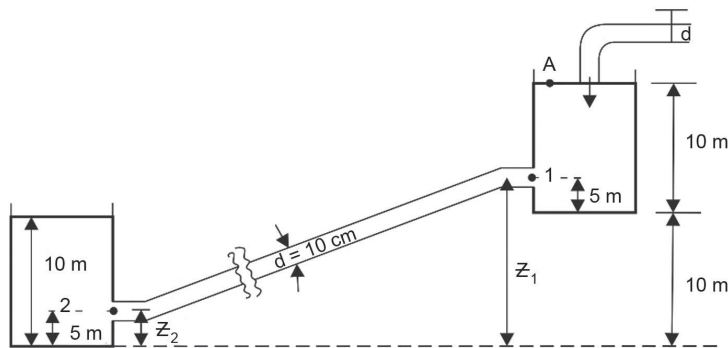


Fig. 1.25

$\frac{P_1}{\gamma} = 5 \text{ m}, \frac{P_2}{\gamma} = 5 \text{ m}$ (applying Bernoulli's equation between the tank water surface and points 1 and 2)

Since the diameter of the pipe is the same, $u_1 = u_2 = Q / \frac{\pi}{4} d^2$
Loss in head, in the pipe

$$h_L = \left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma} \right) + \left(\frac{u_1^2}{2g} - \frac{u_2^2}{2g} \right) + (Z_1 - Z_2)$$

$$= \frac{fL}{D} \times \frac{V^2}{2g}$$

or

$$5 = \frac{0.01 \times 500}{0.15} \times \frac{V^2}{2g}$$

$$V = \text{Velocity through pipe} = \sqrt{2g \times 5} \times \sqrt{\frac{0.15}{0.01 \times 200}}$$

$$= 1.7146 \text{ m/s}$$

$$Q = \frac{\pi}{4} \times d^2 \times V = \frac{\pi}{4} \times (0.15)^2 \times 1.7146 = 0.03 \text{ m}^3/\text{s}$$

$$= 30 \text{ LPS}$$

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