CHAPTER - 1

POWER SYSTEM MODELLING

1.1 REPRESENTATION OF POWER SYSTEM

A typical power system consists of a 3-phase grid to which all generating stations feeds energy and from which all substations taps energy. (A grid is either a 3-phase single circuit or 3-phase two circuit transmission line, running throughout the length and breadth of a country or a state). From the substations electrical energy is transmitted to distribution transformers and from the distribution transformer, the energy is fed to various loads.

The components of power system are Generating stations (Alternators), Power transformers, Transmission lines, Substations (Substation transformers), Distribution transformers and Loads. The various types of loads are Synchronous motors, Induction motors, Heating coils, Lights, etc.,

The various components of power system and their interconnections are usually represented by single line diagram. In a single line diagram the components are represented by standard symbols and their interconnections are shown by single line, eventhough they are three phase circuits.

Single line diagram (One-Line Diagram)

A balanced three-phase system is always analysed on per phase basis by considering one of the three phase lines and neutral. Hence it is enough if we show one phase and neutral in the diagrammatic representation of power system. The diagram is further simplified by omitting the neutral and so the resultant diagram will be a single line diagram.

In single line representation of power system, the components of the system are represented by standard symbols and the transmission lines are represented by straight lines. Hence a single line diagram is diagrammatic representation of power system in which the components are represented by their symbols and the interconnections between them are shown by straight lines. Besides the symbols, the ratings and the impedances of the components are also marked on the single line diagram.

The purpose of the one-line diagram is to supply in concise form the significant information about the system. The various symbols used in single line diagram are shown in table-1.1. A typical single line diagram is shown in fig 1.1.

Table-1.1 Symbols used in single line diagram

Machine or rotating armature	Power circuit breaker, (oil/gas filled)
Two-winding power transformer	Air circuit breaker
Three-winding power transformer	Three-phase, three-wire delta connection
Fuse	Three-phase star, neutral ungrounded
Current transformer	Three-phase star, neutral grounded
Potential transformer or	Ammeter and voltmeter

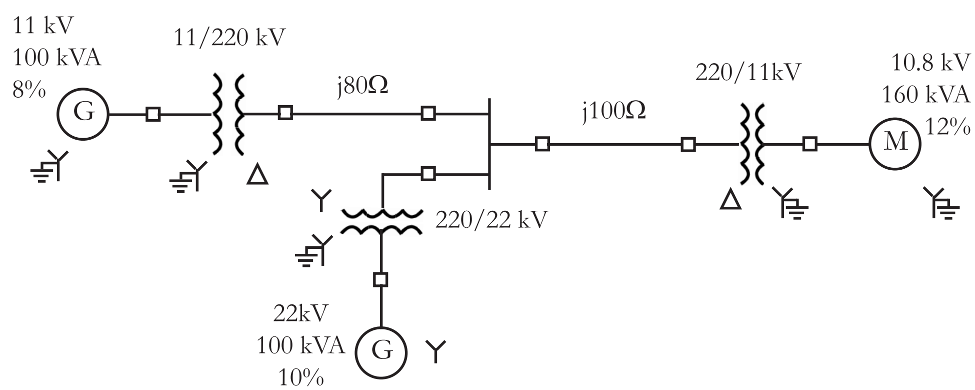


Fig 1.1 : *A typical single line diagram*

1.2 PER UNIT QUANTITIES

The electric power transmission lines are operated at very high voltage levels and transmits large amount of power. Hence the operating voltage of transmission line is expressed in kilovolt (kV) and power transmitted is expressed in kilowatt (kW) or megawatt (MW) and kilovolt ampere (kVA) or megavoltampere (MVA).

The various components of power system like alternators, motors, transformers, etc., have their voltage, power, current and impedance ratings in kV, kVA, kA and Ω respectively.

The components or various sections of power system may operate at different voltage and power levels. It will be convenient for analysis of power system if the voltage, power, current and impedance ratings of components of power system are expressed with reference to a common value called base value. Hence for analysis purpose a base value is chosen for voltage, power, current and impedance.

Then all the voltage, power, current and impedance ratings of the components are expressed as a percent or perunit of the base value.

The per unit value of any quantity is defined as the ratio of the actual value of the quantity to the base value expressed as a decimal. The ratio in percent is 100 times the value in per unit. The base value is an arbitrary chosen value of the quantity.

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}} \quad .....(1.1)$$

$$\%\ \text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}} \times 100 \quad .....(1.2)$$

The power system requires the base values of four quantities and they are Voltage, Power, Current and Impedance. Selection of base values for any two of them determines the base values of the remaining two.

Single phase system

Let, kVA_b = Base kVA

kV_b = Base voltage in kV

I_b = Base current in Amp

Z_b = Base impedance in Ω

The following formulae relate the various quantities

$$\text{Base current, } I_b = \frac{kVA_b}{kV_b} \text{ in amps} \quad .....(1.3)$$

$$\text{Base impedance, } Z_b = \frac{kV_b \times 1000}{I_b} \text{ in } \Omega \quad ....(1.4)$$

On substituting for I_b from equation (1.3) in equ(1.4) we get,

$$\text{Base impedance, } Z_b = \frac{kV_b \times 1000}{\dfrac{kVA_b}{kV_b}} = \frac{(kV_b)^2}{kVA_b/1000} = \frac{kV_b^2}{MVA_b} \quad .....(1.5)$$

$$\text{Per unit impedance} = \frac{\text{Actual impedance}, \Omega}{\text{Base impedance}, \Omega}$$

Three-phase system

The per-unit value of a line-to-neutral (V_{LN}) on the line-to-neutral voltage base ($V_{b,LN}$) is equal to the per-unit value of the line-to-line voltage (V_{LL}) at the same point on the line-to-line voltage base ($V_{b,LL}$) if the system is balanced,

$$\text{i.e., } \frac{V_{LN}}{V_{b,LN}} = \frac{V_{LL}}{V_{b,LL}} \quad .....(1.6)$$

The perunit value of a 3-phase kVA on the 3-phase kVA base is identical to the per unit value of the kVA per phase on the kVA per phase base

$$\text{i.e., } \frac{3-\text{phase kVA}}{3-\text{phase base kVA}} = \frac{\text{kVA per phase}}{\text{Base kVA per phase}} \quad .....(1.7)$$

Therefore in 3-phase systems the line value of voltages and 3-phase kVA are directly used for per unit calculations.

The base impedance and base current of 3-phase system can be computed directly from 3-phase value of base kVA and line value of base kV.

Let kV_b = Line to line base kV

kVA_b = 3-phase base kVA

I_b = Line value of base current

Now, $k\,VA_b = \sqrt{3} \times kV_b \times I_b$ .....(1.8)

$\left(\because \text{In 3 - phase systems, kVA} = \sqrt{3}\ V_L I_L \times 10^{-3} = \sqrt{3} \times (kV_L) I_L\right)$

From equ(1.8) we get,

$$\text{Base current, } I_b = \frac{kVA_b}{\sqrt{3} \times kV_b} \qquad .....(1.9)$$

In a balanced power system the phase voltage is $1/\sqrt{3}$ times, the line voltage. Hence the base impedance per phase is given by

$$\left.\begin{matrix}\text{Base impedance}\\ \text{per phase}\end{matrix}\right\} Z_b = \frac{\left(kV_b/\sqrt{3}\right) \times 1000}{I_b} = \frac{kV_b \times 1000}{\sqrt{3}\ I_b} \qquad .....(1.10)$$

On substituting for I_b from equ(1.9) in equ(1.10) we get,

$$\left.\begin{matrix}\text{Base impedance}\\ \text{per phase}\end{matrix}\right\} Z_b = \frac{kV_b \times 1000}{\sqrt{3} \times \dfrac{kVA_b}{\sqrt{3} \times kV_b}} = \frac{(kV_b)^2}{\dfrac{kVA_b}{1000}} = \frac{(kV_b)}{MVA_b} \qquad .....(1.11)$$

Here, the equ(1.5) and (1.11) looks similar, but in 3-phase system, the kV_b is a line value and MVA_b is a 3-phase MVA.

Note : The impedance is always expressed as phase value.

Changing the base of per-unit quantities

The impedance of a device or component is usually specified in per unit on the base of name plate rating. When a system is formed by interconnecting various devices, it will be convenient for analysis if the impedances are converted to common base. Since all impedance in any one part of a system must be expressed on the common impedance base. It is necessary to have means of converting per-unit impedances from one base to another.

Let, Z = Actual impedance, Ω

Z_b = Base impedance, Ω

$$\text{Per unit impedance of a circuit element} = \frac{Z}{Z_b} = \frac{Z}{\dfrac{(kV_b)^2}{MVA_b}} = \frac{Z \times MVA_b}{(kV_b)^2} \qquad .....(1.12)$$

The equ(1.12) show that per unit impedance is directly proportional to base megavolt amperes and inversely proportional to the square of the base voltage. Using equ(1.12) we can derive an expression to convert the p.u. impedance expressed in one base value (old base) to another base (new base).

Let $kV_{b,\,old}$ and $MVA_{b,\,old}$ represents old base values and $kV_{b,\,new}$ and $MVA_{b,\,new}$ represents new base value.

Let, $Z_{pu,\,old}$ = p.u. impedance of a circuit element calculated on old base.

$Z_{pu,\,new}$ = p.u. impedance of a circuit element calculated on new base.

If old base values are used to compute the p.u. impedance of a circuit element with impedance Z, then equ(1.12) can be written as,

$$Z_{pu,old} = \frac{Z \times MVA_{b,old}}{(kV_{b,old})^2} \qquad .....(1.13)$$

$$Z = Z_{pu,old}\ \frac{(kV_{b,old})^2}{MVA_{b,old}} \qquad .....(1.14)$$

If the new base values are used to compute the p.u. impedance of a circuit element with impedance Z, then equ(1.12) can be written as

$$Z_{pu,new} = \frac{Z \times MVA_{b,new}}{(kV_{b,new})^2} \qquad .....(1.15)$$

On substituting for Z from equ(1.14) in equ(1.15) we get,

$$Z_{pu,new} = Z_{pu,old}\ \frac{(kV_{b,old})^2}{MVA_{b,old}} \times \frac{MVA_{b,new}}{(kV_{b,new})^2}$$

$$= Z_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) \qquad .....(1.16)$$

The equ(1.16) can be used to convert the p.u. impedance expressed on one base value to another base.

Advantages of per-unit computations

1. Manufacturers usually specify the impedance of a device or machine in percent or per unit on the base of the name plate rating.

2. The per-unit impedances of machines of the same type and widely different rating usually lie within a narrow range, although the ohmic values differ widely for machines of different ratings.

3. The per-unit impedance of circuit element connected by transformers expressed on a proper base will be same if it is referred to either side of a transformer.

4. The way in which the transformers are connected in 3-phase circuits (Y or Δ) does not affect the per-unit impedances of the equivalent circuit, although the transformer connection does determine the relation between the voltage bases on the two sides of the transformer.

1.3 EQUIVALENT CIRCUITS OF COMPONENTS OF POWER SYSTEM

The equivalent circuit of a power system is needed to perform analysis like load flow analysis, fault level calculations, etc. It can be obtained from the equivalent circuit of the components of the power system. The various components of power system are generator (alternator), transformer, transmission line, induction motor, synchronous motor, resistive and reactive loads. The equivalent circuits of various electrical machines developed in electrical machine theory can be used in power system modelling with or without approximations.

Equivalent circuit of generator

The equivalent circuit of a 3-phase generator (Alternator) is shown in fig 1.2. It consists of a source representing induced emf per phase, a series reactance representing the armature reactance and leakage reactance and a series resistance representing the armature winding resistance.

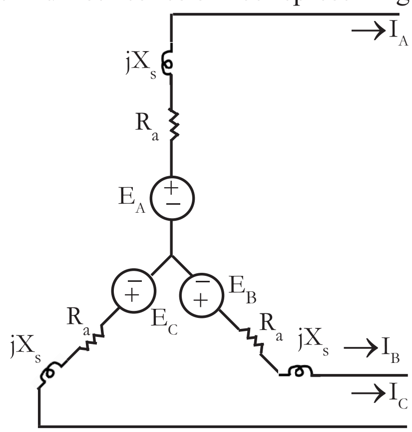


Fig a : *3-Phase equivalent circuit*

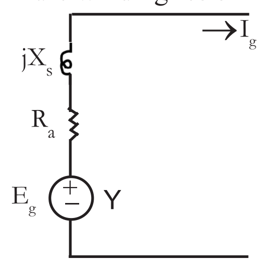


Fig b : *Single phase equivalent circuit*

Fig 1.2 : *Equivalent circuit of generator (3-phase alternator)*

Equivalent circuit of synchronous motor

The equivalent circuit of synchronous motor is shown in fig 1.3. The synchronous motor is similar to a generator in construction, but it performs the reverse action of the generator. (A generator converts mechanical energy to electrical energy, but the motor converts electrical energy to mechanical energy). Therefore the direction of current in motor is opposite to that of generator.

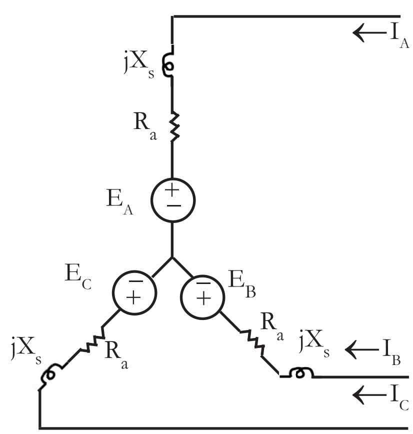


Fig a : *3-Phase equivalent circuit*

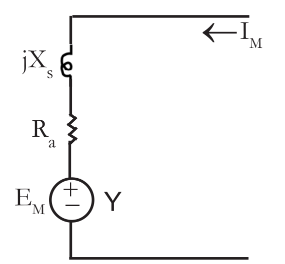


Fig b : *Single phase equivalent circuit*

Fig 1.3 : *Equivalent circuit of synchronous motor*

Equivalent circuit of transformer

The equivalent circuit of a single phase, two winding transformer referred to primary is shown in fig 1.4. It consists of shunt branches to represent magnetising current and core loss, series resistance representing winding resistance referred to primary and the series reactance representing leakage reactance referred to primary.

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} \approx \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$R_{01} = R_1 + R_2' = R_1 + R_2 / K^2$$

$$X_{01} = X_1 + X_2' = X_1 + X_2 / K^2$$

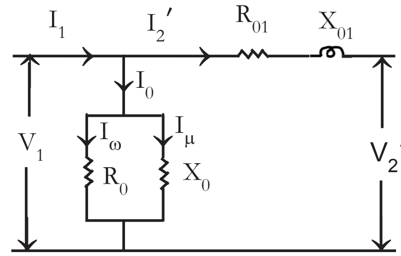


***Fig 1.4 :** Equivalent circuit of a transformer*

The three phase transformer is represented by its single phase equivalent and the equivalent circuit is similar to that of fig 1.4. In three phase transformers, the transformation ratio, K is taken as the ratio of line voltages. This will facilitate the direct conversion of star side impedances to delta side and vice versa.

Equivalent circuit of induction motor

The single phase equivalent circuit of induction motor referred to stator is shown in fig 1.5. It is similar to equivalent circuit of transformer (The induction motor is also called rotating transformer).

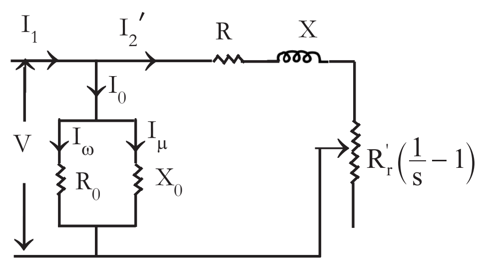


***Fig 1.5 :** Equivalent circuit of induction motor*

s = Slip

$R_r'\left(\frac{1}{s} - 1\right)$ = Resistance representing load.

$R = R_s + R_r'$ = Equivalent resistance referred to stator

$X = X_s + X_r'$ = Equivalent reactance referred to stator

R_s, X_s = Resistance and reactance of stator

R_r, X_r = Resistance and reactance of rotor.

Equivalent circuit of transmission line

The transmission line can be represented by its resistance, inductance and capacitance. The single phase equivalent π-type and T-type model of the transmission line is shown in fig 1.6. The elements R, X_L and X_c are resistance, inductive reactance and capacitive reactance per phase respectively.

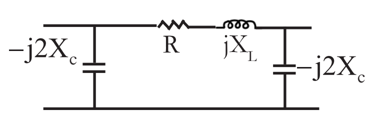


***Fig a :** π - type*

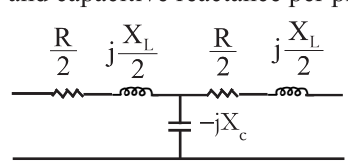


***Fig b :** T - type*

***Fig 1.6 :** Equivalent circuit of transmission line*

Representation of resistive and reactive loads

The resistive and reactive loads can be represented in the equivalent circuit by any one of the following

1. Constant power representation
2. Constant current representation
3. Constant impedance representation

In constant power representation, the load active power (MW) and reactive power (MVAR) are considered to be constant. This method of representation will be useful in load flow studies.

In constant current representation the magnitude of the load current is considered as constant. The constant current of the load can be calculated from the specified voltage, active and reactive powers of the load, as shown below.

Single phase load

Let P = Active power

and Q = Reactive power

The complex power, $S = VI^* = P + jQ$

$$\therefore \; (VI^*) = (P + jQ)$$

$$V^* I = P - jQ$$

$$\therefore I = \frac{P - jQ}{V^*} \qquad .....(1.17)$$

Let, $V = |V| \angle \delta \quad ; \quad V^* = |V| \angle - \delta$ .....(1.18)

and $P - jQ = \sqrt{P^2 + Q^2} \angle - \tan^{-1}\frac{Q}{P} = \sqrt{P^2 + Q^2} \angle - \theta$

where $\theta = \tan^{-1}\frac{Q}{P}$ .....(1.19)

From equations (1.17), (1.18) and (1.19) we can write

$$I = \frac{\sqrt{P^2 + Q^2} \angle - \theta}{|V| \angle - \delta} = \frac{\sqrt{P^2 + Q^2}}{|V|} \angle \delta - \theta = |I| \angle \delta - \theta \qquad .....(1.20)$$

$$\text{where } |I| = \frac{\sqrt{P^2 + Q^2}}{|V|} \qquad .....(1.21)$$

In constant impedance representation the load is represented by its impedance or admittance. The impedance of the load can be calculated from the specified voltage, active and reactive powers of the load as shown below.

Load Impedance, $Z = \frac{V}{I}$ .....(1.22)

On substituting for I from equ(1.17) in equ(1.22) we get,

$$\therefore \text{ Load impedance, } Z = \frac{V}{(P - jQ)/V^*} = \frac{V\,V^*}{P - jQ} = \frac{|V|^2}{P - jQ} \qquad .....(1.23)$$

$$\text{Load admittance, } Y = \frac{1}{Z} = \frac{P - jQ}{|V|^2} \qquad .....(1.24)$$

Three phase load

Balanced star connected load

Let, P = Three phase active power of star connected load in watts.

Q = Three phase reactive power of star connected load in VARs.

V, V_L = Phase & line voltage of load respectively.

I, I_L = Phase & line current of load respectively.

Three phase complex power, $S = 3\,VI^* = P + jQ$

$$\therefore\ (3VI^*)^* = (P + jQ)^*$$

$$3V^*I = P - jQ$$

$$\therefore\ I = \frac{P - jQ}{3V^*} \qquad .....(1.25)$$

Let $V = |V| \angle \delta$; $\therefore V^* = |V| \angle -\delta$

In star connected load, $|V| = \frac{|V_L|}{\sqrt{3}}$ and $I = I_L$

$$\therefore\ V^* = \frac{|V_L|}{\sqrt{3}} \angle -\delta \qquad .....(1.26)$$

Let $P - jQ = \sqrt{P^2 + Q^2} \angle -\tan^{-1}\frac{Q}{P} = \sqrt{P^2 + Q^2} \angle -\theta$ .....(1.27)

where $\theta = \tan^{-1}\frac{Q}{P}$

Using equations (1.26) and (1.27), the equation (1.25) can be written as,

$$I = \frac{P - jQ}{3V^*} = \frac{\sqrt{P^2 + Q^2}}{3\frac{|V_L|}{\sqrt{3}} \angle -\delta} = \frac{\sqrt{P^2 + Q^2}}{\sqrt{3}\,|V_L|} \angle \delta - \theta$$

$$\therefore\ I = I_L = \frac{\sqrt{P^2 + Q^2}}{\sqrt{3}\,|V|} \angle \delta - \theta \qquad .....(1.28)$$

$$\therefore\ |I| = |I_L| = \frac{\sqrt{P^2 + Q^2}}{\sqrt{3}\,|V_L|} \qquad .....(1.29)$$

Load impedance per phase, $Z = \frac{V}{I}$ .....(1.30)

On substituting for I from equ(1.25) in equ(1.30) we get,

$$Z = \frac{V}{\frac{(P - jQ)}{3V^*}} = \frac{3VV^*}{P - jQ} = \frac{3|V|^2}{P - jQ}$$

$$= \frac{3\left(|V_L|/\sqrt{3}\right)^2}{P - jQ} = \frac{|V_L|^2}{P - jQ}$$

$\therefore$ Load impedance per phase, $Z = \frac{|V_L|^2}{P - jQ}$ .....(1.31)

$\therefore$ Load impedance per phase, $Y = \frac{1}{Z} = \frac{P - jQ}{|V_L|^2}$ .....(1.32)

Balanced delta connected Load

Let P = Three phase active power of delta connected load in watts

Q = Three phase reactive power of delta connected load in VARs.

V, V_L = Phase & line voltage of load respectively.

I, I_L = Phase & line current of load respectively.

Three phase complex power, $S = 3\,V\,I^* = P + jQ$

$$\therefore\ (3VI^*)^* = (P + jQ)^*$$

$$3V^*I = P - jQ$$

$$\therefore\ I = \frac{P - jQ}{3V^*} \qquad .....(1.33)$$

Let $V = |V|\angle\,\delta$; $\therefore V^* = |V|\angle -\delta$

In delta connected load, $V = V_L$ and $|I| = |I_L| / \sqrt{3}$ .....(1.34)

Let, $P - jQ = \sqrt{P^2 + Q^2}\angle -\tan^{-1}\dfrac{Q}{P} = \sqrt{P^2 + Q^2}\angle -\theta$ .....(1.35)

where $\theta = \tan^{-1}\dfrac{Q}{P}$

Using equations (1.34) and (1.35), the equation (1.33) can be written as,

$$I = \frac{\sqrt{P^2+Q^2}\angle -\theta}{3|V|\angle -\delta} = \frac{\sqrt{P^2+Q^2}}{3|V|}\angle(\delta-\theta) = \frac{\sqrt{P^2+Q^2}}{3|V_L|}\angle(\delta-\theta)$$

$$= |I|\angle(\delta-\theta)$$

$$\text{where } |I| = \frac{\sqrt{P^2+Q^2}}{3|V_L|}$$

$$\therefore\ |I_L| = \sqrt{3}\,|I| = \sqrt{3}\,\frac{\sqrt{P^2+Q^2}}{3|V_L|} = \frac{\sqrt{P^2+Q^2}}{\sqrt{3}\,|V_L|} \qquad .....(1.36)$$

Load impedance per phase, $Z = \dfrac{V}{I}$ .....(1.37)

On substituting for I from equ(1.33) in equ(1.37) we get,

$$Z = \frac{V}{\dfrac{(P-jQ)}{3V^*}} = \frac{3VV^*}{P-jQ} = \frac{3|V|^2}{P-jQ} = \frac{3|V_L|^2}{P-jQ}$$

Load impedance per phase, $Z = \dfrac{3|V_L|^2}{P-jQ}$ .....(1.38)

Load admittance per phase, $Y = \dfrac{1}{Z} = \dfrac{P-jQ}{3|V_L|^2}$ .....(1.39)

Three winding transformer

In addition to the primary and secondary winding, the transformer may be constructed with a third winding called tertiary winding. In three winding transformers, the two windings are connecting in star and one winding in delta or two windings in delta and one winding in star. The purpose of providing the tertiary winding are the following.

1 To get supply voltage for the substation auxiliary devices. [The auxiliary devices may work at a voltage different from those of the primary and secondary windings].

2. Static capacitors or synchronous condensers may be connected to the tertiary winding for reactive power injection into the system for voltage control.

3. A delta connected tertiary reduces the impedance offered to the zero sequence currents thereby allowing a large earth-fault current to flow for proper protection of protective equipments. Further it limits voltage imbalance when the load is unbalanced. It also permits the third harmonic current to flow thereby reducing third harmonic voltages. For these reasons the tertiary winding is also called stabilization winding.

4. Three windings may be used for interconnecting three transmission line at different voltages.

5. Tertiary can serve the purpose of measuring voltage of an HV testing transformer.

In three winding transformer, the three windings may have different kVA rating. [But the primary and secondary of two winding transformers have the same kVA rating]. The impedance of each winding of a three winding transformer may be given in p.u. calculated by using their own winding rating as bases. But while representing in reactance/impedance diagram it is necessary to convert the p.u. reactances to a common base.

The equivalent circuit of a three winding transformer can be represented by the single phase equivalent circuit shown in fig 1.7.

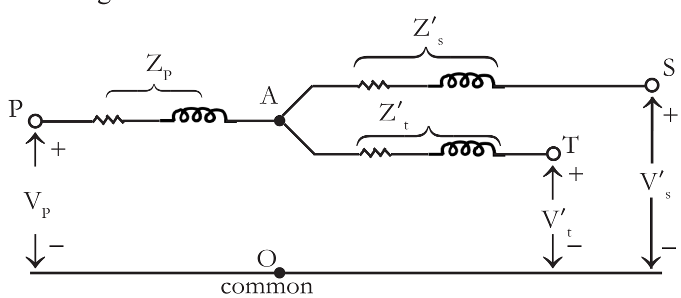


Fig 1.7 : *Single phase equivalnet circuit of three winding transformer*

In this equivalent circuit the impedance per phase of the three windings (referred to one of winding, usually primary) are connected in star to represent the single phase equivalent circuit. For simplicity, the effect of exciting current is ignored in the equivalent circuit. The subscripts p, s and t indicate the primary, secondary and tertiary respectively. Three external circuits are connected between P & O, S & O and T & O, where the terminal O is the common terminal.

The impedances of the three windings are calculated using a common base kVA or MVA. The base voltage is the voltage rating of respective windings. The three impedances can be measured by the standard short-circuit tests.

Z_p = Impedance of primary winding

Z'_s = Impedance of secondary winding referred to primary

Z'_t = Impedance of tertiary winding referred to primary

Z_{ps} = Leakage impedance measured in primary with secondary short-circuited and tertiary open.

Z_{pt} = Leakage impedance measured in primary with tertiary short-circuited and secondary open.

Z'_{st} = Leakage impedance measured in secondary with tertiary short-circuited and primary open and then referred to primary.

The leakage impedances measured by short circuit test are related to winding impedance as follows,

$$Z_{ps} = Z_p + Z_s' \qquad .....(1.40)$$

$$Z_{pt} = Z_p + Z_t' \qquad .....(1.41)$$

$$Z_{st}' = Z_s' + Z_t' \qquad .....(1.42)$$

The equations (1.40) to (1.42) can be expressed in the form of matrix equation as shown in equ(1.43)

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} Z_p \\ Z_s' \\ Z_t' \end{bmatrix} = \begin{bmatrix} Z_{ps} \\ Z_{pt} \\ Z_{st}' \end{bmatrix} \qquad .....(1.43)$$

The matrix equation (1.43) can be solved by cramer's rule as shown below.

$$\Delta = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 1 \times (-1) - 1\,(1) = -2$$

$$\Delta_1 = \begin{vmatrix} Z_{ps} & 1 & 0 \\ Z_{pt} & 0 & 1 \\ Z_{st}' & 1 & 1 \end{vmatrix} = Z_{ps}(-1) - 1\,(Z_{pt} - Z_{st}') = -Z_{ps} - Z_{pt} + Z_{st}'$$

$$\Delta_2 = \begin{vmatrix} 1 & Z_{ps} & 0 \\ 1 & Z_{pt} & 1 \\ 0 & Z_{st}' & 1 \end{vmatrix} = 1 \times (Z_{pt} - Z_{st}') - Z_{ps}(1) = Z_{ps} - Z_{st}' + Z_{pt}$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & Z_{ps} \\ 1 & 0 & Z_{pt} \\ 0 & 1 & Z_{st}' \end{vmatrix} = 1 \times (-Z_{pt}) - 1\,(Z_{st}') + Z_{ps}\,(1) = Z_{pt} - Z_{st}' + Z_{ps}$$

$$Z_p = \frac{\Delta_1}{\Delta} = \frac{1}{-2}\,[-Z_{ps} - Z_{pt} + Z_{st}'\,] = \frac{1}{2}\,[Z_{ps}\ Z_{pt} - Z_{st}'\,] \qquad .....(1.44)$$

$$Z_{st}' = \frac{\Delta_2}{\Delta} = \frac{1}{-2}\,[-Z_{ps} - Z_{st}' + Z_{pt}] = \frac{1}{2}\,[Z_{ps} + Z_{st}' - Z_{pt}] \qquad .....(1.45)$$

$$Z_t' = \frac{\Delta_3}{\Delta} = \frac{1}{-2}\,[-Z_{pt} - Z_{st}' + Z_{ps}] = \frac{1}{2}\,[Z_{pt} + Z_{st}' - Z_{ps}] \qquad ....(1.46)$$

The equations (1.44) to (1.46) can be used to calculate the impedances of the three windings (referred to primary) using the short circuit test data.

EXAMPLE 1.1

A three phase generator with rating 1000 kVA, 33 kV has its armature resistance and synchronous reactance as 20Ω/phase and 70Ω/phase. Calculate p.u. impedance of the generator.

SOLUTION

The generator ratings are chosen as base kV and base kVA.

∴ Base kilovolt, kV_b = 33 kV

Base kilovoltampere, kVA_b = 1000 kVA

$$\left.\begin{array}{l}\text{Base impedance}\\ \text{per phase}\end{array}\right\} Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(33)^2}{1000/1000} = 1089\ \Omega$$

$$\left.\begin{array}{l}\text{Actual impedance}\\ \text{per phase}\end{array}\right\} Z = (20 + j70)\Omega/\text{phase}$$

$$\therefore \text{ p. u. impedance, } Z_{pu} = \frac{\text{Actual impedance}}{\text{Base impedance}} = \frac{Z}{Z_b} = \frac{20 + j70}{1089} = 0.018 + j0.064 \text{ p.u.}$$

EXAMPLE 1.2

A three phase, Δ-Y transformer with rating 100 kVA, 11 kV/400 V has its primary and secondary leakage reactance as 12 Ω/phase and 0.05 Ω/phase respectively. Calculate the p.u. reactance of transformer.

SOLUTION

Case(i)

The high voltage winding (primary) ratings are chosen as base values.

$\therefore$ Base kilovolt, kV_b = 11 kV

Base kilovoltampere, kVA_b = 100 kVA

$$\left.\begin{array}{l}\text{Base impedance}\\ \text{per phase}\end{array}\right\} Z_b = \frac{(kV_b)}{MVA_b} = \frac{(11)^2}{100/1000} = 1210\ \Omega$$

$$\text{Transformer line voltage ratio, } K = \frac{400}{11{,}000} = 0.0364$$

$$\left.\begin{array}{l}\text{Total leakage reactance}\\ \text{referred to primary}\end{array}\right\} X_0 = X_1 + X'_2 = X_1 + \frac{X_2}{K^2} = 12 + \frac{0.05}{(0.0364)^2} = 12 + 37.737$$

$$= 49.737\ \Omega / \text{phase}$$

$$\text{p. u. reactance per phase, } X_{pu} = \frac{\text{Total leakage reactance}}{\text{Base impedance}}$$

$$= \frac{X_{01}}{Z_b} = \frac{49.737}{1210} = 0.0411 \text{ p.u.}$$

Case(ii)

The low voltage winding (secondary) ratings are chosen as base values.

$\therefore$ Base kilovolt, kV_b = 400/1000 = 0.4 kV

Base kilovoltampere, kVA_b = 100 kVA

$$\left.\begin{array}{l}\text{Base impedance}\\ \text{per phase}\end{array}\right\} Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(0.4)^2}{100/1000} = 1.6\ \Omega$$

$$\text{Transformer line voltage ratio, } K = \frac{400}{11{,}000} = 0.0364$$

$$\left.\begin{array}{l}\text{Total leakage reactance}\\ \text{referred to secondary}\end{array}\right\} X_{02} = X'_1 + X_2 = K^2 X_1 + X_2$$

$$= (0.0364)^2 \times 12 + 0.05 = 0.0159 + 0.05 = 0.0659\ \Omega/\text{phase}$$

$$\text{p. u. reactance per phase, } X_{pu} = \frac{\text{Total leakage reactance}}{\text{Base impedance}} = \frac{X_{02}}{Z_b} = \frac{0.0659}{1.6} = 0.0411 \text{ p.u.}$$

Note : *1. It is observed that the p.u. reactance of a transformer referred to primary and secondary are same.*

2. In three phase transformer if the voltage ratio K is obtained using line values then using this value othe phase impedance per phase of star side can be directly transferred to delta side or vice-versa.

EXAMPLE 1.3

A three phase Y-Δ transformer is constructed using three identical single phase transformers of rating 200 kVA, 63.51 kV/11 kV transformer. The impedances of primary and secondary are 20 + j45Ω and 0.1 + j0.2Ω respectively. Calculate the p.u. impedance of the transformer.

SOLUTION

The three phase transformer is formed using three numbers of identical single phase transformers. Hence the kVA rating of three phase transformer is three times that of single phase transformer.

$\therefore$ kVA rating of three phase transformer = 3 × 200 = 600 kVA

Line voltage rating of Y-Δ transformer $= 63.51 \times \sqrt{3}\ \text{kV}/11\ \text{kV}$

$= 110\ \text{kV} / 11\ \text{kV}$

Case(i)

The high voltage winding (primary) ratings are chosen as base values.

$\therefore$ Base kilovolt, $kV_b = 110$ kV

Base kilovoltampere, $kVA_b = 600$ kVA

$$\left.\begin{array}{l}\text{Base impedance}\\ \text{per phase}\end{array}\right\} Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(110)^2}{600/1000} = 20166.7\ \Omega$$

$$\text{Transformer line voltage ratio, } K = \frac{11}{110} = 0.1$$

$$\left.\begin{array}{l}\text{Total impedance}\\ \text{referred to primary}\end{array}\right\} Z_{01} = Z_1 + Z_2' = Z_1 + \frac{Z_2}{K^2} = 20 + j45 + \frac{0.1 + j0.2}{(0.1)^2} = 20 + j45 + 10 + j20$$

$$= 30 + j65\ \Omega / \text{phase}$$

$$\text{p.u. reactance per phase, } Z_{pu} = \frac{\text{Total impedance}}{\text{Base impedance}} = \frac{30 + j65}{20166.7} = 0.0015 + j0.0032\ \text{p.u.}$$

Case(ii)

The low voltage winding (secondary) ratings are chosen as base values.

$\therefore$ Base kilovolt, $kV_b = 11$ kV

Base kilovoltampere, $kVA_b = 600$ k V A

$$\left.\begin{array}{l}\text{Base impedance}\\ \text{per phase}\end{array}\right\} Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(11)^2}{600/1000} = 201.67\ \Omega$$

$$\text{Transformer line voltage ratio, } K = \frac{11}{110} = 0.1$$

$$\left.\begin{array}{l}\text{Total impedance}\\ \text{referred to secondary}\end{array}\right\} Z_{02} = Z_1' + Z_2 = K^2 Z_1 + Z_2$$

$$= (0.1)^2 \times (20 + j45) + 0.1 + j0.2 = 0.2 + j0.45 + 0.1 + j0.2$$

$$= 0.3 + j\ 0.65\ \Omega / \text{phase}$$

$$\text{p.u. reactance per phase, } Z_{pu} = \frac{\text{Total impedance}}{\text{Base impedance}} = \frac{0.3 + j65}{201.67} = 0.0015 + j0.0032\ \text{p.u.}$$

Note : *1. It is observed that the p.u. impedance of a transformer referred to primary and secondary are same.*

2. In three phase transformer if the voltage ratio K is obtained using line values then using this val of K, the impedance per phase of star side can be directly transferred to delta side or vice-versa.

EXAMPLE 1.4

A 50 kW, three phase, Y connected load is fed by a 200 kVA transformer with voltage rating 11 kV/400 V through a feeder. The length of the feeder is 0.5 km and the impedance of the feeder is 0.1+j0.2 Ω/km. If the load p.f. is 0.8, calculate the p.u. impedance of the load and feeder.

SOLUTION

Let us choose the secondary winding rating of transformer as base values.

∴ Base kilovolt, kV_b = 400/1000 = 0.4 kV

Base kilovoltampere, kVA_b = 200 kVA

$$\left.\begin{array}{l}\text{Base impedance}\\\text{per phase}\end{array}\right\} Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{(0.4)^2}{200/1000} = 0.8\,\Omega$$

$$\left.\begin{array}{l}\text{Actual impedance}\\\text{of feeder}\end{array}\right\} Z_{fed} = (0.1 + j0.2) \times 0.5 = 0.05 + j0.1\,\Omega\text{ / phase}$$

$$\left.\begin{array}{l}\text{p.u. impedance}\\\text{of feeder}\end{array}\right\} Z_{pu,fed} = \frac{\text{Actual impedance}}{\text{Base impedance}} = \frac{Z_{fed}}{Z_b}$$

$$= \frac{0.05 + j0.1}{0.8} = 0.0625 + j0.125\text{ p.u.}$$

Given that P = 50 kW and pf = cosϕ = 0.8

∴ $\sin\phi = \sin(\cos^{-1} 0.8) = 0.6$

$$\text{Reactive power, } Q = \frac{P}{\text{Cos}\,\phi} \times \sin\phi = \frac{50}{0.8} \times 0.6 = 37.5\text{ kVAR}$$

$$\left.\begin{array}{l}\text{Load impedance}\\\text{per phase}\end{array}\right\} Z_L = \frac{|V_L|^2}{P - jQ} = \frac{400^2}{(50 - j37.5) \times 10^3} = \frac{400^2 \times 10^{-3}}{50 - j37.5}$$

$$= \frac{160}{62.5\angle -36.87^{\circ}} = 2.56\angle 36.87^{\circ} = 2.048 + j1.536\text{ W/phase}$$

$$\left.\begin{array}{l}\text{p.u value of}\\\text{load impedance}\end{array}\right\} Z_{L,pu} = \frac{\text{Load impedance}}{\text{Base impedance}} = \frac{Z_L}{Z_b} = \frac{2.048 + j1.536}{0.8} = 2.56 + j1.92\text{ p.u.}$$

EXAMPLE 1.5

The three-phase ratings of a three winding transformer are

Primary	:	Y-connected, 110 kV, 20 MVA
Secondary	:	Y-connected, 13.2 kV, 15 MVA
Tertiary	:	Δ - connected, 2.1 kV, 0.5 MVA

Three short-circuit tests performed on this transformer yielded the following results (i) primary excited, secondary shorted : 2290 V, 52.5 A (ii) Primary excited, Tertiary shorted : 1785 V, 52.5 A (iii) Secondary excited, Tertiary shorted : 148 V, 328 A.

Find the p.u. impedances of the star-connected single-phase equivalent circuit for a base of 20 MVA, 110 kV in the primary circuit. Neglect resistances.

SOLUTION

Test (i) and (ii) are performed on primary winding, hence the p.u. impedances of Z_{ps} and Z_{pt} can be obtained directly using the primary winding ratings as base values.

The test (iii) is performed on secondary winding, hence the p.u. impedance Z_{st} is obtained using secondary winding rating as bases and then it can be converted to primary winding base.

To find p.u. value of Z_{ps} and Z_{pt}

Base kilovolt of primary circuit, $kV_{b,\,py}$ = 110 kV

Base megavoltampere of primary circuit, $MVA_{b,\,py}$ = 20 MVA

$$\therefore \left.\begin{matrix}\text{Base impedance}\\ \text{of primary circuit}\end{matrix}\right\} Z_{b,py} = \frac{(kV_{b,py})^2}{MVA_{b,py}} = \frac{(110)^2}{20} = 605\ \Omega/\text{phase}$$

The readings of test (i) can be used to calculate the value of Z_{ps} in Ω/phase.

$$Z_{ps} \text{ in } \Omega/\text{phase} = \frac{2290/\sqrt{3}}{52.5} = 25.1835\ \Omega/\text{phase}$$

Note : *The primary is star connected.*

$$\text{p.u. value of } Z_{ps} = \frac{Z_{ps}\text{ in }\Omega/\text{phase}}{Z_{b,py}} = \frac{25.1835}{605} = 0.0416 \text{ p.u.}$$

The readings of test (ii) can be used to calculate the value of Z_{pt} in Ω/phase.

$$Z_{pt} \text{ in } \Omega/\text{phase} = \frac{1785/\sqrt{3}}{52.5} = 19.6299\ \Omega/\text{phase}$$

Note : *The primary is star connected.*

$$\text{p.u. value of } Z_{pt} = \frac{Z_{pt}\text{ in }\Omega/\text{phase}}{Z_{b,py}} = \frac{19.6299}{605} = 0.0324 \text{ p.u.}$$

To find p.u. value of Z_{st}'

Base kilovolt of secondary circuit, $kV_{b,\,sec}$ = 13.2 kV

Base megavoltampere of secondary circuit, $MVA_{b,\,sec}$ = 15 MVA

$$\therefore \left.\begin{matrix}\text{Base impedance}\\ \text{of primary circuit}\end{matrix}\right\} Z_{b,sec} = \frac{(kV_{b,sec})^2}{MVA_{b,sec}} = \frac{(13.2)^2}{15} = 11.616\ \Omega/\text{phase}$$

The readings of test (iii) can be used to calculate the value of Z_{st} in Ω/phase.

$$Z_{st} \text{ in } \Omega/\text{phase} = \frac{148/\sqrt{3}}{328} = 0.2605\ \Omega/\text{phase}$$

Note : *The secondary is star connected.*

$$\left.\begin{matrix}\text{p.u. value of } Z_{st} \text{ on the base}\\ \text{values of secondary circuit}\end{matrix}\right\} Z_{st} = \frac{Z_{st}\text{ in }\Omega/\text{phase}}{Z_{b,sec}} = \frac{0.2605}{11.616} = 0.0224 \text{ p.u}$$

The p.u. value of Z_{st} on the secondary circuit base values can be converted to primary circuit base using the following formula

$$Z_{pu,new} = Z_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

Here new refers to primary and old refers to secondary.

$$\left.\begin{matrix}\text{p.u. value of } Z_{st} \text{ on the base}\\ \text{values of primary circuit}\end{matrix}\right\} Z'_{st} = 0.0224 \times \left(\frac{110}{110}\right)^2 \times \left(\frac{20}{15}\right) = 0.0299 \text{ p.u.}$$

Note : *The $kV_{b,\,sec}$ when referred to primary side will be (110/13.2)×13.2 = 110 kV*

To compute Z_p, Z_s' and Z_t'

$$Z_p = \frac{1}{2}[Z_{ps} + Z_{pt} - Z'_{st}] = \frac{1}{2}[0.0416 + 0.0324 - 0.0299] = 0.0221 \text{ p.u.}$$

$$Z'_s = \frac{1}{2}[Z_{ps} + Z'_{st} - Z_{pt}] = \frac{1}{2}[0.0416 + 0.0299 - 0.0324] = 0.0196 \text{ p.u.}$$

$$Z'_t = \frac{1}{2}[Z_{pt} + Z'_{st} - Z_{ps}] = \frac{1}{2}[0.0324 + 0.0299 - 0.0416] = 0.01035 \text{ p.u.}$$

The single phase star connected equivalent circuit of three winding transformer is shown in fig 1.5.1. Since the resistances are neglected the impedances are represented as pure reactances.

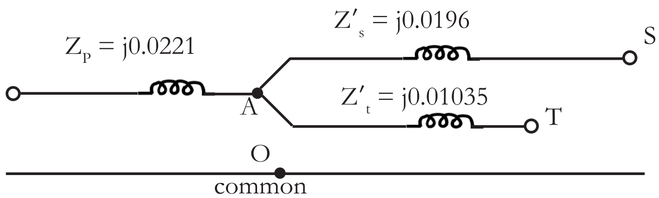


Fig 1.5.1 : *Single phase equivalnet circuit of three winding transformer*

1.4 IMPEDANCE AND REACTANCE DIAGRAM

The Impedance or Reactance diagram of a power system is the equivalent circuit of the power system in which the various components of the power system are represented by their approximate or simplified equivalent circuits. This equivalent circuit of power system is used to analyse the performance of a system under load conditions (load flow studies) or to analyse the condition of the system under fault.

Impedance diagram

The impedance of a power system diagram is used for load flow studies. The impedance diagram can be obtained from the single line diagram by replacing all the components of the power system by their single phase equivalent circuit. The following approximations are made while forming impedance diagram.

1. The current limiting impedances connected between the generator neutral and ground are neglected since under balanced conditions no current flows through neutral.
2. Since the magnetizing current of a transformer is very low when compared to load current the shunt branches in the equivalent circuit of the transformer can be neglected.
3. If the inductive reactance of a component is very high when compared to resistance then the resistance can be omitted, which introduces a little error in calculations.

Reactance diagram

The reactance diagram is used for fault calculations. The following approximations are made in constructing reactance diagram, (when the system is balanced).

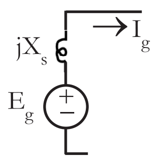
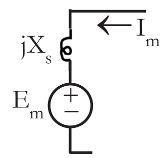
1. The neutral to ground impedance of the generator is neglected for symmetrical faults.
2. Shunt branches in the equivalent circuits of transformer are neglected.
3. The resistances in the equivalent circuits of various components of the system are omitted.
4. All static loads are neglected.
5. Induction motors are neglected in computing fault current a few cycles after the fault occurs, because the current contributed by an induction motor dies out very quickly after the induction motor is short-circuited.

6. The capacitance of the transmission lines are neglected.

The reactance diagram can be obtained from impedance diagram if we omit all static loads, all resistances, shunt branches of transformer and capacitance of transmission lines in the impedance diagram. The simplified representation of various components of power system in reactance diagram are shown in table 1.2.

The impedance and reactance diagram obtained from the above approximations are also called positive sequence impedance diagram and positive sequence reactance diagram.

Table-1.2 : Representation of components of power system in reactance diagram

Component	Equivalent circuit	Component	Equivalent circuit
3-phase generator		Synchronous motor	
Transformer	% X 	Transmission line	Inductive Reactance 

A sample single-line diagram of a power system, its impedance and reactance diagram are shown in fig 1.8.

The impedance or reactances of various components of power system in a single line diagram are expressed in percentage value or per unit calculated by taking their ratings as base values. When the impedance or reactance diagram is formed, all the impedances or reactances should be expressed in per unit calculated on a common base value. Hence it is necessary to convert all the p.u. reactances to a common base. The conversion of per unit reactances from one base to another can be performed using the following equation which is obtained from equation (1.16) after replacing Z by X.

$$X_{pu,new} = X_{pu,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) \qquad .....(1.47)$$

Procedure to form reactance diagram from single line diagram

1. Select a base kilovoltampere or megavoltampere (kVA_b or MVA_b). The kVA_b or MVA_b will be same for all sections of the power system. In case of three phase power system, the kVA_b or MVA_b is three phase power rating.
2. Select a base kilovolt (kV_b) for one section of power system. In case of three phase power system, the kV_b is a line value. The various sections of power system works at different voltage levels and the voltage conversion is achieved by means of transformers. Hence the kV_b of one section of power system should be converted to a kV_b corresponding to another section using the transformer voltage ratio. In case of three phase transformer, line-to-line voltage ratio is used to transfer the kV_b on one section to another section.

$$kV_b \text{ on LT section} = k\,V_b \text{ on HT section} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}} \qquad .....(1.48)$$

$$kV_b \text{ on HT section} = kV_b \text{ on LT section} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}} \qquad .....(1.49)$$

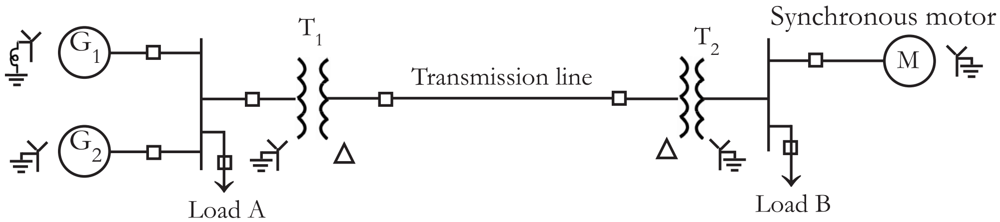


Fig a : *Single line diagram*

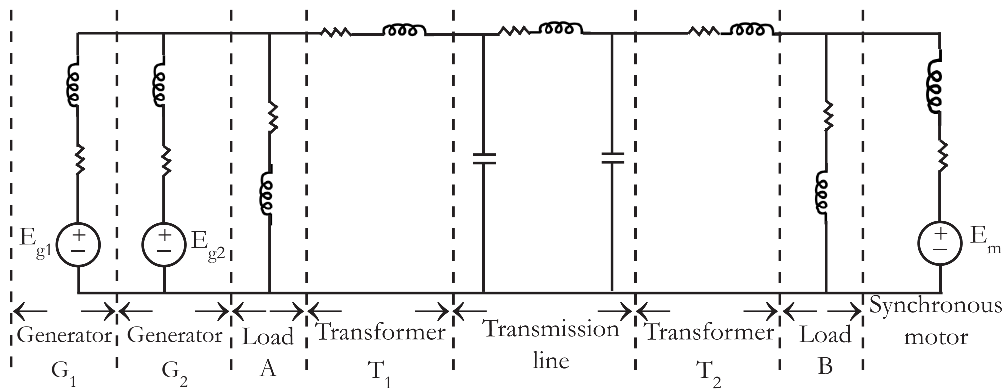


Fig b : *Impedance diagram*

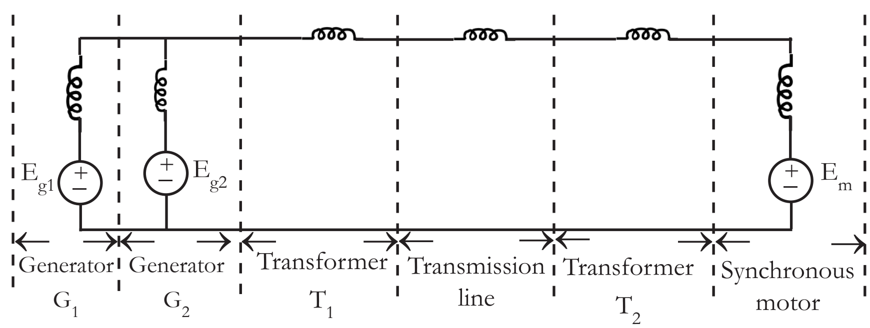


Fig c : *Reactance diagram*

Fig 1.8 : *Single line diagram, impedance diagram and reactance diagram of a sampled power system*

3. The impedance of the components of power system are expressed either in ohms (actual impedance) or in p.u. which is calculated using the component rating as base values. In reactance diagram, the resistances are neglected and the reactances of all the components are expressed on a common base. Hence starting from one end of power system the reactances of each component should be converted to p.u. reactances on the selected new base. When the specified reactance of the component is in ohms then

$$\text{p.u. reactance} = \frac{\text{Actual reactance in ohms}}{\text{Base impedance}}$$

when the specified reactance of the component is in p.u. on the component rating as base values, then consider the component rating as old base values and selected base values as new bases. Now the p.u. reactance on new base can be calculated using the formula.

$$X_{pu,new} = X_{pu,old} \times \left(\frac{k\,V_{b,old}}{k\,V_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

EXAMPLE 1.6

a) A generator is rated 500 MVA, 22 kV. Its Y-connected winding has a reactance of 1.1 p.u. Find the ohmic value of the reactance of winding.

b) If the generator is working in a circuit for which the bases are specified as 100 MVA, 20 kV. Then find the p.u. value of reactance of generator winding on the specified base.

SOLUTION

(a) **The generator p.u. reactance will be specified by taking its rating as base values**

$\therefore$Base kilovolt, kV_b = 22 kV and Base megavoltampere, MVA_b = 500 MVA

$$\text{Base impedance, } Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{22^2}{500} = 0.968\ \Omega$$

$$\text{Per unit reactance, } X_{pu} = \frac{\text{Actual reactance}, \Omega}{\text{Base impedance}, \Omega} = \frac{X}{Z_b}$$

$$\therefore\ \text{Actual reactance, } X = X_{pu} \times Z_b = 1.1 \times 0.968 = 1.0648\ \Omega/\text{phase}$$

(b) **The formula used to convert the p.u. reactance specified on a base value to another base is given below**

$$X_{pu,new} = X_{pu,old} \times \left(\frac{KV_{b,old}}{KV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

The new base values are, $kV_{b,\ new}$ = 20 kV and $MVA_{b,\ new}$ = 100 MVA.

The old base values are $kV_{b,\ old}$ = 22 kV and $MVA_{b,\ old}$ = 500 MVA

$$\therefore\ X_{pu,new} = 1.1 \times \left(\frac{22}{20}\right)^2 \times \left(\frac{100}{500}\right) = 0.2662 \text{ p.u.}$$

EXAMPLE 1.7

A 300 MVA, 20 kV, 3ϕ generator has a subtransient reactance of 20%. The generator supplies 2 synchronous motors through a 64 km transmission line having transformers at both ends as shown in fig 1.7.1. In this, T_1 is a 3ϕ transformer and T_2 is made of 3 single phase transformer of rating 100 MVA, 127/13.2 kV, 10% reactance. Series reactance of the transmission line is 0.5 Ω/km. Draw the reactance diagram with all the reactances marked in p.u. Select the generator rating as base values.

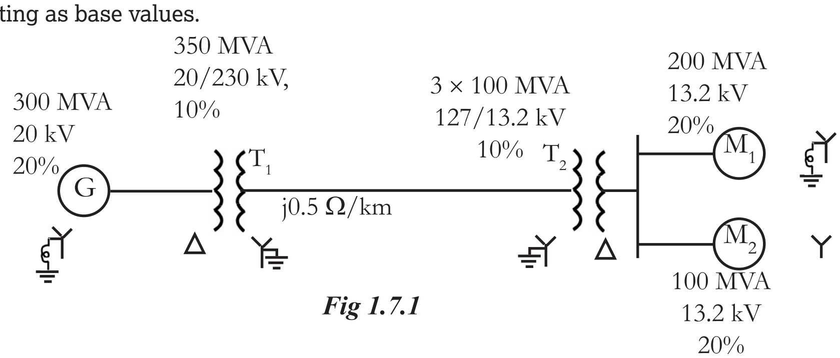


Fig 1.7.1

SOLUTION

Base megavoltampere, $MVA_{b,\ new}$ = 300 MVA

Base kilovolt, $kV_{b,\ new}$ = 20 kV

Reactance of Generator G

Since the generator rating and the base values are same, the generator p.u. reactance does not change.

$\therefore$ p.u. reactance of generator = 20% = 0.2 p.u.

Reactance of Transformer T_1

$$\left.\begin{array}{l}\text{The new p.u. reactance}\\ \text{of Transformer } T_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{20}{20}\right)^2 \times \frac{300}{350} = 0.0857 \text{ p.u}$$

Reactance of Transmission line

Reactance of transmission line = 0.5 Ω/km.

Total reactance of transmission line = 0.5 × 64 = 32 Ω.

$$\left.\begin{array}{l}\text{Base kV on HT side}\\ \text{of transformer } T_1\end{array}\right\} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}} = 20 \times \frac{230}{20} = 230 \text{ kV}$$

$$\text{Base impedance, } Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{230^2}{300} = 176.33\,\Omega$$

$$\left.\begin{array}{l}\text{per unit reactance of}\\ \text{Transmission line}\end{array}\right\} = \frac{\text{Actual reactance}}{\text{Base impedance}} = \frac{32}{176.33} = 0.1815 \text{ p.u.}$$

Reactance of Transformer T_2

The transformer T_2 is a 3-phase transformer bank formed using three numbers of single phase transformers with voltage rating 127/13.2 kV. In this the high voltage side is star connected and low-voltage side is delta connected.

$$\therefore \text{ Voltage ratio of line voltage of 3 - phase transformer bank} = \frac{\sqrt{3} \times 127}{13.2} = \frac{220}{13.2} \text{ kV}$$

$$\text{Base kV on LT side of transformer } T_2 = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 230 \times \frac{13.2}{220} = 13.8 \text{ kV}$$

$$\text{The new p.u. reactance of transformer } T_2 = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{13.2}{13.8}\right)^2 \times \left(\frac{300}{3 \times 100}\right) = 0.0915 \text{ p.u.}$$

Reactance of M_1

$$\text{p.u. reactance of } M_1 \text{ on new base} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{13.2}{13.8}\right)^2 \times \frac{300}{200} = 0.2745 \text{ p.u.}$$

Reactance of M_2

$$\text{p.u. reactance of } M_2 \text{ on new base} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{13.2}{13.8}\right)^2 \times \frac{300}{100} = 0.549 \text{ p.u.}$$

Reactance diagram

The reactance diagram is shown in fig 1.7.2.

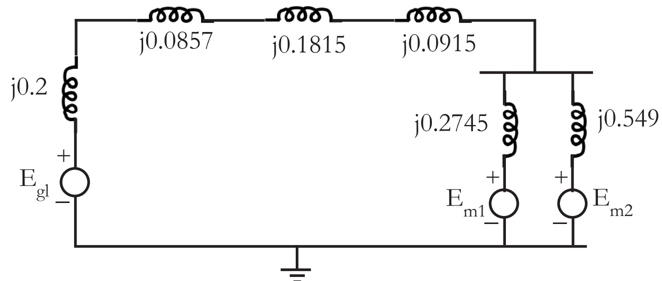


Fig 1.7.2 : *Reactance diagram of the system shown in fig 1.7.1. (all reactance values are in p.u.)*

EXAMPLE 1.8

A 120 MVA, 19.5 kV generator has a synchronous reactance of 0.15 p.u. and it is connected to a transmission line through a transformer rated 150 MVA, 230/18 kV (Y/Δ) with X = 0.1 pu.

(a) Calculate the p.u. reactances by taking generator rating as base values.

(b) Calculate the p.u. reactance by taking transformer rating as base values.

(c) Calculate the p.u. reactances for a base value of 100 MVA and 220 kV on HT side of transformer.

SOLUTION

(a) Base megavoltampere, $MVA_{b,\,new}$ = 120 MVA

Base kilovolt, $kV_{b,\,new}$ = 19.5 kV

Since the generator ratings are chosen as base values, its p.u. reactance will not change.

$\therefore$ Reactance of generator = 0.15 p.u.

$$\text{New p.u. reactance of transformer} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{18}{19.5}\right)^2 \times \frac{120}{150} = 0.0682 \text{ p.u.}$$

(b) Base megavoltampere, $MVA_{b,\,new}$ = 150 MVA

Base kilovolt, $kV_{b,\,new}$ = 18 kV

Since the transformer ratings are chosen as base values, its pu reactance will not change.

$\therefore$ Reactance of transformer = 0.1 p.u.

$$\text{New p.u. reactance of generator} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{19.5}{18}\right)^2 \times \frac{150}{120} = 0.22 \text{ p.u.}$$

(c) Base megavoltampere, $MVA_{b,\,new}$ = 100 MVA

Base kilovolt, $kV_{b,\,new}$ = 220 kV

In this case the base values are neither generator ratings nor transformer ratings. Hence both the p.u. reactances should be converted to new base.

$$\text{New p.u. reactance of transformer} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,new}}\right)$$

$$= 0.1 \times \left(\frac{230}{220}\right)^2 \times \frac{100}{150} = 0.0729 \text{ p.u.}$$

The generator is connected to LT side of transformer.

$$\therefore \left.\begin{matrix}\text{Base kV referred of} \\ \text{LT side of transformer}\end{matrix}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 220 \times \frac{18}{230} = 17.22 \text{ KV}$$

New, k $V_{b,new}$ = 17.22 kV

$$\text{New p.u. reactance of generator} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{19.5}{17.22}\right)^2 \times \left(\frac{100}{120}\right) = 0.1603 \text{ p.u.}$$

RESULT

a. kV_b = 19.5 kV ; MVA_b = 120 MVA, $X_{pu,\,gen}$ = 0.15 p.u. ; $X_{pu,\,tr}$ = 0.0682 p.u.

b. kV_b = 18 kV ; MVA_b = 150 MVA, $X_{pu,\,gen}$ = 0.22 p.u. ; $X_{pu,\,tr}$ = 0.1 p.u.

c. kV_b = 220 kV ; MVA_b = 100 MVA, $X_{pu,\,gen}$ = 0.1603 p.u. ; $X_{pu,\,tr}$ = 0.0729 p.u.

EXAMPLE 1.9

The single line diagram of an unloaded power system is shown in fig 1.9.1. The generator and transformers are rated as follows.

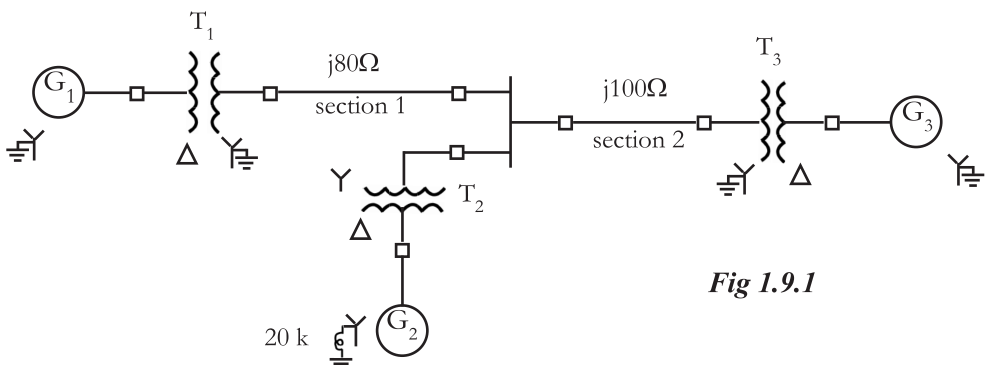


Fig 1.9.1

Generator, G_1 = 20 MVA, 13.8 kV, X″ = 20%
Generator, G_2 = 30 MVA, 18 kV, X″ = 20%
Generator, G_3 = 30 MVA, 20 kV, X″ = 20%
Transformer, T_1 = 25 MVA, 220/13.8 kV, X = 10%
Transformer, T_2 = 3 single phase units each rated at 10 MVA, 127/18 kV, X = 10%.
Transformer, T_3 = 35 MVA, 220/22 kV, X = 10%.

Draw the reactance diagram using a base of 50 MVA and 13.8 kV on the generator G_1.

SOLUTION

Base megavoltampere, $MVA_{b,\,new}$ = 50 MVA

Base kilovolt, $kV_{b,\,new}$ = 13.8 kV

Reactance of Generator G_1

$$\left.\begin{matrix}\text{New p.u. reactance} \\ \text{of generator } G_1\end{matrix}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{13.8}{13.8}\right)^2 \times \left(\frac{50}{20}\right) = 0.5 \text{ p.u.}$$

Reactance of Transformer T_1

$$\left.\begin{array}{l}\text{New p.u. reactance} \\ \text{of transformer } T_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) = 0.1 \times \left(\frac{13.8}{13.8}\right)^2 \times \left(\frac{50}{25}\right) = 0.2 \text{ p.u.}$$

Reactance of Transmission lines

$$\left.\begin{array}{l}\text{Base kV on HT side} \\ \text{of transformer } T_1\end{array}\right\} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}} = 13.8 \times \frac{220}{13.8} = 220 \text{ kV}$$

Now, $kV_{b,new} = 220$ kV

$$\left.\begin{array}{l}\text{Base impedance on HT side} \\ \text{of transformer}\end{array}\right\} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{220^2}{50} = 968\ \Omega$$

$$\left.\begin{array}{l}\text{p.u. reactance of section 1} \\ \text{of transmission line}\end{array}\right\} = \frac{\text{Actual impedance}, \Omega}{\text{Base impedance}, \Omega} = \frac{80}{968} = 0.0826 \text{ p.u.}$$

$$\left.\begin{array}{l}\text{p.u. reactance of section 2} \\ \text{of transmission line}\end{array}\right\} = \frac{\text{Actual impedance}, \Omega}{\text{Base impedance}, \Omega} = \frac{100}{968} = 0.1033 \text{ p.u.}$$

Reactance of Transformer T_2

The transformer T_2 is a 3-phase transformer bank formed using three numbers of single phase transformer with voltage rating 127/18 kV. In this the HT side is star connected and LT side is delta connected.

$$\therefore \left.\begin{array}{l}\text{Voltage ratio of line voltage} \\ \text{of 3 – phase transformer back}\end{array}\right\} = \frac{\sqrt{3} \times 127}{18} = \frac{220}{18} \text{ kV}$$

$$\left.\begin{array}{l}\text{Base kV on LT side} \\ \text{of transformer } T_2\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}} = 220 \times \frac{18}{220} = 18 \text{ kV}$$

Now, $kV_{b,\,new} = 18$ kV

$$\left.\begin{array}{l}\text{New p.u. reactance} \\ \text{of transformer } T_2\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{3 \times 10}\right) = 0.1667 \text{ p.u.}$$

Reactance of Generator G_2

$$\left.\begin{array}{l}\text{New p.u. reactance} \\ \text{of Generator } G_2\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{30}\right) = 0.3333 \text{ p.u.}$$

Reactance of Transformer T_3

$$\left.\begin{array}{l}\text{Base kV on LT side} \\ \text{of transformer } T_3\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}} = 220 \times \frac{22}{220} = 22 \text{ kV}$$

Now, $kV_{b,new} = 220\text{ kV}$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of transformer } T_3\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) = 0.1 \times \left(\frac{22}{22}\right)^2 \times \left(\frac{50}{35}\right) = 0.149\text{ p.u.}$$

Reactance of Generator G_3

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of generator } G_3\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{20}{22}\right)^2 \times \left(\frac{50}{30}\right) = 0.2755\text{ p.u.}$$

Reactance diagram

The reactance diagram is shown in fig 1.9.2.

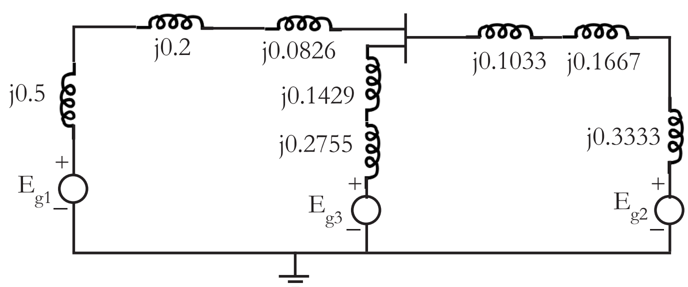


***Fig 1.9.2 :** Reactance diagram of the system shown in fig 1.9.1 (all reactance values are in p.u.)*

EXAMPLE 1.10

Draw the reactance diagram for the power system shown in fig 1.10.1. Neglect resistance and use a base of 100 MVA, 220 kV in 50 Ω line. The ratings of the generator, motor and transformer are given below.

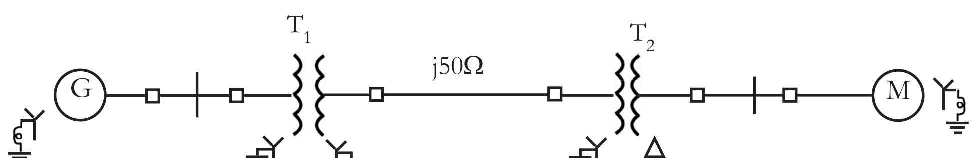


Fig 1.10.1

Generator : 40 MVA, 25 kV, X″ = 20%

Synchronous motor : 50 MVA, 11 kV, X″ = 30%

Y-Y Transformer : 40 MVA, 33/220 kV, X = 15%

Y-Δ Transformer : 30 MVA, 11/220 kV (Δ/Y), X = 15%

SOLUTION

Base megavoltampere, $MVA_{b,\,new} = 100\text{ MVA}$

Base kilovolt, $kV_{b,\,new} = 220\text{ kV}$

Reactance of Transmission line

$$\text{Base impedance} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{220^2}{100} = 484\ \Omega$$

$$\left.\begin{array}{l}\text{p.u. reactance}\\\text{of transmission line}\end{array}\right\} = \frac{\text{Actual reactance}, \Omega}{\text{Base impedance}, \Omega} = \frac{50}{484} = 0.1033 \text{ p.u.}$$

Reactance of Transformer T_1

$$\left.\begin{array}{l}\text{Base kV on LT side}\\\text{of Transformer } T_1\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}} = 220 \times \frac{33}{220} = 33 \text{ kV}$$

Now, $kV_{b,new} = 33 \text{ kV}$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\\text{of transformer } T_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{33}{33}\right)^2 \times \left(\frac{100}{40}\right) = 0.375 \text{ p.u.}$$

Reactance of Generator G

$$\left.\begin{array}{l}\text{New p.u. reactance}\\\text{of generator G}\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{25}{33}\right)^2 \times \left(\frac{100}{40}\right) = 0.287 \text{ p.u.}$$

Reactance of Transformer T_2

$$\left.\begin{array}{l}\text{Base kV on LT side}\\\text{of Transformer } T_2\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}} = 220 \times \frac{11}{220} = 11 \text{ kV}$$

Now, $kV_{b,new} = 11 \text{ kV}$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\\text{of transformer } T_2\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{30}\right) = 0.5 \text{ p.u.}$$

Reactance of Synchronous motor

$$\left.\begin{array}{l}\text{New p.u. reactance}\\\text{of synchronous motor}\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.3 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{50}\right) = 0.6 \text{ p.u.}$$

Reactance diagram

The reactance diagram is shown in fig 1.10.2.

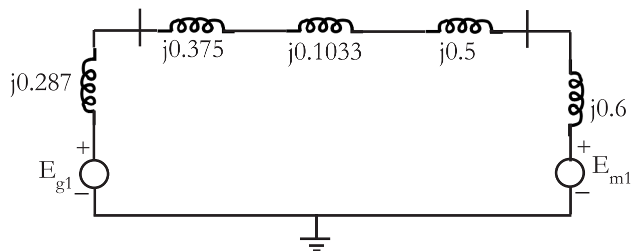


Fig 1.10.2 : *Reactance diagram of the system shown in fig 1.10.1 (all reactance values are in p.u.)*

EXAMPLE 1.11

Draw the reactance diagram for the power system shown in fig 1.11.1. The ratings of generator, motor and transformers are given below. Neglect resistance and use a base of 50 MVA, 138 kV in the 40 Ω line.

Generator G_1 : 20 MVA, 18 kV, X″ = 20%

Generator G_2 : 20 MVA, 18 kV, X″ = 20%

Synchronous motor : 30 MVA, 13.8 kV, X″ = 20%

3-phase, Y-Y Transformer : 20 MVA, 138/20 kV, X = 10%

3-phase, Y-Δ Transformer : 15 MVA, 138/13.8 kV, X = 10%

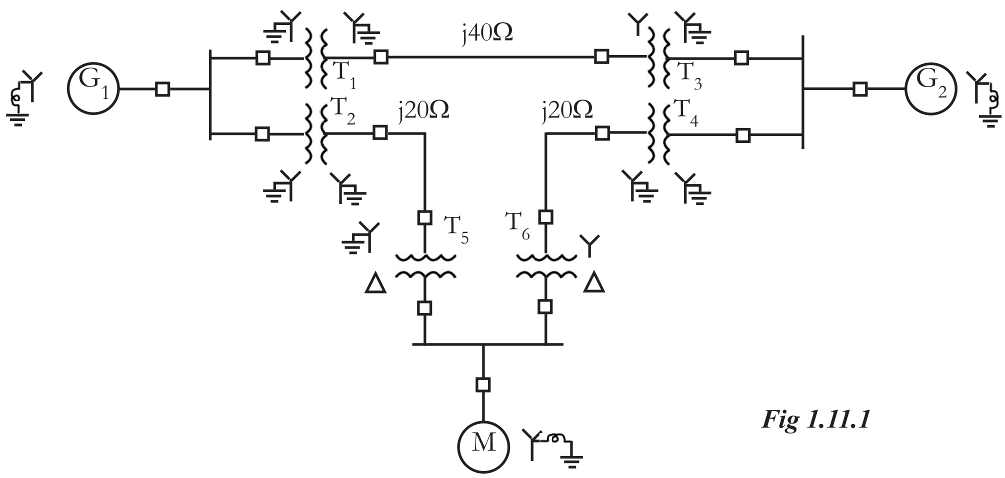


Fig 1.11.1

SOLUTION

Base megavoltamere, $MVA_{b,\, new}$ = 50 MVA

Base kilovolt, $kV_{b,\, new}$ = 138 kV

Reactance of j40Ω Transmission line

$$\text{Base impedance} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{138^2}{50} = 380.88\ \Omega$$

$$\left.\begin{array}{l}\text{p.u. reactance of}\\ 40\Omega\ \text{Transmission line}\end{array}\right\} = \frac{\text{Actual reactance}, \Omega}{\text{Base impedance}, \Omega} = \frac{40}{380.88} = 0.105\ \text{p.u.}$$

Reactance of Transformer T_1

$$\left.\begin{array}{l}\text{Base kV on LT side}\\ \text{of Transformer } T_1\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}} = 138 \times \frac{20}{138} = 20\ \text{kV}$$

Now, $kV_{b,new} = 20\text{ kV}$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of Transformer } T_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) = 0.1 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{20}\right) = 0.25\text{ p.u.}$$

Reactance of Generator G_1

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of generator } G_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) = 0.2 \times \left(\frac{18}{20}\right)^2 \times \left(\frac{50}{20}\right) = 0.405\text{ p.u.}$$

Reactance of Transformer T_2

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of Transformer } T_2\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{20}\right) = 0.25\text{ p.u.}$$

Reactance of j20Ω Transmission line

$$\left.\begin{array}{l}\text{Base kV on LT side}\\ \text{of Transformer } T_2\end{array}\right\} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$= 20 \times \frac{138}{20} = 138\text{ kV}$$

Now, $kV_{b,new} = 138\text{ kV}$

$$\text{Base impedance} = \frac{kV_b^2}{MVA_b} = \frac{138^2}{50} = 380.88\ \Omega$$

$$\left.\begin{array}{l}\text{p.u. reactance of}\\ 20\Omega\text{ Transmission line}\end{array}\right\} = \frac{\text{Actual reactance}, \Omega}{\text{Base impedance}, \Omega} = \frac{20}{380.88} = 0.0525\text{ p.u.}$$

Here it is observed that both the sections of j20Ω transmission lines have same values of reactances and base kV's. Hence their p.u. reactances will be same.

Reactance of Transformer T_5

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of Transformer } T_5\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{138}{138}\right)^2 \times \left(\frac{50}{15}\right) = 0.333\text{ p.u.}$$

Reactance of Synchronous motor

$$\left.\begin{array}{l}\text{Base kV on LT side}\\ \text{of Transformer } T_2\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}} = 138 \times \frac{13.8}{138} = 138\text{ kV}$$

Now, $kV_{b,new} = 138\text{ kV}$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of synchronous motor}\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{138}{138}\right)^2 \times \left(\frac{50}{30}\right) = 0.333 \text{ p.u.}$$

Reactances of T_6, T_4, T_3 and G_2

1. The transformer T_6 is identical to that of T_5. Hence p.u. reactance of T_5 and T_6 are same.
2. The transformers T_1, T_2, T_3 and T_4 are identical. Hence their p.u. reactances are same.
3. The generator G_2 is identical to that of G_1. Hence their p.u. reactances are same.

Reactance diagram

The reactance diagram of the power system is shown in fig 1.11.2.

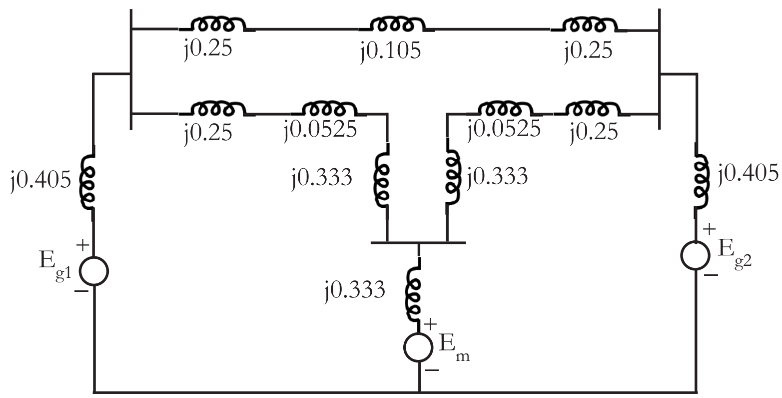


***Fig 1.11.2 :** Reactance diagram of the system shown in fig 1.11.1 (all reactance values are in p.u.)*

EXAMPLE 1.12

A 15 MVA, 8.5 kV, 3-phase generator has a subtransient reactance of 20%. It is connected through a Δ-Y transformer to a high voltage transmission line having a total series reactance of 70 ohms. The load end of the line has Y-Y step down transformer. Both transformer banks are composed of single phase transformers connected for 3-phase operation. Each of three transformers composing three phase bank is rated 6667 kVA, 10/100 kV, with a reactance of 10%. The load represented as impedance, is drawing 10 MVA at 12.5 kV and 0.8 pf lagging. Draw the single line diagram of the power network. Choose a base of 10 MVA, 12.5 kV in the load circuit and determine the reactance diagram. Determine also the voltage at the terminals of the generator.

SOLUTION

The single line diagram of the power system is shown in fig 1.12.1.

Base values

Base megavoltampere, $MVA_{b, new} = 10$ MVA

Base kilovolt, $kV_{b, new} = 12.5$ kV

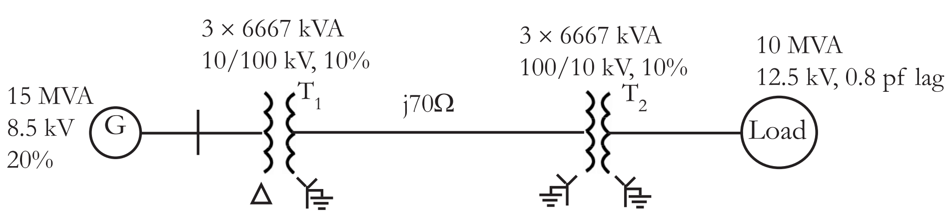


***Fig 1.12.1 :** Single line diagram*

Reactance of Transformer T_2

Voltage ratio of line voltage of transformer $T_2 = \frac{100 \times \sqrt{3}\ \text{kV}}{10 \times \sqrt{3}\ \text{kV}} = \frac{173.2\ \text{kV}}{17.32\ \text{kV}}$

3 - phase k V A rating of transformer $T_2 = 3 \times 6667 = 20{,}000\ \text{kVA} = 20\ \text{MVA}$

$\therefore\ \text{k}\,V_{b,old} = 17.32\ \text{kV}$ (on LT side)

$\text{MVA}_{b,old} = 20\ \text{MVA}$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of Transformer } T_2\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{17.32}{12.5}\right)^2 \times \left(\frac{10}{20}\right) = 0.096\ \text{p.u.}$$

Reactance of Transmission line

$$\left.\begin{array}{l}\text{Base kV on HT side}\\ \text{of Transformer } T_2\end{array}\right\} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}} = 12.5 \times \frac{173.2}{17.32} = 125\ \text{kV}$$

Now, $kV_{b,new} = 125\ \text{kV}$

Base impedance $Z_b = \frac{(\text{k}\,V_{b,new})^2}{MVA_{b,new}} = \frac{(125)^2}{10} = 1562.5\ \Omega$

$$\left.\begin{array}{l}\text{p.u. reactance of}\\ \text{Transmission line}\end{array}\right\} = \frac{\text{Actual reactance}, \Omega}{\text{Base impedance}, \Omega} = \frac{70}{1562.5} = 0.0448\ \text{p.u.}$$

Reactance of Transformer T_1

Voltage ratio of line voltage of transformer $T_1 = \frac{10\ \text{kV}}{100 \times \sqrt{3}\ \text{kV}} = \frac{10\ \text{kV}}{173.2\ \text{kV}}$

3 - phase k V A rating of transformer $T_1 = 3 \times 6667 = 20{,}000\ \text{kVA} = 20\ \text{MVA}$

$\therefore\ \text{k}\,V_{b,old} = 173.2\ \text{kV}$ (on HT side)

$\text{MVA}_{b,old} = 20\ \text{MVA}$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of Transformer } T_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{173.2}{125}\right)^2 \times \left(\frac{10}{20}\right) = 0.096\ \text{p.u.}$$

Reactance of Generator

$$\left.\begin{array}{l}\text{Base kV on LT side}\\ \text{of Transformer } T_1\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}} = 125 \times \frac{10}{173.2} = 7.217\ \text{kV}$$

Now, $kV_{b,new} = 7.217\ \text{kV}$

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of generator}\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{8.5}{7.217}\right)^2 \times \left(\frac{10}{15}\right) = 0.185\ \text{p.u.}$$

Load

This can be represented as constant current load.

p.f of load = 0.8 lag

$\therefore$ p.f. angle = $-\cos^{-1} 0.8 = -36.87°$

Complex load power = $10 \angle -36.87°$ MVA

$$\text{p. u. value of load (power)} = \frac{\text{Actual load MVA}}{\text{Base value of MVA}} = \frac{10\angle -36.87^\circ}{10} = 1\angle -36.87^\circ \text{ p.u.}$$

$$\text{p. u. value of load voltage} = \frac{\text{Actual load voltage}}{\text{Base voltage}} = \frac{12.5\text{ kV}}{12.5\text{ kV}} = 1.0 \text{ p.u.}$$

Let, I = Load current in p.u.

V = Load voltage in p.u.

we know that, V × I = p. u. value of load (power)

$$\therefore\ I = \frac{\text{p.u. value of load}}{V} = \frac{1\angle -36.87^\circ}{1.0} = 1\angle -36.87^\circ$$

Reactance diagram

The reactance diagram of the system is shown in fig 1.12.2.

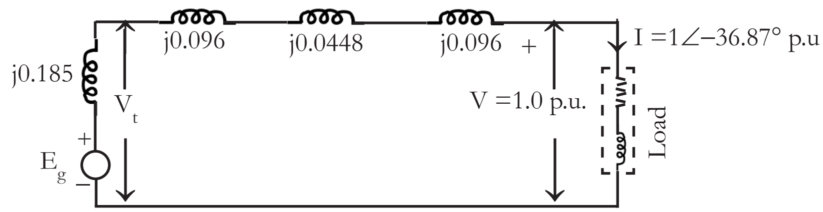


Fig 1.12.2 : *Reactance diagram of the system shown in fig 1.12.1 (all reactance values are in p.u.)*

To find the terminal voltage of the generator

With reference to fig 1.12.2.

The terminal voltage of the generator, $V_t = V + I(j0.096 + j0.0448 + j0.096)$

$$= 1.0 + 1\angle -36.87^\circ (j0.2368)$$

$$= 1.0 + 1\angle -36.87^\circ \times 0.2368\angle 90^\circ$$

$$= 1.0 + 0.2368\angle 53.13^\circ$$

$$= 1.0 + 0.1421 + j0.1894$$

$$= 1.1421 + j0.1894$$

$$= 1.1577\angle 9.4^\circ \text{ p.u.}$$

$$\left.\begin{array}{l}\text{Actual value of generator}\\ \text{terminal voltage}\end{array}\right\} = \text{p.u. value of voltage} \times \begin{pmatrix}\text{Base kV on}\\ \text{LT side of}\\ \text{Transformer T}\end{pmatrix}$$

$$= 1.1577\angle 9.4^\circ \times 7.217 = 8.355\angle 9.4^\circ \text{ kV}$$

1.5 NODE EQUATIONS AND BUS ADMITTANCE MATRIX

The meeting point of various components in a power system is called bus. (practically, the element used to connect one component to another is called bus). The bus or bus bar is a conductor made of copper or aluminium having negligible resistance. Hence the bus bar will have zero voltage drop when it conducts the rated current. Therefore the buses are considered as points of constant voltage in a power system.

When the power system is represented by impedance/reactance diagram, it can be considered as a circuit or network. The buses can be treated as nodes and the voltages of all buses (nodes) can be solved by conventional node analysis technique.

Let N be the number of major or jprincipal nodes in a circuit or network. (The principal nodes are meeting points of more than two elements). Since the voltage of a node can be measured only with respect to a reference point, one of the node is considered as reference node. Now the network will have (N-1) independent voltages. In nodal analysis, the independent voltages are solved by writing Kirchoff's Current Law (KCL) equation for (N-1) nodes in the circuit. For writing KCL equations, the voltage sources in the circuit should be converted to equivalent current sources.

Let, $V_1, V_2, V_3, ..., V_n$ = Node voltages of nodes 1, 2, 3 ,......, n respectively.

$I_{11}, I_{22}, I_{33}, ..., I_{nn}$ = Sum of current sources connected (or injecting current) to nodes 1, 2, 3 ,......., n respectively.

Y_{jj} = Sum of admittances connected to node-j.

Y_{jk} = Negative of sum of admittances connected between node-j and node-k.

> ***Note:*** *If the direction of current in current source is towards the node then the source is considered positive and if it is away from the node then the source is negative.*

Now the n-number of nodal equations for N-bus system will be in the form shown below (Here n = N-1)

$$Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + ... + Y_{1n}V_n = I_{11}$$

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + ... + Y_{2n}V_n = I_{22}$$

$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + ... + Y_{3n}V_n = I_{33}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad\qquad \vdots \qquad \vdots$$

$$Y_{n1}V_1 + Y_{n2}V_2 + Y_{n3}V_3 + ... + Y_{nn}V_n = I_{nn}$$

The above n-number of equations can be arranged in the matrix form as shown in equ(1.50).

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & .... & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & .... & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & .... & Y_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & Y_{n3} & .... & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_{11} \\ I_{22} \\ I_{33} \\ \vdots \\ I_{nn} \end{bmatrix} \quad .....(1.50)$$

In matrix notation the equ(1.50) can be written as,

$$\mathbf{YV = I} \quad .....(1.51)$$

In power system the Y-matrix is designated as $\mathbf{Y_{bus}}$, and called bus admittance matrix. The node voltages are called bus voltages.

Therefore the equ(1.51) can be written as shown in equ(1.52)

$$\mathbf{Y_{bus}\,V = I} \qquad .....(1.52)$$

where, $\mathbf{Y_{bus}}$ = Bus admittances matrix of order (n × n)

$\mathbf{V}$ = Bus voltage matrix of order (n × 1)

$\mathbf{I}$ = Current sources matrix of order (n × 1)

$\mathbf{n}$ = Number of independent buses in the system.

$$\text{Bus admittance matrix, } \mathbf{Y_{bus}} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \dots\dots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \dots\dots & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & \dots\dots & Y_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & Y_{n3} & \dots\dots & Y_{nn} \end{bmatrix} \qquad .....(1.53)$$

The bus admittance matrix, Y_{bus} is symmetrical around the principal diagonal. The admittances Y_{11}, Y_{22}, Y_{33} ,......, Y_{nn} are called self admittances at the buses and the other admittances are called mutual admittances.

In general,

Y_{jj} = Sum of all admittances connected to bus-j.

Y_{jk} = Negative of sum of all admittances connected between bus-j and bus-k.

Also, $Y_{jk} = Y_{kj}$

Note: *If the power system is represented by reactance diagram, then all the elements of the networkare inductive susceptances (which are negative). In this case, Y_{jj} will be negative and Y_{jk} will be positive.*

Solution of bus voltages

Consider the node basis matrix equation [equ(1.52)] of N-bus system.

$$\mathbf{Y_{bus}\,V = I} \qquad .....(1.54)$$

On premultiplying the equ(1.54) by $\mathbf{Y_{bus}^{-1}}$ we get,

$$\mathbf{V = Y_{bus}^{-1}\; I} \qquad .....(1.55)$$

we know that, $\mathbf{Y_{bus}^{-1}} = \dfrac{\text{Adjoint of } \mathbf{Y}_{\text{bus}}}{\text{Determinant of } \mathbf{Y}_{\text{bus}}}$

Let Δ = Determinant of $\mathbf{Y_{bus}}$

and Δ_{ij} = Cofactor of $\mathbf{Y_{jk}}$

$$\text{Now, Adjoint of } \mathbf{Y_{bus}} = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \dots\dots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \dots\dots & \Delta_{2n} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \dots\dots & \Delta_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ \Delta_{n1} & \Delta_{n2} & \Delta_{n3} & \dots\dots & \Delta_{nn} \end{bmatrix}^T = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \dots\dots & \Delta_{1n} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \dots\dots & \Delta_{2n} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \dots\dots & \Delta_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ \Delta_{1n} & \Delta_{2n} & \Delta_{3n} & \dots\dots & \Delta_{nn} \end{bmatrix} \qquad .....(1.56)$$

$$\text{Also, } \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} \quad \text{and} \quad \mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{bmatrix} \qquad .....(1.57)$$

Using equations (1.56) and (1.57), the equation (1.55) can be written as shown in equ(1.58).

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \cdots\cdots & \Delta_{n1} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \cdots\cdots & \Delta_{n2} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \cdots\cdots & \Delta_{n3} \\ \vdots & \vdots & \vdots & & \vdots \\ \Delta_{1n} & \Delta_{2n} & \Delta_{3n} & \cdots\cdots & \Delta_{nn} \end{bmatrix} \begin{bmatrix} I_{11} \\ I_{12} \\ I_{13} \\ \vdots \\ I_{nn} \end{bmatrix} \qquad .....(1.58)$$

By matrix multiplication the equation (1.58) can be expressed as n-number of linear independent equations shown below.

$$V_1 = \frac{1}{\Delta}[\Delta_{11}I_{11} + \Delta_{21}I_{22} + \Delta_{31}I_{33} + ... + \Delta_{n1}I_{nn}]$$

$$V_2 = \frac{1}{\Delta}[\Delta_{12}I_{11} + \Delta_{22}I_{22} + \Delta_{32}I_{33} + ... + \Delta_{n2}I_{nn}]$$

$$V_3 = \frac{1}{\Delta}[\Delta_{31}I_{11} + \Delta_{23}I_{22} + \Delta_{33}I_{33} + ... + \Delta_{n3}I_{nn}]$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$V_n = \frac{1}{\Delta}[\Delta_{1n}I_{11} + \Delta_{2n}I_{22} + \Delta_{3n}I_{33} + ... + \Delta_{nn}I_{nn}]$$

In general the k^{th} bus voltage is given by,

$$V_k = \frac{1}{\Delta}[\Delta_{1k}I_{11} + \Delta_{2k}I_{22} + \Delta_{3k}I_{33} + ... + \Delta_{nk}I_{nn}] \qquad .....(1.59)$$

$$\therefore\ V_k = \frac{1}{\Delta}\sum_{j=1}^{n} \Delta_{jk} I_{jj}$$

where, Δ = Determinant of $\mathbf{Y_{bus}}$ matrix.

I_{jj} = Sum of current sources injecting current to node-j.

Δ_{jk} = Cofactor of the element $\mathbf{Y_{jk}}$ of bus admittance matrix.

Note: *The equ(1.59) is Cramer's rule and this equation can be expressed in another simpler form as shown below.*

Let Δ = Determinant of $\mathbf{Y_{bus}}$ matrix.

Δ_k = Determinant of $\mathbf{Y_{bus}}$ matrix after replacing k^{th} column by current source vector I.

Now, k^{th} bus coltage, $V_k = \dfrac{\Delta_k}{\Delta}$

Bus or node elimination by matrix algebra

The buses or nodes which does not have any current sources can be eliminated by matrix manipulation of the standard node equation.

Consider the general form of node basis matrix equation,

$$\mathbf{Y}_{\mathbf{bus}}\mathbf{V} = \mathbf{I} \qquad .....(1.60)$$

Now the matrices in the equ(1.60) can be partitioned using the guidelines given below.

1. *The column matrices **V** and **I** are rearranged such that the element associated with buses to be eliminated are in the lower rows of the matrices.*
2. *The square matrix $\boldsymbol{Y}_{bus}$ is rearranged such that the elements associated with buses to be eliminated are in the last rows and columns of the matrices.*
3. *The bus admittance matrix is partitioned so that elements identified only with nodes to be eliminated are separated from the other elements by horizontal and vertical lines.*

Consider a bus admittance matrix of order (5×5). Now the matrix equ(1.60) for n = 5 can be written as shown in equ(1.61). Let us assume that the buses 4 and 5 does not have any current source and so they can be eliminated.

$$\left[\begin{array}{ccc|cc} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} \\ \hline Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ \hline V_4 \\ V_5 \end{array}\right] = \left[\begin{array}{c} I_{11} \\ I_{22} \\ I_{33} \\ \hline I_{44} \\ I_{55} \end{array}\right] \qquad .....(1.61)$$

Let us partition the matrix equ(1.61) and define the following submatrices.

$$\mathbf{K} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \quad \mathbf{L} = \begin{bmatrix} Y_{14} & Y_{15} \\ Y_{24} & Y_{25} \\ Y_{34} & Y_{35} \end{bmatrix} \quad \mathbf{L}^{\mathbf{T}} = \begin{bmatrix} Y_{41} & Y_{42} & Y_{43} \\ Y_{51} & Y_{52} & Y_{53} \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} Y_{44} & Y_{45} \\ Y_{54} & Y_{55} \end{bmatrix} \quad \mathbf{V}_A = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad \mathbf{V}_X = \begin{bmatrix} V_4 \\ V_5 \end{bmatrix} \quad \mathbf{I}_A = \begin{bmatrix} I_{11} \\ I_{22} \\ I_{33} \end{bmatrix} \quad \mathbf{I}_X = \begin{bmatrix} I_{44} \\ I_{55} \end{bmatrix}$$

where, $\mathbf{I}_X$ = Submatrix composed of the currents entering the buses to be eliminated.

$\mathbf{V}_X$ = Submatrix composed of the voltages of the buses to be eliminated.

K = Submatrix composed of self and mutual admittances identified only with buses to be retained.

M = Submatrix composed of self and mutual admittances identified only with buses to be eliminated.

L = Submatrix composed of only mutual admittances between buses to be retained and eliminated.

$\mathbf{L}^{\mathbf{T}}$ = Transpose of **L**.

Using the submatrices as defined above, the matrix equation (1.61) can be written as shown in equ (1.62).

$$\begin{bmatrix} \mathbf{K} & \mathbf{L} \\ \mathbf{L}^{T} & \mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{V}_A \\ \mathbf{V}_X \end{bmatrix} = \begin{bmatrix} \mathbf{I}_A \\ \mathbf{I}_X \end{bmatrix} \qquad .....(1.62)$$

By matrix multiplication the equ(1.62) can be written as,

$$\mathbf{K\,V_A + LV_X = I_A} \qquad .....(1.63)$$

$$\mathbf{L^T V_A + MV_X = I_X} \qquad .....(1.64)$$

From equ(1.64) we get,

$$\mathbf{M\,V_X \;=\; I_X - L^T V_A} \qquad .....(1.65)$$

Here all the elements of the submatrix I_X are zero, because the buses to be eliminated does not have any current source.

$$\therefore\ \mathbf{M\,V_X \;= - L^T V_A} \qquad .....(1.66)$$

On premultiplying equ(1.66) by M^{-1} we get,

$$\mathbf{V_X \;= - M^{-1}\, L^T\; V_A} \qquad .....(1.67)$$

On substituting for V_X from equ (1.67) in equ(1.63) we get,

$$\mathbf{K\,V_A - L\,M^{-1}\,L^T\;V_A = I_A}$$

$$\therefore\ \mathbf{[\,K - L\,M^{-1}\,L^T\,]\;V_A = I_A}$$

$$\mathbf{Y_{bus,\,new}\;V_A = I_A} \qquad .....(1.68)$$

where, $\mathbf{Y_{bus,\,new} = K - L\,M^{-1}\,L^T}$ .....(1.69)

The new bus admittance matrix $\mathbf{Y_{bus,\,new}}$ used to reconstruct the circuit with the unwanted buses eliminated.

The matrix partitioning method discussed above is a general procedure and it is suitable for computer solutions. When large number of buses are to be eliminated, then the size of matrix **M** will be large and finding $\mathbf{M^{-1}}$ will be tedious.

The process of bus elimination can be simplified if one bus is eliminated at a time. In this case the matrix **M** will have only one element and $\mathbf{M^{-1}}$ is the reciprocal of this element. Here, the bus to be eliminated must be the highest numbered bus and renumbering may be required. This can be achieved by row interchange and column interchange. When bus-p has to be eliminated in a system with n-independent buses, the pth row is interchanged with nth row and pth column is interchanged with nth column in the bus admittance matrix..

Consider a bus admittance matrix of order (n×n), in which the nth bus has to be eliminated

$$\mathbf{Y_{bus}} = \left[\begin{array}{cccc|c}
\multicolumn{4}{c|}{\overbrace{\hspace{6cm}}^{\mathbf{K}}} & \overbrace{\hspace{1cm}}^{\mathbf{L}} \\
Y_{11} & Y_{12} & .... & Y_{1(n-1)} & Y_{1n} \\
Y_{21} & Y_{22} & .... & Y_{2(n-1)} & Y_{2n} \\
\vdots & \vdots & & \vdots & \vdots \\
Y_{(n-1)1} & Y_{(n-2)2} & .... & Y_{(n-1)(n-1)} & Y_{(n-1)n} \\
\hline
Y_{n1} & Y_{n2} & .... & Y_{n(n-1)} & Y_{nn} \\
\multicolumn{4}{c|}{\underbrace{\hspace{6cm}}_{\mathbf{L^T}}} & \underbrace{\hspace{1cm}}_{\mathbf{M}}
\end{array}\right] \qquad .....(1.70)$$

Let us partition the bus admittance matrix as shown in equ(1.70). The submatrices are given below.

$$\mathbf{K} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots\cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots\cdots & Y_{2(n-1)} \\ \vdots & \vdots & & \vdots \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots\cdots & Y_{(n-1)(n-1)} \end{bmatrix}; \quad \mathbf{L} = \begin{bmatrix} Y_{1n} \\ Y_{2n} \\ \vdots \\ Y_{(n-1)n} \end{bmatrix}$$

$$\mathbf{L}^T = \begin{bmatrix} Y_{n1} & Y_{n2} & \cdots\cdots & Y_{n(n-1)} \end{bmatrix}; \quad \mathbf{M} = Y_{nn}\,; \quad \mathbf{M}^{-1} = \frac{1}{Y_{nn}}$$

From equ(1.69) we get,

$$\mathbf{Y}_{bus,new} = K - \mathbf{L}\,\mathbf{M}^{-1}\,\mathbf{L}^T \qquad .....(1.71)$$

On substituting the submatrices in equ(1.71) we get,

$$\mathbf{Y}_{bus,new} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots\cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots\cdots & Y_{2(n-1)} \\ \vdots & \vdots & & \vdots \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots\cdots & Y_{(n-1)(n-1)} \end{bmatrix} - \begin{bmatrix} Y_{1n} \\ Y_{2n} \\ \vdots \\ Y_{(n-1)n} \end{bmatrix} \begin{bmatrix} \frac{1}{Y_{nn}} \end{bmatrix} \begin{bmatrix} Y_{n1} & Y_{n2} & \cdots\cdots & Y_{n((n-1))} \end{bmatrix}$$

$$\mathbf{Y}_{bus,new} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots\cdots & Y_{1(n-1)} \\ Y_{21} & Y_{22} & \cdots\cdots & Y_{2(n-1)} \\ \vdots & \vdots & & \vdots \\ Y_{(n-1)1} & Y_{(n-2)2} & \cdots\cdots & Y_{(n-1)(n-1)} \end{bmatrix} - \begin{bmatrix} \frac{1}{Y_{nn}} \end{bmatrix} \begin{bmatrix} Y_{n1}Y_{n1} & Y_{1n}Y_{n2} & \cdots\cdots & Y_{1n}Y_{n(n-1)} \\ Y_{2n}Y_{n1} & Y_{2n}Y_{n2} & \cdots\cdots & Y_{2n}Y_{(n-1)} \\ \vdots & \vdots & & \vdots \\ Y_{(n-1)n}Y_{n1} & Y_{(n-1)n}Y_{n2} & \cdots\cdots & Y_{(n-1)n}Y_{n(n-1)} \end{bmatrix} \qquad .....(1.72)$$

From equation (1.72) the element Y_{jk} (i.e. the element in row-k and column-j) of the resulting $(n-1) \times (n-1)$ new bus admittance matrix is given by

$$\mathbf{Y}_{jk,new} = Y_{jk} - \frac{Y_{jn}Y_{nk}}{Y_{nn}} \qquad .....(1.73)$$

for j = 1, 2, 3 ,...., (n−1) and k = 1, 2, 3 ,...., (n−1).

whereY_{jk}, $Y_{jn,}$ Y_{nk} and Y_{nn} are elements of original or given bus admittance matrix.

The equ(1.73) can be used to compute the elements of new bus admittance matrix directly from the elements of original bus admittance matrix.

The following procedure can be used to compute an element of bus admittance matrix using equ(1.73).

Step 1: Consider an element Y_{jk} of original bus admittance matrix.

Step 2: Get the product of last element of row-j and last element of column-k.

Step 3: Divide the product obtained in step-2 by Y_{nn} of original bus admittance matrix.

Step 4: On substracting the resultant value obtained in step-3 from Y_{jk} of original bus admittance matrix we get, the new value of Y_{jk}.

Note: *Since bus admittance matrix is symmetrical we can take $Y_{kj,\,new} = Y_{jk,\,new}$ and can avoid m(m-1)/2 calculations for a new bus admittance matrix of order (mxm), where m = n-1.*

EXAMPLE 1.13

Solve the node voltages V_1 and V_2 in the network shown in fig 1.13.1. The voltages and impedances are in p.u.

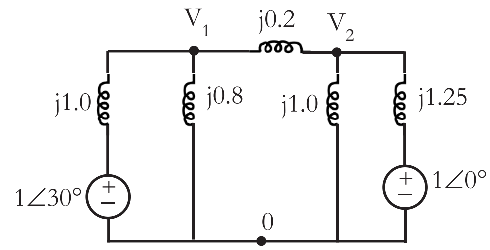


Fig 1.13.1.

SOLUTION

The voltage sources in the network can be converted to current sources by source transformation as shown in fig 1.13.2.

The node basis matrix equation is formed as shown below.

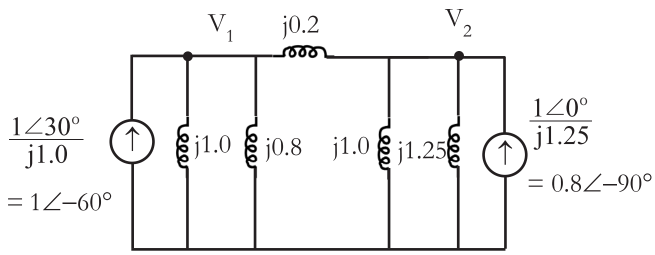


Fig 1.13.2.

$$\begin{bmatrix} \frac{1}{j1}+\frac{1}{j0.8}+\frac{1}{j0.2} & -\frac{1}{j0.2} \\ -\frac{1}{j0.2} & \frac{1}{j0.2}+\frac{1}{j1}+\frac{1}{j1.25} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} \frac{1\angle 30^\circ}{j1} \\ \frac{1\angle 0^\circ}{j1.25} \end{bmatrix}$$

$$\begin{bmatrix} -j1-j1.25-j5 & j5 \\ j5 & -j5-j-j0.8 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} \frac{1\angle 30^\circ}{j1} \\ \frac{1\angle 0^\circ}{1.25\angle 90^\circ} \end{bmatrix}$$

$$\begin{bmatrix} -j7.25 & j5 \\ j5 & -j6.8 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} 1\angle -60^\circ \\ 0.8\angle -90^\circ \end{bmatrix}$$

$$\Delta=\begin{vmatrix} -j7.25 & j5 \\ j5 & -j6.8 \end{vmatrix}=(-j7.25)\,(-j6.8)-(j5)(j5)$$

$$= j^2\,(7.25\times 6.8)-j^2 25=-(7.25\times 6.8)+25=-24.3$$

In node basis analysis, k^{th} node voltage V_k, is given by,

$$V_k=\frac{1}{\Delta}\sum_{j=1}^{n}\Delta_{jk}\,I_{jj}$$

where, Δ_{jk} = Cofactor of Y_{jk}

Δ = Determinant of Y

I_{jj} = Sum of current sources connected to node j.

Here $n = 2$

$$\therefore\ V_1=\frac{1}{\Delta}[\Delta_{11}I_{11}+\Delta_{21}I_{22}]$$

$$=\frac{-1}{24.3}[(-j6.8\times 1\angle -60^\circ)+(-j5\times 0.8\angle -90^\circ)]$$

$$= -\frac{1}{24.3}\left[(6.8\angle-90^{\circ}\times 1\angle-60^{\circ})+(5\angle-90^{\circ}\times 0.8\angle-90^{\circ})\right]$$

$$= -\frac{1}{24.3}\left[(6.8\angle-150^{\circ}+4\angle-180^{\circ})\right] = -\frac{1}{24.3}\left[-5.88-j3.4-4\right]$$

$$= \frac{-9.88-j3.4}{-24.3} = \frac{9.88}{24.3}+j\frac{3.4}{24.3} = 0.4066+j0.1399 = 0.43\angle 19^{\circ}\text{ p.u.}$$

$$V_2 = \frac{1}{\Delta}\left[\Delta_{12}I_{11}+\Delta_{22}I_{22}\right]$$

$$= \frac{-1}{24.3}\left[(-j5\times 1\angle-60^{\circ})+(-j7.25\times 0.8\angle-90^{\circ})\right]$$

$$= -\frac{1}{24.3}\left[(5\angle-90^{\circ}\times 1\angle-60^{\circ})+(7.25\angle-90^{\circ}\times 0.8\angle-90^{\circ})\right]$$

$$= \frac{5\angle-150^{\circ}+5.8\angle-180^{\circ}}{-24.3} = \frac{-4.33-j2.5-5.8}{-24.3}$$

$$= \frac{-10.13-j2.5}{-24.3} = \frac{10.13}{24.3}+j\frac{25}{24.3} = 0.4169+j0.1029 = 0.43\angle 14^{\circ}\text{ p.u.}$$

EXAMPLE 1.14

For the network shown in fig 1.14.1. form the bus admittance matrix. Determine the reduced admittance matrix by eliminating node 4. The values are marked in p.u.

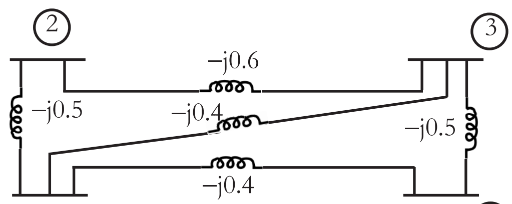


Fig 1.14.1.

SOLUTION

The $\mathbf{Y_{bus}}$ matrix of the network is,

$$\mathbf{Y_{bus}} = \begin{bmatrix} -(j0.5+j0.4+j0.4) & j0.5 & j0.4 & j0.4 \\ j0.5 & -(j0.5+j0.6) & j0.6 & 0 \\ j0.4 & j0.6 & -(0.6+j0.5+j0.4) & j0.5 \\ j0.4 & 0 & j0.5 & -(j0.5+j0.4) \end{bmatrix}$$

$$\mathbf{Y_{bus}} = \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

The elements of new bus admittance matrix after eliminating the 4th row and 4th column is given by

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn}\,Y_{nk}}{Y_{nn}}\ ; \text{ where } n = 4\ ;\ j = 1, 2, 3 \text{ and } k = 1, 2, 3,$$

The bus admittance matrix is symmetrical , $\therefore\ Y_{kj,\,new} = Y_{jk,\,new}$

$$Y_{11,new} = Y_{11} - \frac{Y_{14}Y_{41}}{Y_{44}} = -j1.3 - \frac{(j0.4)(j0.4)}{-j0.9} = -j1.12$$

$$Y_{12,new} = Y_{12} - \frac{Y_{14}Y_{42}}{Y_{44}} = j0.5 - \frac{(j0.4\times 0)0}{-j0.9} = j0.5$$

$$Y_{13,new} = Y_{13} - \frac{Y_{14}Y_{43}}{Y_{44}} = j0.4 - \frac{(j0.4)(j0.5)}{-j0.9} = j0.622$$

$$Y_{21,new} = Y_{12,new} = j0.5$$

$$Y_{22,new} = Y_{22} - \frac{Y_{24}Y_{42}}{Y_{44}} = -j1.1 - \frac{(0)(0)}{-j0.9} = -j1.1$$

$$Y_{23,new} = Y_{23} - \frac{Y_{24}Y_{43}}{Y_{44}} = j0.6 - \frac{(0)(j0.5)}{-j0.9} = j0.6$$

$$Y_{31,new} = Y_{13,new} = j0.622$$

$$Y_{32,new} = Y_{23,new} = j0.6$$

$$Y_{33,new} = Y_{33} - \frac{Y_{34}Y_{43}}{Y_{44}} = -j1.5 - \frac{(j0.5)(j0.5)}{-j0.9} = j1.222$$

The reduced bus admittance matrix after eliminating 4th row is shown below.

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j1.12 & j0.5 & j0.622 \\ j0.5 & -j1.1 & j0.6 \\ j0.622 & j0.6 & -j1.222 \end{bmatrix}$$

EXAMPLE 1.15

Eliminate buses 3 and 4 in the given bus admittance matrix and form new bus admittance matrix.

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j9.8 & 0.0 & j4.0 & j5.0 \\ 0.0 & -j8.3 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j14 & j8.0 \\ j5.0 & j5.0 & j8.0 & -j18.0 \end{bmatrix}$$

SOLUTION

First let us eliminate 4th bus, $\therefore Y_{nn} = Y_{44} = -j18.0$

The elements of new bus admittance after eliminating 4th row and 4th column is given by

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn}\,Y_{nk}}{Y_{nn}} \;; \text{ where } n = 4 \;; j = 1, 2, 3 \text{ and } k = 1, 2, 3,$$

$$Y_{11,new} = Y_{11} - \frac{Y_{14}Y_{41}}{Y_{44}} = -j9.8 - \frac{j5.0 \times j5.0}{-j18.0} = -j8.411$$

$$Y_{12,new} = Y_{12} - \frac{Y_{14}Y_{42}}{Y_{44}} = 0.0 - \frac{j5.0 \times j5.0}{-j18.0} = j1.388$$

$$Y_{13,new} = Y_{13} - \frac{Y_{14}Y_{43}}{Y_{44}} = j4.0 - \frac{j5 \times j8}{-j18.0} = j6.222$$

$$Y_{21,new} = Y_{12,new} = j1.388$$

$$Y_{22,new} = Y_{22} - \frac{Y_{24}Y_{42}}{Y_{44}} = -j8.3 - \frac{j5 \times j5}{-j18.0} = -j6.911$$

$$Y_{23,new} = Y_{23} - \frac{Y_{24}Y_{43}}{Y_{44}} = j2.5 - \frac{j5 \times j8}{-j18} = j4.722$$

$$Y_{31,new} = Y_{13,new} = j6.222$$

$$Y_{32,new} = Y_{23,new} = j4.722$$

$$Y_{33,new} = Y_{33} - \frac{Y_{34}Y_{43}}{Y_{44}} = -j14 - \frac{j8 \times j8}{-j18.0} = -j10.444$$

The reduced bus admittance matrix after eliminating 4th node is given by

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j8.411 & j1.3889 & j6.222 \\ j1.388 & -j6.911 & j4.722 \\ j6.222 & j4.722 & -j10.444 \end{bmatrix}$$

Elimination of node 3 : $Y_{nn} = Y_{33} = -j10.444$

The other elements of the reduced bus admittance matrix can be formed from the equation

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn}\,Y_{nk}}{Y_{nn}} \;;\; \text{where } n = 3\;;\; j = 1, 2, \quad \text{and} \quad k = 1, 2,$$

$$Y_{11,new} = Y_{11} - \frac{Y_{13}Y_{31}}{Y_{33}} = -j8.411 - \frac{j6.222 \times j6.222}{-j10.444} = -j4.7043$$

$$Y_{12,new} = Y_{12} - \frac{Y_{13}Y_{32}}{Y_{33}} = j1.388 - \frac{j6.222 \times j4.722}{-j10.444} = j4.2011$$

$$Y_{21,new} = Y_{12,new} = j4.2011$$

$$Y_{22,new} = Y_{22} - \frac{Y_{23}Y_{32}}{Y_{33}} = -j6.911 - \frac{j4.722 \times j4.722}{-j10.444} = -j4.7761$$

The reduced bus admittance matrix after eliminating node 3 and 4 is

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j4.7043 & j4.2011 \\ j4.2011 & -j4.7761 \end{bmatrix}$$

EXAMPLE 1.16

Determine the bus admittance matrix of the system whose reactance diagram is shown in fig 1.16.1. The currents and admittances are given in p.u. Determine the reduced bus admittance matrix after eliminating node-3.

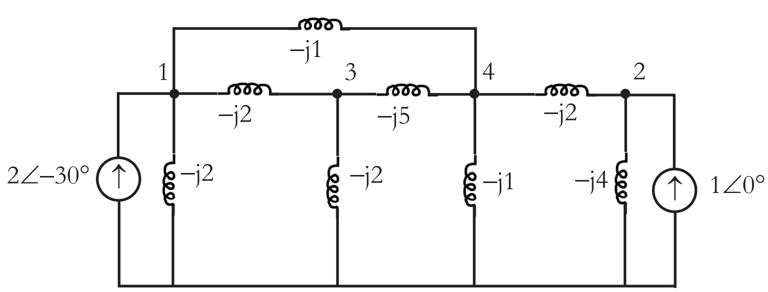


Fig 1.16.1.

SOLUTION

The bus admittance matrix can be formed by inspection using the following guidelines.

1. The diagonal element Y_{jj} is given by sum of all the admittances connected to node-j.
2. The off-diagonal element Y_{jk} is given by negative of the sum of all the admittances connected between node-j and node-k.

$$\therefore\ \mathbf{Y}_{bus} = \begin{bmatrix} -j2-j2-j1 & 0 & j2 & j1 \\ 0 & -j2-j4 & 0 & j2 \\ j2 & 0 & -j2-j2-j5 & j5 \\ j1 & j2 & j5 & -j1-j5-j2-j1 \end{bmatrix}$$

$$\therefore\ \mathbf{Y}_{bus} = \begin{bmatrix} -j5 & 0 & j2 & j1 \\ 0 & -j6 & 0 & j2 \\ j2 & 0 & -9 & j5 \\ j1 & j2 & j5 & -j9 \end{bmatrix} \quad .....(1.16.1)$$

For eliminating node-3, the bus admittance matrix is rearranged by interchanging row-3 & row-4, and then interchanging column-3 & column - 4.

After interchanging row - 3 & row-4 of $\mathbf{Y}_{bus}$ matrix of equ (1.16.1) we get,

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j5 & 0 & j2 & j1 \\ 0 & -j6 & 0 & j2 \\ j1 & j2 & j5 & -j9 \\ j2 & 0 & -j9 & j5 \end{bmatrix} \quad .....(1.16.2)$$

After interchanging column - 3 & column - 4 of $\mathbf{Y}_{bus}$ matrix of equ (1.16.2) we get,

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j5 & 0 & j1 & j2 \\ 0 & -j6 & j2 & 0 \\ j1 & j2 & -j9 & j5 \\ j2 & 0 & j5 & -j9 \end{bmatrix} \quad .....(1.16.3)$$

Now the last row and last column [i.e., 4^{th} row and 4^{th} column] of $\mathbf{Y}_{bus}$ matrix of equ(1.16.3) can be eliminated.

The elements of new bus admittance matrix after eliminating 4^{th} row and 4^{th} column is given by

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn}\,Y_{nk}}{Y_{nn}}\ ;\ \text{where n = 4 ; j = 1, 2, 3 and k = 1, 2, 3,}$$

$$Y_{11,new} = Y_{11} - \frac{Y_{14}Y_{41}}{Y_{44}} = -j5 - \frac{(j2)\,(j2)}{-j9} = -j4.556$$

$$Y_{12,new} = Y_{12} - \frac{Y_{14}Y_{42}}{Y_{44}} = 0 - \frac{j2 \times 0}{-j9} = 0$$

$$Y_{13,new} = Y_{13} - \frac{Y_{14}Y_{43}}{Y_{44}} = j1 - \frac{(j2)(j5)}{-j9} = j2.111$$

$$Y_{21,new} = Y_{12,new} = 0$$

$$Y_{22,new} = Y_{22} - \frac{Y_{24}Y_{42}}{Y_{44}} = -j6 - \frac{0 \times 0}{-j9} = -j6$$

$$Y_{23,new} = Y_{23} - \frac{Y_{24}Y_{43}}{Y_{44}} = j2 - \frac{0 \times j5}{-j9} = j2$$

$$Y_{31,new} = Y_{13,new} = j2.111$$

$$Y_{32,new} = Y_{23,new} = j2$$

$$Y_{33,new} = Y_{33} - \frac{Y_{34}Y_{43}}{Y_{44}} = -j9 - \frac{j5 \times j5}{-j9} = -j6.222$$

The reduced bus admittance matrix after eliminating bus-3 is given by,

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j4.556 & 0 & j2.111 \\ 0 & -j6 & j2 \\ j2.111 & j2 & -j6.222 \end{bmatrix}$$

1.6 BUS IMPEDANCE MATRIX

Consider the node basis matrix equation representing the power system

$$\mathbf{Y}_{\text{bus}}\,\mathbf{V} = \mathbf{I} \qquad \text{......(1.74)}$$

On premultiplying the equ (1.74) by $\mathbf{Y}_{\text{bus}}^{-1}$ we get,

$$\mathbf{V} = \mathbf{Y}_{\text{bus}}^{-1}\,\mathbf{I} \qquad \text{......(1.75)}$$

Now the elements of $\mathbf{Y}_{\text{bus}}^{-1}$ will be impedances and so the matrix $\mathbf{Y}_{\text{bus}}^{-1}$ can be represenred by an impedance matrix called bus impedance matrix, $\mathbf{Z}_{\text{bus}}$.

$$\mathbf{Z}_{\text{bus}} = \mathbf{Y}_{\text{bus}}^{-1} \qquad \text{......(1.76)}$$

$$\text{and } \mathbf{V} = \mathbf{Z}_{\text{bus}}\,\mathbf{I} \qquad \text{......(1.77)}$$

Since the bus admittance matrix is symmetrical, the bus impedance matrix is also symmetrical around the principal diagonal. In bus impedance matrix, the elements on the main diagonal are called driving point impedance of the buses or nodes and the off-diagonal elements are called the transfer impedances of the buses or nodes. The bus impedance matrix is very useful in fault analysis or calculations.

Note : *The equ (1.77) resembles a mesh basis matrix equation but it is not so,because the matrix* ***V*** *is a node voltage matrix and the matrix* ***I*** *is a current source matrix.*

The bus impedance matrix can be determined by two methods. In one method we can form the bus admittance matrix and then taking its inverse to get the bus impedance matix. In another method the bus impedance matrix can be directly formed from the reactance diagram and this method requires the knowledge of the modifications of existing bus impedance matrix due to addition of new bus or addition of new line (or impedance) between existing buses.

Modification of an existing bus impedance matrix

Let us denote the orginal $\mathbf{Z}_{\text{bus}}$ of a system with n- number of independent buses as $\mathbf{Z}_{\text{orig}}$. When a branch of impedance $\mathbf{Z}_b$ is added to the system the $\mathbf{Z}_{\text{orig}}$ gets modified. The branch impedance $\mathbf{Z}_b$ can be added to the original system in the following four different ways.

Case 1 : *Adding a branch of impedance Z_b from a new bus-p to the reference bus.*

Case 2 : *Adding a branch of impedance Z_b from a new bus-p to an existing bus-q.*

Case 3 : *Adding a branch of impedance Z_b from an existing bus-q to the reference bus.*

Case 4 : *Adding a branch of impedance Z_b between two existing buses h and q.*

The modification of $\mathbf{Z}_{\text{orig}}$ for the above four cases have been presented here without proof

Case 1 : Adding Z_b from a new bus-p to the reference bus.

Consider a n-bus system as shown in fig 1.9. Let us add a bus-p through an impedance Z_b to the reference bus. The addition of a bus will increase the order of the bus impedance.

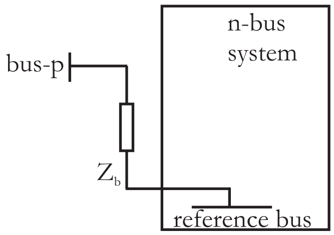


Fig 1.9 : Adding a new bus through an impedance to reference bus.

$$Z_{bus,new} = \left[\begin{array}{cccc|c} & & & & 0 \\ & & & & 0 \\ & Z_{orig} & & & \vdots \\ & & & & 0 \\ \hline 0 & 0 & \ldots\ldots & 0 & Z_b \end{array}\right] \qquad \ldots\ldots(1.78)$$

In this case the elements of $(n + 1)^{th}$ column and row are all zeros except the diagonal. The diagonal element is the added branch impedance Z_b. The elements of original Z_{bus} matrix are not altered. The new bus impedance matrix will be as shown in equ (1.78).

Case 2 : Adding Z_b from a new bus-p to an existing bus-q.

Consider a n-bus system as shown in fig 1.10. in which a new bus-p is added through an impedance Z_b to an existing bus-q. The addition of a bus will increase the order of the bus impedance matrix by one.

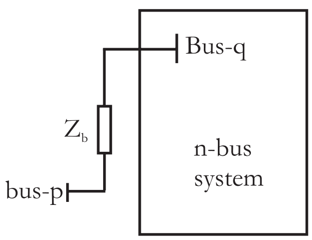


Fig 1.10 : Adding a new bus through an impedance to an existing bus.

$$Z_{bus,new} = \left[\begin{array}{cccc|c} & & & & Z_{1q} \\ & & & & Z_{2q} \\ & Z_{orig} & & & \vdots \\ & & & & Z_{nq} \\ \hline Z_{q1} & Z_{q2} & \ldots\ldots & Z_{qn} & Z_{qq} + Z_b \end{array}\right] \qquad \ldots\ldots(1.79)$$

In this elements of $(n + 1)^{th}$ column are the elements of q^{th} column and elements of $(n + 1)^{th}$ row are the elements q^{th} row. The diagonal element is given by sum of Z_{qq} and Z_b. The elements of original Z_{bus} matrix are not altered. The new bus impedance matrix will be as shown in equ (1.79).

Case 3 : Adding Z_b from an existing bus-q to the reference bus

Consider a n-bus system shown in fig 1.11 in which an impedance Z_b is added from an existing bus-q to the reference bus. Let us consider as if the impedance Z_b is connected from a new bus-p and existing bus-q. Now it will be an addition as that of case-2. The new bus impedance matrix order (n + 1) can be formed as that of case-2. Then we can short-circuit the bus-q to reference bus. This is equivalent to eliminating $(n + 1)^{th}$ bus (i.e., bus-p in this case) and so the bus impedance matrix has to be modified by eliminating $(n + 1)^{th}$ row and $(n + 1)^{th}$ column. The reduced bus impedance matric can be formed by a procedure similar to that of bus elimination in bus admittance matrix, developed in section 1.5. This reduced bus impedance matrix is the actual new bus impedance matrix. Every element of actual new bus impedance matrix can be determined using the equation (1.80).

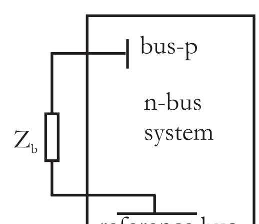


Fig 1.11 : Adding an impedance between existing bus and reference.

$$\mathbf{Z}_{jk,act} = \mathbf{Z}_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad \ldots\ldots(1.80)$$

Note: *1. $Z_{jk,act}$ is the impedance corresponding to row-j and column-k of actual new bus impedance matrix.*

2. Z_{jk}, $Z_{(n+1)k}$, $Z_{j(n+1)}$, $Z_{(n+1)(n+1)}$ are impedance of new bus impedance matrix of order (n + 1).

3. Since bus impedance matix is symmetrical

$$Z_{jk,act} = Z_{kj,act}$$

<u>Case 4 : Adding Z_b between two existing buses h and q</u>

Consider a n-bus system shown in fig 1.12, in which an impedance Z_b is added between two existing buses h and q.

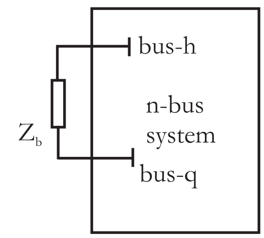


Fig 1.12 : *Adding an impedance between bus-h and bus-q.*

In this case the bus impedance matrix is formed as shown in equ(1.81). Here the elements of $(n+1)^{th}$ column is the difference between the elements of column-h and column-q. The elements of $(n+1)^{th}$ row is the difference between the elements of row-h and row-q. The diagonal element is given by equ(1.82).

$$Z_{bus,new} = \left[\begin{array}{c|c} Z_{orig} & \begin{matrix} Z_{1h} - Z_{1q} \\ Z_{2h} - Z_{2q} \\ \vdots \\ Z_{nh} - Z_{nq} \end{matrix} \\ \hline Z_{h1} - Z_{q1} \; Z_{h2} - Z_{q2} \; \ldots\ldots Z_{hn} - Z_{qn} & Z_{(n+1)(n+1)} \end{array}\right] \qquad ....(1.81)$$

$$Z_{(n+1)(n+1)} = Z_b + Z_{hh} + Z_{qq} - 2\,Z_{hq} \qquad ....(1.82)$$

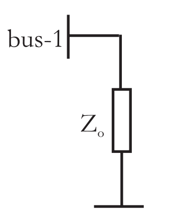
Since the modification does not involve addition of new bus, the order of new bus impedance matrix has to be reduced to nxn by eliminating the $(n + 1)^{th}$ column and $(n + 1)^{th}$ row.

This reduced bus impedance matrix is the actual new bus impedance matrix.Every element of this actual new bus impedance matrix can be determined using equ (1.80) which is also given below for reference.

$$\mathbf{Z}_{jk,act} = \mathbf{Z}_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

<u>Direct dtemination of a bus impedance matriux</u>

The bus impedance matrix can be directly obtained from impedance or reactance diagram instead of forming $\mathbf{Y}_{bus}$ and then inverting it. In direct determination of $\mathbf{Z}_{bus}$ first consider an impedance Z_a connected from bus-1 to reference bus. Now the bus impedance matrix will have a single elements as shown below.



$$\mathbf{Z}_{bus} = [Z_a]$$

Then add each elemnt of impedance or reactance diagram one by one and modify the $\mathbf{Z}_{bus}$ in each step. Each modification of $\mathbf{Z}_{bus}$ involve any one of the four cases discussed above.

EXAMPLE 1.17

Determine $\mathbf{Z}_{bus}$ for system whose reactance diagram is shown in fig 1.17.1. where the impedance is given in p.u. preserve all the three nodes.

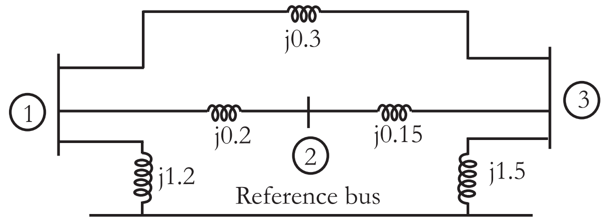


Fig 1.17.1

SOLUTION

Step 1: Consider the branch with impedance j1.2 p.u. connected between bus-1 and reference as shown in fig 1.17.2. The system shown in fig 1.17.2. has a single bus and so the order of bus impedance matrix is on, as shown in below.

$$\mathbf{Z}_{bus} = [j1.2]$$

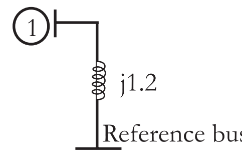


Fig 1.17.2

Step 2: Connect bus-2 to bus-1 through an impedance j0.2 as shown in fig 1.17.3. This is case-2 modification and so the order of bus impedance matrix increases by one. In the new bus impedance matrix, the elements of 1st column are copied as elements of 2nd column and the elements of 1st row are copied as elements of 2nd row.The diagonal elements is given by $Z_{11} + Z_b$ where $Z_b = j0.2$.

$$\therefore\ \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.2 + j0.2 \end{bmatrix} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.4 \end{bmatrix}$$

Step 3: Connect the bus -3 to bus-2 through an impedance j0.15 as shown in fig 1.17.4. This is case-2 modification and so the order of the bus impedance matrix increases by one. In the new bus impedance matrix the elements of 2nd row are copied as elements of 3rd row. The diagonal element is given by $Z_{22} + Z_b$ where $Z_b = j0.15$.

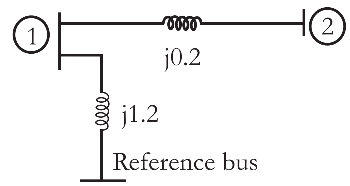


Fig 1.17.3

$$\therefore\ \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.4 + j0.15 \end{bmatrix}$$

$$= \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix}$$

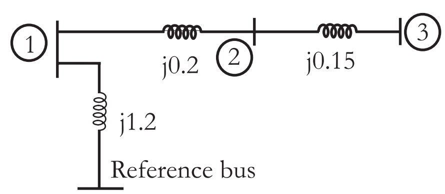


Fig 1.17.4

Step 4 : Connect the impedance j1.5 from bus-3 to reference bus as shown in fig 1.17.5. This is case-3 modification. In case-2 and then the last row and column are eliminated by node elimination techniques.

In new bus impedance matrix the elements of 3rd column are copied as elements of 4th column and the elements of 3rd row are copied as elements of 4th row. The diagonal element is given by $Z_{33} + Z_b$ where $Z_b = j1.5$.

$$\therefore\ \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j1.55 + j1.5 \end{bmatrix} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j3.05 \end{bmatrix}$$

The actual new bus impedance matrix is obtained by eliminating the 4th row and 4th column. The element $\mathbf{Z}_{jk}$ of the actual new bus impedance matrix is given by,

$$\therefore\ \mathbf{Z}_{jk,act} = \mathbf{Z}_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad \text{where } n = 3\ ;\ j = 1, 2, 3 \ \text{ and } k = 1, 2, 3$$

$$Z_{11,act} = Z_{11} - \frac{Z_{14}\,Z_{41}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.2}{j3.05} = j0.728$$

$$Z_{12,act} = Z_{12} - \frac{Z_{14}\,Z_{42}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.4}{j3.05} = j0.649$$

$$Z_{13,act} = Z_{13} - \frac{Z_{14}\,Z_{43}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.55}{j3.05} = j0.590$$

$$Z_{21,act} = Z_{12,act} = j0.649$$

$$Z_{22,act} = Z_{22} - \frac{Z_{24}\,Z_{42}}{Z_{44}} = j1.4 - \frac{j1.4 \times j1.4}{j3.05} = j0.757$$

$$Z_{23,act} = Z_{23} - \frac{Z_{24}\,Z_{43}}{Z_{44}} = j1.4 - \frac{j1.4 \times j1.55}{j3.05} = j0.689$$

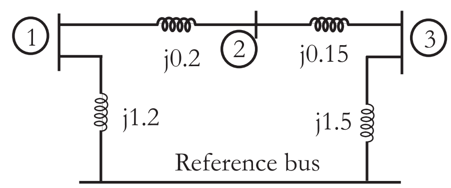


Fig 1.17.5

$$Z_{31,act} = Z_{13,act} = j0.590$$

$$Z_{32,act} = Z_{23,act} = j0.689$$

$$Z_{33,act} = Z_{33} - \frac{Z_{34}\,Z_{43}}{Z_{44}} = j1.55 - \frac{j1.55 \times j1.55}{j3.05} = j0.762$$

$$\therefore\ \mathbf{Z}_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 \\ j0.649 & j0.757 & j0.689 \\ j0.590 & j0.689 & j0.762 \end{bmatrix}$$

Step 5: Connect the impedance j0.3 between bus-1 and bus-3 as shown in fig 1.17.6. This is case-4 modification.

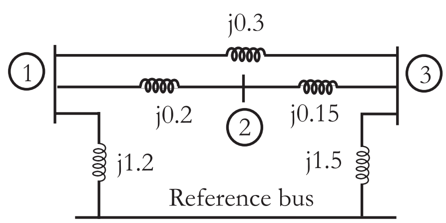


Fig 1.17.6

In new bus impedance matrix, the elements of 4th column are obtained by substracting the elements of 3rd column from 1st column and the elements of 4th row are obtained by substracting the elements of 3rd row from 1st row.The diagonal element Z_{44} is given by the following equation.

$$Z_{44} = Z_b + Z_{11} + Z_{33} - 2Z_{13}$$

where $Z_b = j0.3$

$$\therefore\ Z_{44} = j0.3 + j0.728 + j0.762 - 2(j0.59) = j0.61$$

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.728 - j0.59 \\ j0.649 & j0.757 & j0.689 & j0.649 - j0.689 \\ j0.59 & j0.689 & j0.762 & j0.59 - j0.762 \\ j0.728 - j0.59 & j0.649 - j0.689 & j0.59 - j0.762 & j0.61 \end{bmatrix}$$

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.138 \\ j0.649 & j0.757 & j0.689 & -0.04 \\ j0.59 & j0.689 & j0.762 & -j0.172 \\ j0.138 & -0.04 & -j0.172 & j0.61 \end{bmatrix}$$

Since this modification does not add a new bus, the 4th row and column has to be eliminated using node elimination technique, to determine the actual new bus impedance matrix. The element Z^{jk} of actual new bus impedance matrix is given by.

$$\therefore\ \mathbf{Z}_{jk,act} = \mathbf{Z}_{jk} - \frac{Z_{j(n+1)}Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad \text{where } n = 3\ ;\ j = 1, 2, 3 \text{ and } k = 1, 2, 3$$

$$Z_{11,act} = Z_{11} - \frac{Z_{14}\,Z_{41}}{Z_{44}} = j0.728 - \frac{j0.138 \times j0.138}{j0.61} = j0.697$$

$$Z_{12,act} = Z_{12} - \frac{Z_{14}\,Z_{42}}{Z_{44}} = j0.649 - \frac{j0.138 \times (-j0.04)}{j0.61} = j0.658$$

$$Z_{13,act} = Z_{13} - \frac{Z_{14}\,Z_{43}}{Z_{44}} = j0.59 - \frac{j0.138 \times (-j0.172)}{j0.61} = j0.629$$

$$Z_{21,act} = Z_{12,act} = j0.658$$

$$Z_{22,act} = Z_{22} - \frac{Z_{24}\,Z_{42}}{Z_{44}} = j0.757 - \frac{(-j0.04)(-j0.04)}{j0.61} = j0.754$$

$$Z_{23,act} = Z_{23} - \frac{Z_{24}\,Z_{43}}{Z_{44}} = j0.689 - \frac{(-j0.04)\,(-j0.172)}{j0.61} = j0.678$$

$$Z_{31,act} = Z_{13,act} = j0.629$$

$$Z_{32,act} = Z_{23,act} = j0.678$$

$$Z_{33,act} = Z_{33} - \frac{Z_{34}\,Z_{43}}{Z_{44}} = j0.762 - \frac{(-j0.172)\,(-j0.172)}{j0.61} = j0.714$$

$$\therefore\ \mathbf{Z}_{bus} = \begin{bmatrix} j0.697 & j0.658 & j0.629 \\ j0.658 & j0.754 & j0.678 \\ j0.629 & j0.678 & j0.714 \end{bmatrix}$$

EXAMPLE 1.18

Find the bus impedance matrix for the system whose reactance diagram is shown in fig 1.18.1. All the impedance are in p.u.

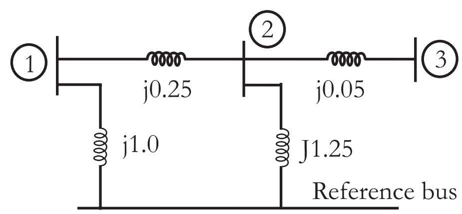


Fig 1.18.1

SOLUTION

Step 1 : Consider the branch with impedance j1 p.u. connected between bus-1 and refence as shown in fig 1.18.2

The system shown in fig 1.18.2. has a single bus and so the order of bus impedance matrixis one, as shown below.

$$Z_{bus} = [j1.0]$$

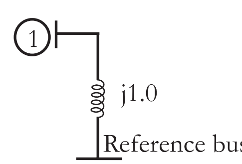


Fig 1.18.2

Step 2 : Connect bus -2 to bus-1 through an impedance j0.25 as shown in fig 1.18.3. This is case-2 modification and so the order of bus impedance matrix increases by one.

In the new bus impedance matrix, the elemenets of 1st column are copied as elements of 2nd column and the elements of 1st row are copied as elements of 2nd row. The diagonal element is given by $Z_{11} + Z_b$ where $Z_b = j0.25$.

$$\therefore\ \mathbf{Z}_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.0 + j0.25 \end{bmatrix} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.25 \end{bmatrix}$$

Step 3 : Connect the impedance j1.25 from bus-2 to reference bus as shown in fig 1.18.4. This is case-3 modification. In case-3 modification thenew bus impedance matrix is formed as that of case-2 and then the last row and column are eliminated by node elimination technique.

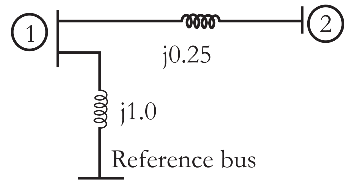


Fig 1.18.3

In the new bus impedance matrix the elements of 2nd column are copied as elements of 3rd column and the elements of 2nd row are copied as elements of 3rd row. The diagonal element is given by $Z_{22} + Z_b$ where Z_b = j1.25

$$\therefore\ \mathbf{Z}_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j1.25 + j1.25 \end{bmatrix}$$

$$= \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.5 \end{bmatrix}$$

The actual new bus impedance matrix is obtaned by eliminating the 3rd row and 3rd column. The element Z_{jk} of the actual new bus impedance matrix is given by,

$$\therefore\ \mathbf{Z}_{jk,act} = \mathbf{Z}_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad \text{where n = 2 ; j = 1, 2, and k = 1, 2,}$$

$$Z_{11,act} = Z_{11} - \frac{Z_{13}\,Z_{31}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.0}{j2.5} = j0.6$$

$$Z_{12,act} = Z_{13} - \frac{Z_{13}\,Z_{32}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.25}{j2.5} = j0.5$$

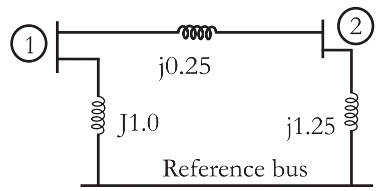


Fig 1.18.4

$$Z_{21,act} = Z_{12,act} - j0.5$$

$$Z_{22,act} = Z_{22} - \frac{Z_{23}\,Z_{32}}{Z_{33}} = j1.25 - \frac{j1.25 \times j1.25}{j2.5} = j0.625$$

$$\therefore\ \mathbf{Z}_{bus} = \begin{bmatrix} j0.6 & j0.5 \\ j0.5 & j0.625 \end{bmatrix}$$

Step 4 : Connect the bus-3 to bus-2 through an impedance j0.05 as shown in fig 1.18.5. This is case-2 modification and so the order of the bus impedance matrix increases by one.

In the new bus impedance matrix, the elements of 2nd column are copied as elements of 3rd column and the elements of 2nd row are copied as elements of 3rd row. The diagonal elements is given by Z_{22} + Z_b where Z_b = j0.5.

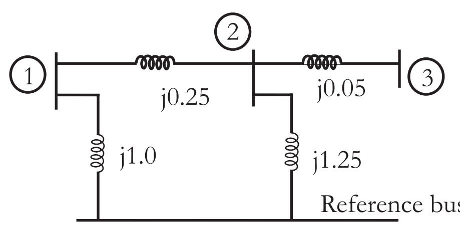


Fig 1.18.5

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.625 + j0.05 \end{bmatrix}$$

$$\therefore\ \mathbf{Z}_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.675 \end{bmatrix}$$

1.7 SYMMETRICAL COMPONENTS

The analysis of unsymmetrical polyphase network by the method of symmetrical components was introduced by *Dr. C. Fortesque.* He proved that an unbalanced system of *n* related vectors can be resolved into n system of balanced vectors called symmetrical components of original vectors. The *n* vectors of each set of components are equal in length and the phase angles between adjacent vectors of the set are equal.

In three phase system, the three unbalanced vectors [either V_a, V_b & V_c or I_a, I_b & I_c] can be resolved into three balanced system of vectors. The vectors of the balanced system are called symmetrical components of the original system. The symmetrical components of three phase system are,

1. Positive sequence components
2. Negative sequence components
3. Zero sequence components.

The positive sequence components consists of three vectors equal in magnitude, displaced form each other by 120^o in phase, and having the same phase sequence as the original vectors.

The negative sequence components consists of three vectors equal in magnitude, displaced form each other by 120^o in phase, and having the phase sequence opposite to that of the original vectors.

The zero sequence components consists of three vectors equal in magnitude and with zero phase displacement from each other.

Let V_{a1}, V_{b1} & V_{c1} = Positive sequence components of V_a, V_b & V_c respectively with phase sequence abc.

V_{a2}, V_{b2} & V_{c2} = Negative sequence components of V_a, V_b & V_c respectively with phase sequence acb.

V_{a0}, V_{b0} & V_{c0} = Zero sequence components of V_a, V_b & V_c respectively.

The vector diagrams of positive, negative and zero sequence components are shown in fig 1.13.

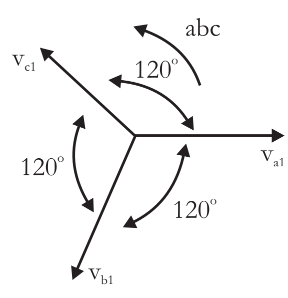


Fig a : *Positive sequence components*

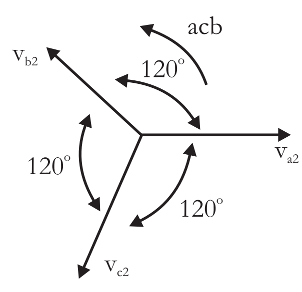


Fig b : *Negative sequence components*

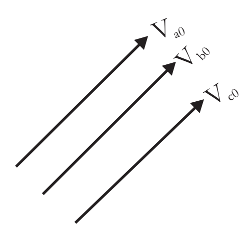


Fig c : *Zero sequence components*

Fig 1.13 : *Vector diagram of symmetrical components of unbalanced 3-phase voltage vectors.*

From the vector diagram of symmetrical components the following conclusions can be made.

1. On rotating the vector V_{a1} by 120^o in anticlockwise direction we get V_{c1}
2. On rotating the vector V_{a1} by 120^o in anticlockwise direction we get V_{b1}

3. On rotating the vector V_{a2} by 120° in anticlockwise direction we get V_{b2}
4. On rotating the vector V_{a2} by 240° in anticlockwise direction we get V_{c2}

Therefore, on rotating the symmetrical component of one vector by 120° or multiples of 120° we get the symmetrical components of other vectors. Hence we can define an operator which causes a rotation of 120° in the anticlockwise direction, such an operator is denoted by the letter "**a**".

The operator "**a**" is defined as,

$$a = 1\angle 120^o = 1\, e^{+j2\pi/3} = \cos 2\pi/3 + j \sin 2\pi/3 = -0.5 + j0.866$$

Since, $a = 1\angle 120^o = -0.5 + j0.866$ ......(1.83)

$$a^2 = 1\angle 240^o = -0.5 - j0.866 \qquad ......(1.84)$$

$$a^3 = 1\angle 360^o = 1 \qquad ......(1.85)$$

$$1 + a + a^2 = 1 + (-0.5 + j0.866) + (-0.5 - j0.866) = 0$$

$$\therefore \; 1 + a + a^2 = 0 \qquad ......(1.86)$$

Computation of unbalanced vectors from their symmetrical components

Each of the original unbalanced vector is the sum of its positive, negative and zero sequence component. Therefore the original unbalanced three phase voltage vectors can be expressed in terms of therir components as shown below.

$$V_a = V_{a0} + V_{a1} + V_{a2} \qquad ......(1.87)$$

$$V_b = V_{b0} + V_{b1} + V_{b2} \qquad ......(1.88)$$

$$V_c = V_{c0} + V_{c1} + V_{c2} \qquad ......(1.89)$$

From the vector diagram shown in figure (1.11), we get the following relations between various symmetrical components.

$$V_{b0} = V_{a0}\,; \qquad V_{b1} = a^2 V_{a1}\,; \qquad V_{b2} = aV_{a2} \qquad ......(1.90)$$

$$V_{c0} = V_{a0}\,; \qquad V_{c1} = aV_{a1}\,; \qquad V_{c2} = a^2 V_{a2} \qquad ......(1.91)$$

Using equations (1.90) and (1.91), the equations (1.87) to (1.89) can be written as shown below.

$$V_a = V_{a0} + V_{a1} + V_{a2} \qquad ......(1.92)$$

$$V_b = V_{a0} + a^2\, V_{a1} + a\, V_{a2} \qquad ......(1.93)$$

$$V_c = V_{a0} + a\, V_{a1} + a^2\, V_{a2} \qquad ......(1.94)$$

The equation (1.90) and (1.91) can be arranged in the matrix form as shown in equ (1.95).

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \qquad ......(1.95)$$

The equation (1.95) can be used to compute the unbalanced voltage vectors from the knowledge of symmetrical components.

Computation of symmetrical components of unbalanced vectors

The matrix equation (1.95) can be written in the vector notification as shown in equ(1.96).

$$\mathbf{V} = \mathbf{A}\,\mathbf{V}_{sy} \qquad ......(1.96)$$

$$\text{where, } \mathbf{V} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}; \quad \mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \quad \text{and} \quad \mathbf{V}_{sy} = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

On premultiplying the equ(1.96) by $\mathbf{A}^{-1}$ we get

$$\mathbf{A}^{-1}\mathbf{V} = \mathbf{V}_{sy}$$

$$\therefore\ \mathbf{V}_{sy} = \mathbf{A}^{-1}\mathbf{V} \qquad ......(1.97)$$

$$\mathbf{A}^{-1} = \frac{\text{Adjoint of } \mathbf{A}}{\text{Determinant of } \mathbf{A}}$$

Let Δ = Determinant of A

$$\therefore\ \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a & a^2 \end{vmatrix} = 1\times(a^4 - a^2) - 1\times(a^2 - a) + 1(a - a^2)$$

$$= (a - a^2) + (a - a^2) + (a - a^2) = 3(a - a^2)$$

$$\boxed{\begin{pmatrix} \because a^4 = a^2.a = a \\ a^3 = 1 \end{pmatrix}}$$

Let Δ_{jj} = Cofactor of A_{ij}

$$\therefore\ \Delta_{11} = a^4 - a^2 = a - a^2 \qquad \Delta_{23} = -(a-1) = 1 - a$$

$$\Delta_{12} = -(a^2 - a) = a - a^2 \qquad \Delta_{31} = a - a^2$$

$$\Delta_{13} = a - a^2 \qquad \Delta_{32} = -(a-1) = 1 - a$$

$$\Delta_{21} = -(a^2 - a) = a - a^2 \qquad \Delta_{33} = a^2 - 1$$

$$\Delta_{22} = a^2 - 1$$

$$\therefore\ \mathbf{A}^{-1} = \frac{1}{\Delta}\text{ adjoint of } \mathbf{A} = \frac{1}{\Delta}\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{bmatrix}^T$$

$$= \frac{1}{\Delta}\begin{bmatrix} \Delta_{11} & \Delta_{21} & \Delta_{31} \\ \Delta_{12} & \Delta_{22} & \Delta_{32} \\ \Delta_{13} & \Delta_{23} & \Delta_{33} \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{3(a - a^2)}\begin{bmatrix} a - a^2 & a - a^2 & a - a^2 \\ a - a^2 & a^2 - 1 & 1 - a \\ a - a^2 & 1 - a & a^2 - 1 \end{bmatrix}^T$$

$$\mathbf{A}^{-1} = \frac{(a-a^2)}{3(a-a^2)}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \frac{a^2-1}{a-a^2} & \frac{1-a}{a-a^2} \\ 1 & \frac{1-a}{a-a^2} & \frac{a^2-1}{a-a^2} \end{bmatrix} \qquad ......(1.98)$$

$$\frac{a^2-1}{a-a^2} = \frac{a^2-a^3}{a-a^2} = \frac{a(a-a^2)}{a-a^2} = a \qquad ......(1.99)$$

$$\frac{1-a}{a-a^2} = \frac{a^2(1-a)}{a^2(a-a^2)} = \frac{a^2(1-a)}{(a^3-a^4)} = \frac{a^2(1-a)}{1-a} = a^2 \qquad ......(1.100)$$

Note: $a^3 = 1$; $a^4 = a^3 \,.\, a = a$

Using equations (1.99) and (1.100) the equation (1.98) can be written as

$$\mathbf{A^{-1}} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \qquad ......(1.101)$$

On substituting for A^{-1} from equ(1.101) in equ(1.97) we get

$$\mathbf{V}_{sy} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\mathbf{V}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad ......(1.102)$$

The matrix equ(1.102) can be expressed as three independent linear equations as shown below.

$$\therefore \quad V_{a0} = \frac{1}{3}[V_a + V_b + V_c] \qquad ......(1.103)$$

$$V_{a1} = \frac{1}{3}[V_a + a\,V_b + a^2\,V_c] \qquad ......(1.104)$$

$$V_{a2} = \frac{1}{3}[V_a + a^2\,V_b + a\,V_c] \qquad ......(1.105)$$

The equations(1.102) to (1.105) can be used to compute the symmetrical components of the unbalanced voltages.

Symmetrical components of unbalanced current vectors

The symmetrical components of unbalanced current vectors can be obtained by an analysis similar to that of voltage vectors. All the equations developed for voltages can be used for current if we replace **V** by **I**.

Let I_a, I_b, I_c = Unbalanced current vectors with phase sequence abc

I_{a1}, I_{b1}, I_{c1} = Positive sequence components of I_a,I_b and I_c respectively with phase sequence abc.

I_{a2}, I_{b2}, I_{c2} = Negative sequence components of I_a,I_b and I_c respectively with phase sequence acb.

I_{a0}, I_{b0}, I_{c0} = Zero sequence components of I_a, I_b and I_c respectively.

The vector diagram of positive, negative and zero sequence components are shown in fig 1.14.

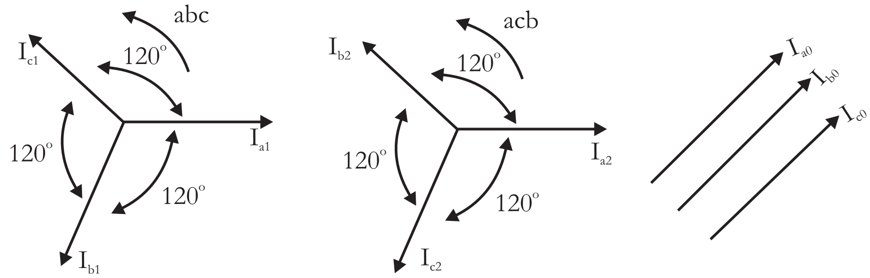


Fig a : *Positive sequence components*

Fig b : *Negative sequence components*

Fig c : *Zero sequence components*

Fig 1.14 : *Vector diagram of symmetrical components of unbalanced 3-phase current vectors.*

The following equations are used to compute the unbalanced current vectors from the knowledge of their symmetrical components [Refer equations(1.92) to (1.95)].

$$I_a = I_{a0} + I_{a1} + I_{a2} \quad ......(1.106)$$

$$I_b = I_{a0} + a^2 I_{a1} + a\, I_{a2} \quad ......(1.107)$$

$$I_c = I_{a0} + a\, I_{a1} + a^2 I_{a2} \quad ......(1.108)$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{b1} \\ I_{c2} \end{bmatrix} \quad ......(1.109)$$

The following equations are used to compute the symmetrical components of unbalanced current vectors [Refer equations (1.102) to (1.105)].

$$I_{a0} = \frac{1}{3}\,[I_a + I_b + I_c] \quad ......(1.110)$$

$$I_{a1} = \frac{1}{3}\,[I_a + a\, I_b + a^2 I_c] \quad ......(1.111)$$

$$I_{a2} = \frac{1}{3}\,[I_a + a^2 I_b + a\, I_c] \quad ......(1.112)$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad ......(1.113)$$

EXAMPLE 1.19

The voltages across a 3 phase unbalanced load are $V_a = 300\angle 20°V$, $V_b = 360\angle 90°V$ and $V_c = 500\angle -140°\ V$ Determine the symmetrrical components of voltages, phase sequenceis abc.

SOLUTION

The symmetrical components of Va are given by the following matrix equations.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\therefore \quad V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$$

$$V_{a1} = \frac{1}{3}\left[V_a + a\,V_b + a^2\,V_c\right]$$

$$V_{a2} = \frac{1}{3}\left[V_a + a^2\,V_b + a\,V_c\right]$$

Given that $V_a = 300\angle 20^\circ V = 281.91 + j102.61\ V$

$$V_b = 360\angle 90^\circ V = 0 + j360\ V$$

$$V_c = 500\angle -140^\circ V = -383.02 - j321.39\ V$$

$$\therefore \quad aV_b = 1\angle 120^\circ \times 360\angle 90^\circ = 360\angle 210^\circ = -311.77 - j180\ V$$

$$a^2V_b = 1\angle 240^\circ \times 360\angle 90^\circ = 360\angle 330^\circ = 311.77 - j180\ V$$

$$aV_c = 1\angle 240^\circ \times 500\angle -140^\circ = 500\angle -20^\circ = 469.85 - j171.01\ V$$

$$a^2V_c = 1\angle 240^\circ \times 500\angle -140^\circ = 500\angle 100^\circ = -86.82 + j492.40\ V$$

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c] = \frac{1}{3}(281.91 + j102.61 + 0 + j360 - 383.02 - j321.39)$$

$$= \frac{1}{3}(-10.111 + j141.22) = -33.70 + j47.07 = 57.89\angle 126^\circ\ V$$

$$V_{a1} = \frac{1}{3}[V_a + aV_b + a^2V_c] = \frac{1}{3}(281.91 + j102.61 - 311.77 - j180 - 86.82 + j492.40)$$

$$= \frac{1}{3}(-116.68 + j415.01) = -38.89 + j138.34 = 143.0\angle 106^\circ\ V$$

$$V_{a2} = \frac{1}{3}[V_a + a^2V_b + aV_c] = \frac{1}{3}(281.91 + j102.61 + 311.77 - j180 + 469.85 - j171.01)$$

$$= \frac{1}{3}(1063.53 - j248.40) = 354.51 - j82.80 = 364.05\angle -13^\circ\ V$$

We know that $V_{a0} = V_{b0} = V_{c0}$

$\therefore$ The Zero sequence components are

$$V_{a0} = 57.89\angle 126^\circ V$$

$$V_{b0} = 57.89\angle 126^\circ V$$

$$V_{c0} = 57.89\angle 126^\circ V$$

we know that, $V_{b1} = a^2\,V_{a1}$; $V_{c1} = a\,V_{a1}$

$\therefore$ The positive sequence components are

$$V_{a1} = 143.70\angle 106^\circ\ V$$

$$V_{b1} = a\,V_{a2} = 1\angle 240^\circ \times 143.70\angle 106^\circ = 143.70\angle 346^\circ\ V$$

$$V_{c1} = a\,V_{a1} = 1\angle 120^\circ \times 143.70\angle 106^\circ = 143.70\angle 226^\circ\ V$$

we know that, $V_{b2} = a\,V_{a2}$; $V_{c2} = a^2\,V_{a2}$

$\therefore$ The negative sequence components are

$$V_{a2} = 364.05 \angle -13^\circ \text{ V}$$

$$V_{b2} = a\,V_{a2} = 1\angle 120^\circ \times 364.05 \angle -13^\circ = 364.05 \angle 107^\circ \text{ V}$$

$$V_{c2} = a^2\,V_{a2} = 1 < 240^\circ \times 364.05 < -13^\circ = 364.05 < 227^\circ \text{ V}$$

EXAMPLE 1.20

The symmetrical components of phase-a voltage in a 3-phase unbalanced system are $V_{a0} = 10\angle 180^\circ\text{V}$, $V_{a1} = 50\angle 0^\circ\text{V}$ and $V_{a2} = 20\angle 90^\circ \text{ V}$. Determine the phase voltages V_a, V_b and V_c.

SOLUTION

The phase voltages of V_a, V_b and V_c are given by the following matrix equations.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\therefore\ V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + a^2\,V_{a1} + a\,V_{a2}$$

$$V_c = V_{a0} + a\,V_{a1} + a^2\,V_{a2}$$

Given that $V_{a0} = 10\angle 180^\circ\text{V} = -10 + j0$

$$V_{a1} = 50\angle 0^\circ\text{V} = 50 + j0$$

$$V_{a2} = 20\angle 90^\circ\text{V} = 0 + j20$$

$$\therefore\ aV_{a1} = 1\angle 120^\circ \times 50\angle 0^\circ = 50\angle 120^\circ = -25 + j43.30$$

$$a^2V_{a1} = 1\angle 240^\circ \times 50\angle 0^\circ = 50\angle 240^\circ = -25 - j43.30$$

$$aV_{a2} = 1\angle 120^\circ \times 20\angle 90^\circ = 20\angle 210^\circ = -17.32 - j10$$

$$a^2V_{a2} = 1\angle 240^\circ \times 20\angle 90^\circ = 20\angle 330^\circ = 17.32 - j10$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = -10 + 50 + j20 = 40 + j20 = 44.72\angle 27^\circ \text{ V}$$

$$V_b = V_{a0} + a^2\,V_{a1} + a\,V_{a2} = -10 - 25 - j43.30 - 17.32 - j10$$

$$= -52.32 - j53.30 = 74.69 \angle -134^\circ \text{ V}$$

$$V_c = V_{a0} + a\,V_{a1} + a^2\,V_{a2} = -10 - 25 + j43.30 + 17.32 - j10$$

$$= -17.68 + j33.3 = 37.70\angle 118^\circ \text{ V}$$

EXAMPLE 1.21

The Symmetrical components of phase-a faulit current in a 3-phase unbalanced system are $I_{a0} = 350\angle 90^o$ A, $I_{a1} = 600\angle -90^o$ A and $I_{a2} = 250\angle 90^o$ A. Determine the phase currents I_a, I_b, and I_c .

SOLUTION

The currents I_a, I_b, and I_c are given by the following matrix equations.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\therefore\ I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

Given that $I_{a0} = 350\angle 90^o = 0 + j350$

$$I_{a1} = 600\angle -90^o = 0 - j600$$

$$I_{a2} = 250\angle 90^o = 0 + j250$$

$$\therefore\ aI_{a1} = 1\angle 120^o \times 600\angle -90^o = 600\angle 30^o = 519.62 + j300$$

$$a^2 I_{a1} = 1\angle 240^o \times 600\angle -90^o = 600\angle 150^o = -519.62 + j300$$

$$aI_{a2} = 1\angle 120^o \times 250\angle 90^o = 250\angle 210^o = -216.51 - j125$$

$$a^2 I_{a2} = 1\angle 240^o \times 250\angle 90^o = 250\angle 330^o = 216.51 - j125$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = j350 - j600 + j250 = 0$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2} = j350 - 519.62 + j300 - 216.51 - j125$$

$$= -736.13 + j525 = 904.16\angle 145^o \text{ A}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2} = j350 + 519.62 + j300 + 216.51 - j125$$

$$= 736.13 + j525 = 904.16\angle 35^o \text{ A}$$

EXAMPLE 1.22

Determine the symmetrical components of the unbalanced three phase currents $I_{a0} = 10\angle 0^o$ A, $I_{a1} = 12\angle 230^o$ A and $I_{a2} = 10\angle 130^o$ A.

SOLUTION

The symmetrical components of Ia given by the following matrix equations.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\therefore\ I_{a0} = \frac{1}{3}(I_a + I_b + I_c)$$

$$I_{a1} = \frac{1}{3}\left(I_a + a\,I_b + a^2\,I_c\right)$$

$$I_{a2} = \frac{1}{3}\left(I_a + a^2\,I_b + a\,I_c\right)$$

Given that $I_a = 10\angle 0^o = 10 + j0$

$$I_b = 12\angle 230^o = -7.71 - j9.19$$

$$I_c = 10\angle 130^o = -6.43 + j7.66$$

$$\therefore\ aI_b = 1\angle 120^o \times 12\angle 230^o = 12\angle 350^o = 11.82 - j2.08$$

$$a^2 I_b = 1\angle 240^o \times 12\angle 470^o = 12\angle 110^o = -4.10 + j11.28$$

$$a\,I_c = 1\angle 120^o \times 10\angle 130^o = 10\angle 250^o = -3.42 - j9.40$$

$$a^2\,I_c = 1\angle 240^o \times 10\angle 130^o = 10\angle 370^o = 10\angle 10^o = 9.85 + j1.74$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}(10 - 7.71 - j9.19 - 6.43 + j7.66)$$

$$= \frac{1}{3}(-4.14 - j1.53) = -1.38 - j0.51 = 1.47\angle -160^o$$

$$I_{a1} = \frac{1}{3}(I_a + a\,I_b + a^2\,I_c) = \frac{1}{3}(10 + 11.82 - j2.08 + 9.85 + j1.74)$$

$$= \frac{1}{3}(31.67 - j0.34) = 10.56 - j0.11 = 10.56\angle -0.6^o \approx 10.56\angle 0^o$$

$$I_{a2} = \frac{1}{3}(I_a + a^2\,I_b + a\,I_c) = \frac{1}{3}(10 - 4.10 + j11.28 - 3.42 - j9.40)$$

$$= \frac{1}{3}(2.48 + j1.88) = 0.83 + j0.63 = 1.04\angle 37^o$$

We know that $I_{a0} = I_{b0} = I_{c0}$

$\therefore$ The Zero sequence components are

$$I_{a0} = 1.47\angle -160^o\ A$$

$$I_{b0} = 1.47\angle -160^o\ A$$

$$I_{c0} = 1.47\angle -160^o\ A$$

we know that, $I_{b1} = a^2\,I_{a1}$; $I_{c1} = a\,I_{a1}$

$\therefore$ The positive sequence components are

$$I_{a1} = 10.56\angle 0^o\ A$$

$$I_{b1} = a^2 I_{a1} = 1\angle 240^o \times 10.56\angle 0^o = 10.56\angle 240^o\ A$$

$$I_{c1} = a\,I_{a1} = 1\angle 120^o \times 10.56\angle 0^o = 10.56\angle 120^o\ A$$

we know that, $I_{b2} = a\,I_{a2}$; $I_{c2} = a^2\,I_{a2}$

$\therefore$ The negative sequence components are

$$I_{a2} = 1.04\angle 37^o\ A$$

$$I_{b2} = a\,I_{a2} = 1\angle 120^o \times 1.04\angle 37^o = 1.04\angle 157^o\ A$$

$$I_{c2} = a^2\,I_{a2} = 1\angle 240^o \times 1.04\angle 37^o = 1.04\angle 277^o\ A$$

1.8 SEQUENCE IMPEDANCE AND SEQUENCE NETWORKS

The sequence impedance are impedance offered by the circuit elements (or power system components) to positive, negative, and zero sequence currents. In any element of a circuit, the voltage drop caused by current of a certain sequence depends on the impedance of the element to that sequence current.

The impedance of a circuit element when positive sequence currents alone are flowing is called the positive sequence impedance. Similarly, when only negative sequence currents are present, the impedance is called negative sequence impedance. When only zero sequence currents are present the impedance is called zero sequence impedance. The impedance of any element of a balanced circuit to current of one sequence may be different from impedance to current of another sequence.

The single phase equivalent circuit of power system (impedance or reactance diagram) formed using the impedances of any one sequence only is called the sequence network for that particular sequence. Therefore the impedance or reactance diagram formedusing positive sequence impedance is called positive sequence network. Similarly the impedance or reactance diagram formed using negative sequence impedance is called negative sequence network. The impedance or reactance diagram formed using zero sequence impedance is called zero sequence network.

The sequence impedances and networkes are useful in the analysis of unsymmetrical faults in the power system. In unsymmetrical fault analysis of a power system, the positive, negative and zero sequence networks of the system are determined and then they are interconnected to represent the various unbalanced fault conditions. Each sequence network includes the generated emfs and impedances of like sequence. Also, the sequence network carries only the current of like sequence.

Sequence Impedances and networks of generator

Consider the three phase equivalent circuit of generator shown in fig 1.15. The neutral of the generator is grounded through a reactance, Z_n.

When the generator is delivering a balanced load or under symmetrical fault the neutral current is zero.But when the generator is delivering an unbalanced load or during unsymmetrical faults the neutral current flows through Z_n.

The generator is designed to supply balanced three phase voltages.Therefore the generated emfs are of positive sequence only.

Let E_a, E_b, E_c = Generated emf per phase in phase a, b and c respectively. (Positive sequence emf)

Z_1 = Positive sequence impedance per phase of generator.

Z_2 = Negative sequence impedance per phase of generator.

Z_{g0} = Zero sequence impedance per phase of generator.

Z_n = Neutral reactance.

Z_0 = Total zero sequence impedance per phase of zero-sequence network of generator.

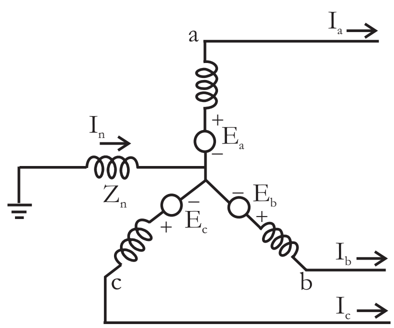


Fig 1.15 : *Three phase equivalent circuit of a generator grounded through a reactance*

The positive sequence network consists of an emf in series with positive sequence impedance of the generator. The negative and zero sequence network will not have any sources but include their respective sequence impedance. The positive, negative and zero sequence current paths are shown in fig 1.16. The positive, negative and zero sequence networks of the generator are shown in fig 1.17.

> ***Note :*** *the positive and negative sequence currents are balanced currents and so they will not pass through neutral reactance.*

The reactances in positive sequence network is subtransient, transient or synchronous reactance depending on whether subtransient, transient or steady state conditions are being studied. Under no load condition the emf E_a is the induced emf per phase. Under load or fault condition E_a is replaced by E'_a for transient state and E_a is replaced by E''_g for subtransient state.

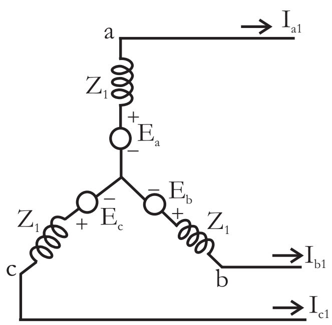


Fig (a) : *Positive-sequence current paths*

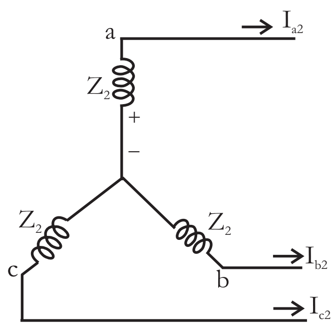


Fig (b) : *Negative-sequence current paths.*

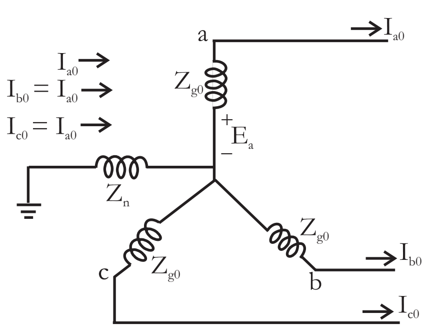
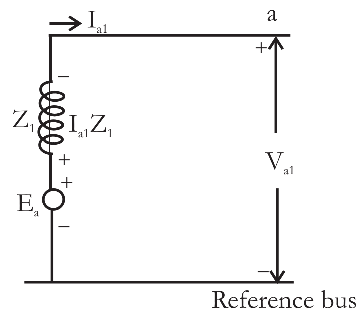
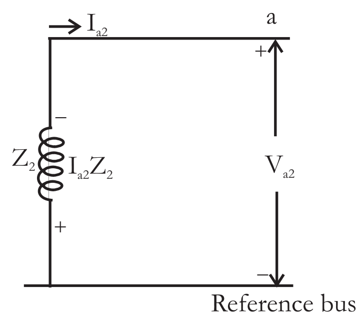


Fig (c) : *Zero-sequence current paths.*

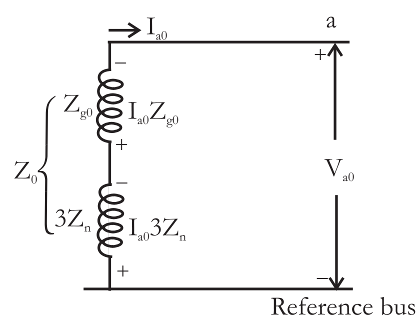
Fig 1.16 : *Sequence current paths in a generator*



Fig(a) : *Positive-sequence network*



Fig(a) : *Negative-sequence network*



Fig(c) : *Zero-sequence network*

Fig 1.17 : *Sequence networks of a generator*

On examining the zero sequence current paths (refer fig 1.16-c) we can say that the current through neutral reactance is $3\ I_{a0}$. The zero-sequence voltage drop from point-a to ground is $-3\ I_{ao}\ Z_n - I_{a0}\ Z_{g0}$.

The zero sequence network is a single phase network and assumed to carry only the zero sequence current of one phase. Hence the zero sequence current of one phase, must have an impedance of $3\ Z_n + Z_{g0}$.

$$\left.\begin{array}{l}\therefore \text{ Total zero sequence impedance per phase} \\ \text{of a generator grounded through reactance}\end{array}\right\} Z_0 = 3\ Z_n + Z_{g0} \qquad ......(1.114)$$

With reference to fig 1.17, the equations for the phase-a component voltage are,

$$V_{a1} = E_a - I_{a1}\ Z_1 \qquad ......(1.115)$$

$$V_{a2} = -\ I_{a2}\ Z_2 \qquad ......(1.116)$$

$$V_{a3} = -\ I_{a0}\ Z_0 \qquad ......(1.117)$$

The zero sequence network of generator when the neutral is solidly grounded (i.e. directly grounded) and when the neutral is ungrounded are shown in fig 1.18 and fig 1.19 respectively. In these cases there is no change in positive and negative sequence network.

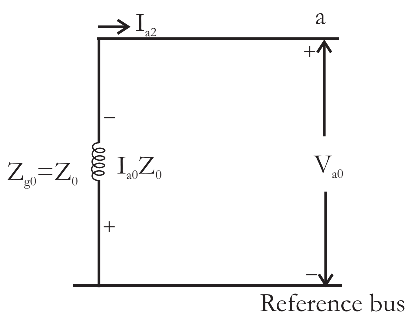


Fig 1.18 : *Zero-sequence network of a generator when the neutral is solidly grounded*

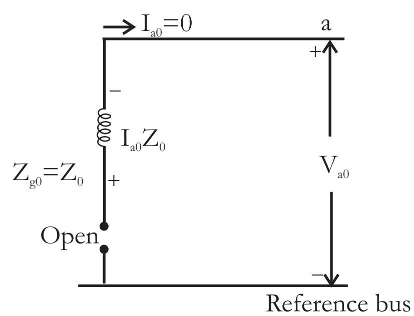


Fig 1.19 : *Zero-sequence network of a generator when the neutral is ungrounded*

> ***Note :*** *The sequence networks of synchronous motor is same as that of generator when the directions of currents in the sequence network of generator are reversed.*

Sequence impedance and networks of transmission lines

The impedance per phase of transmission line for balanced currents in independent of phase sequence. This is due to the symmetry of transposed transmission lines. Therefore, the impedances offered by the transposed transmission lines for positive and negative sequence currents are identical.

The zero sequence current is identical (both in magnitude and phase) in each phase conductor and returns through the ground, through overhead ground wires or through both. The ground wires being grounded at several towers, the return currents in the ground wire may not be uniform along the entire length of transmission line. But for positive or negative sequence currents there is no return current and they have a phase difference of 120^o. Therefore the magnetic field due to zero sequence current is different from the magnetic field caused by either positive or negative sequence current. Due to the difference in the magnetic field, the zero sequence inductive reactance is 2 to 3.5. times the positive sequence reactance.

Let, Z_1 = Positive sequence impedance of transmission line

Z_2 = Negative sequence impedance of transmission line

Z_3 = Zero sequence impedance of transmission line

The positive, negative and zero sequence impedances of transmission lines are represented as a series impedance in their respective sequence networks as shown in fig 1.20.

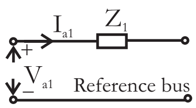


Fig a : *Positive sequence network*

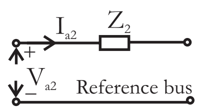


Fig b : *Negative sequence network*

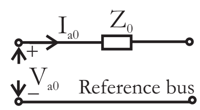


Fig c : *Zero sequence network*

Fig 1.20 : *Positive, Negative and Zero sequence networks of transmission line*

Sequence impedances and networks of transformer

When the applied voltage is balanced, the positive and negative sequence of linear, symmetrical, static devices are identical. Therefore in a transformer the positive and negative sequence impedances are identical. Eventhough the zero sequence impedance may slightly differ from positive and negative sequence impedance, it is normal practice to assume the zero sequence impedance as equal to positive or negative sequence impedance. [For all types of transformers the series impedance of all sequences are assumed equal].

> ***Note :*** *When the neutral of star connection is grounded through reactance Z_n then $3Z_n$ should be added to zero sequence impedance of transformer to get the total zero sequence impedance.*

Let Z_1 = Positive sequence impedance of transformer

Z_2 = Negative sequence impedance of transformer

Z_3 = Zero sequence impedance of transformer

The positive and negative sequence impedances of transformer are represented as a series impedance in their respective sequence networks as shown in fig 1.21.

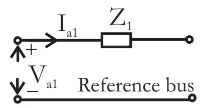


Fig a : *Positive sequence network*

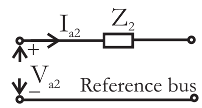


Fig b : *Negative sequence network*

Fig 1.21 : *Positive and Negative sequence networks of transformer*

The zero-sequence network of the transformer depends on the type of connections (**Y** or $\triangle$) of the primary and secondary windings and also on the grounding of neutral in **Y** connection.

The following general observations can be made for zero sequence currents in transformers.

1. When magnetizing current is neglected, the primary winding will carry current only if there is a current flow on the secondary winding. Therefore the zero sequence current can flow in the primary winding of a transformer only if there is a path for zero sequence current in secondary winding or vice-versa.

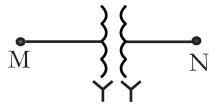
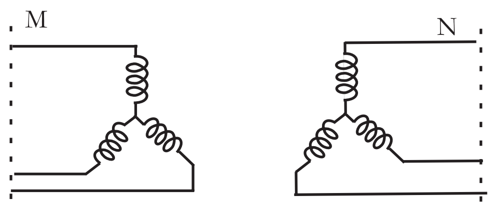
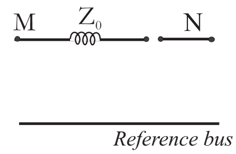
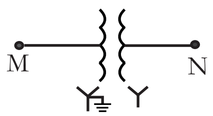
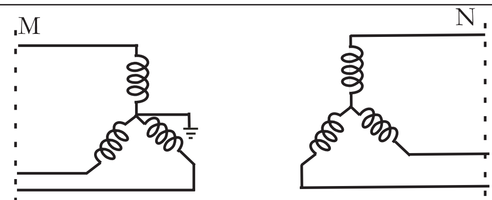
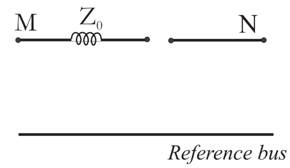
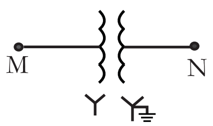
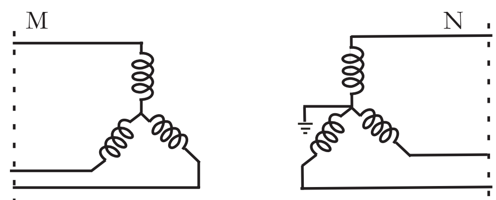
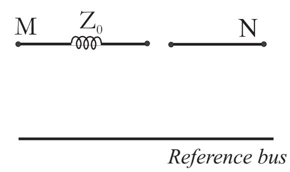
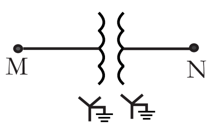
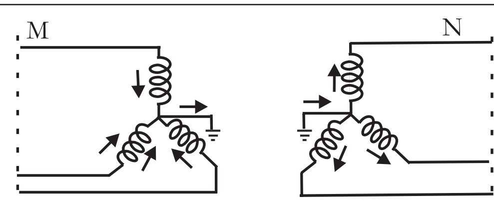
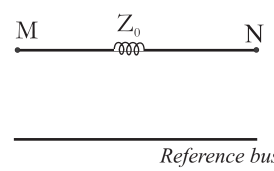
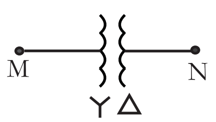
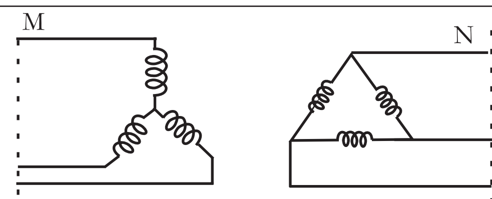
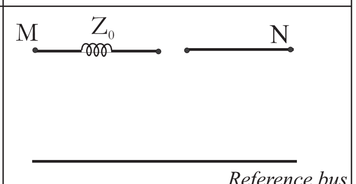
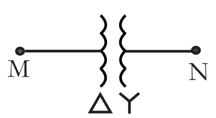
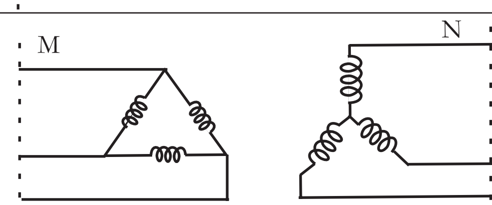
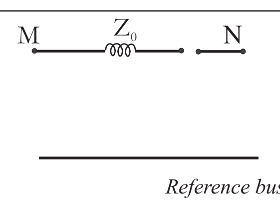
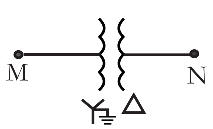
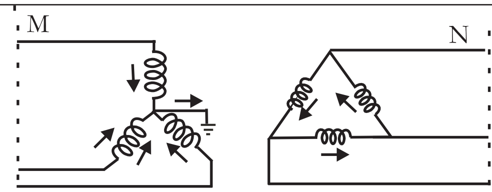
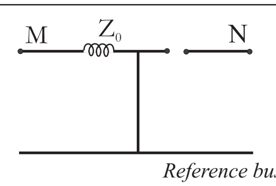
2. If the neutral point in the **Y** connected winding is not grounded then there is no path zero sequence current in star connected winding.

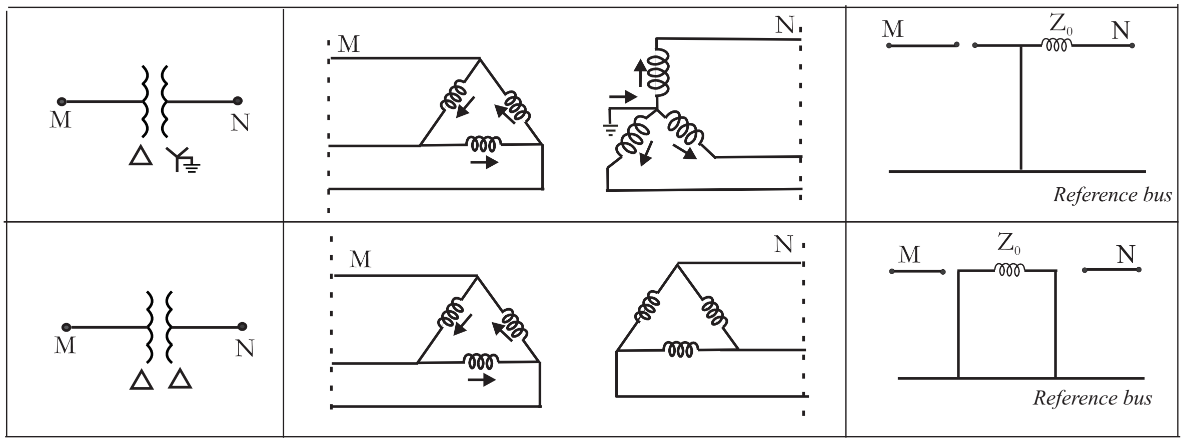
3. The zero sequence current flows in the star connected winding and in the lines connected to the winding only when the neutral point is grounded.

4. The zero sequence current can circulate in the delta connected winding but the zero sequence current cannot flow through the lines connected to the delta connected winding.

Based on the above observations the zero sequence network of 3-phase transformer can be obtained for any configuration. The zero sequence network for nine possible configuration are presented in table-1.3. The arrows on the windings, indicate path for zero sequence current and the absence of arrows indicate that there is no path for zero sequence currents.

Table-1.3 : Zero Sequence network of three phase transformer

Configuration	**Winding Connection Diagram**	**Zero Sequence Network**
		
		
		
		
		
		
		



Sequence impedances and networks of loads

In balanced **Y** or Δ connected loads the positive, negative and zero sequence impedances are equal. When the neutral point of star connected loads is grounded through a reactance Z_n the $3Z_n$ is added to the zero sequence impedance of load to get the total zero sequence impedance of load.

Let Z_{L1} = Positive sequence impedance of load

Z_{L2} = Negative sequence impedance of load

Z_{L0} = Zero sequence impedance of load

The positive and negative sequence impedances of load are represented as a shunt impedance in their respective sequence networks as shown in fig 1.22.

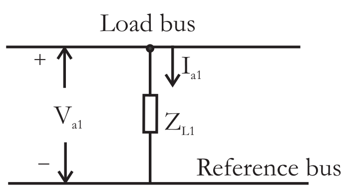


Fig a : *Positive sequence network*

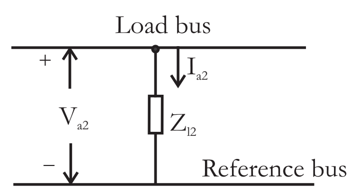
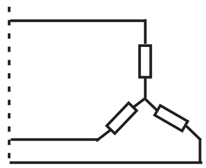
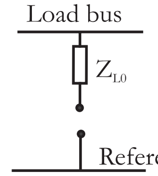
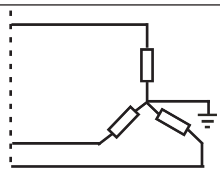
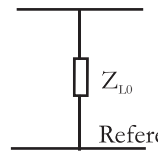
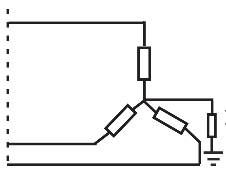
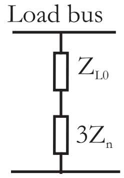
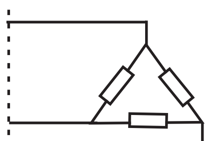
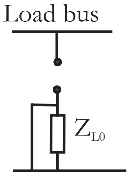


Fig b : *Negative sequence network*

Fig 1.22 : *Positive and Negative sequence network of load*

The zero sequence network of the 3-phase load depends on the type of connection, i.e., **Y** or Δ connection. The zero sequence current will flow in network only if a return path exists for it. The zero sequence network for various types of loads are shown in table-4.

Table - 1.4 : Zero Sequence networks of loads

Connection Diagram of load	Zero sequence network
	
	
	
	

EXAMPLE 1.23

Draw the positive, negative and zero sequence reactance diagram of the power system shown in fig 1.23.1.

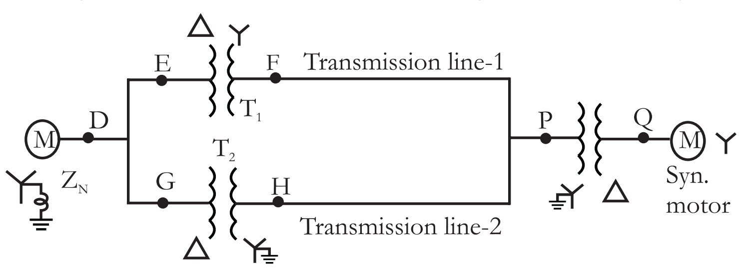


Fig 1.23.1

SOLUTION

The positive, negative and zero sequence reactance diagram (networks) of the power system are shown in fig (1.23.2), (1.23.3) and (1.23.4) respectively.

Let, $X_{G,1}$ = Positive sequence reactance of generator G

$X_{M,1}$ = Positive sequence reactance of motor M

$X_{T1,1}$ = Positive sequence reactance of Transformer T_1

$X_{T2,1}$ = Positive sequence reactance of Transformer T_2

$X_{T3,1}$ = Positive sequence reactance of Transformer T_3

$X_{TL,1}$ = Positive sequence reactance of Transmission line-1

$X_{TL,1}$ = Positive sequence reactance of Transmission line-2

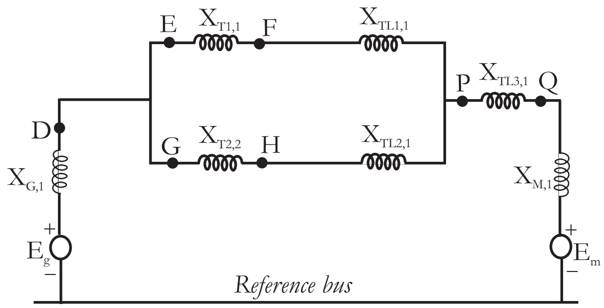


Fig 1.23.2 : *Positive sequence reactance diagram of power system shown in fig 1.23.1*

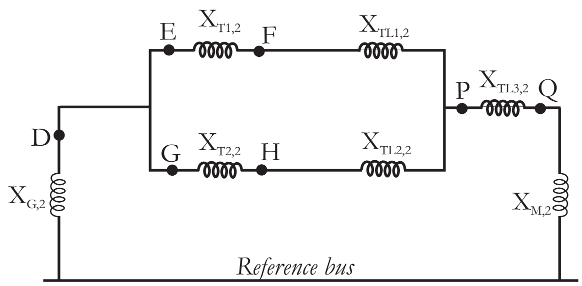
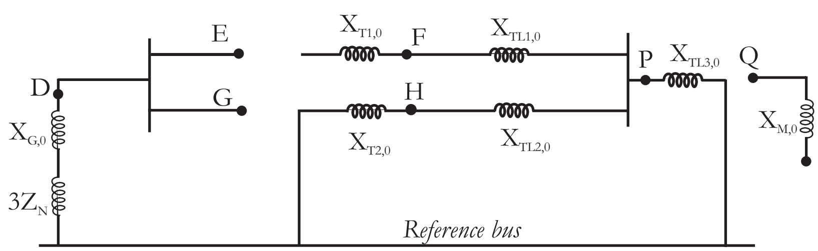


Fig 1.23.3 : *Negative sequence reactance diagram of power system shown in fig 1.23.1*



Note : *The suffix"0"denotes zero sequence reactance*

Fig 1.23.4 : *Zero sequence reactance diagram of power system shown in fig 1.23.1*

EXAMPLE 1.24

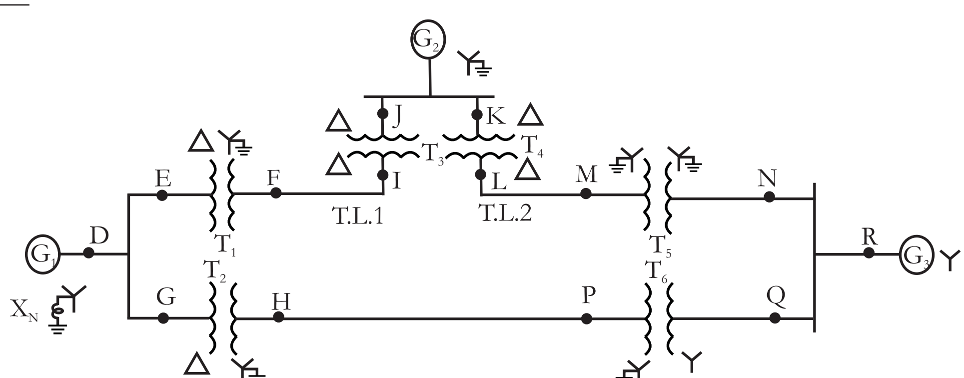


Fig 1.24.1

For the power system shown in fig 1.24.1, draw the positive, negative and zero sequence reactance diagrams.

SOLUTION

The positive, negative and zero sequence reactance diagram (networks) of the power system are shown in fig (1.24.2), (1.24.3) and (1.24.4) respectively.

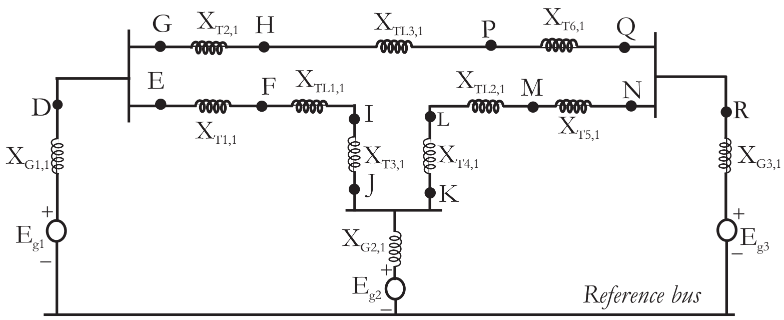


Fig 1.24.2 : *Positive sequence reactance diagram of power system shown in fig 1.24.1*

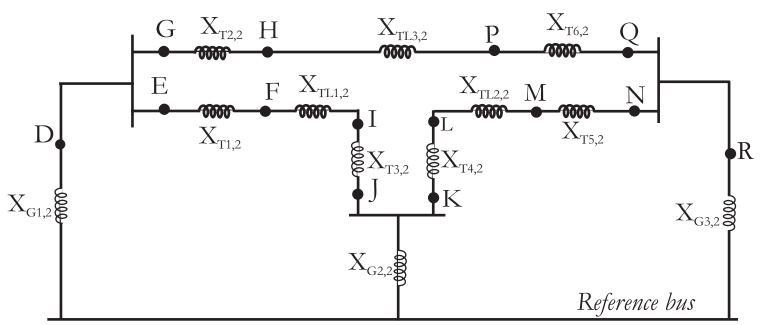


Fig 1.24.3 : *Negative sequence reactance diagram of power system shown in fig 1.24.1*

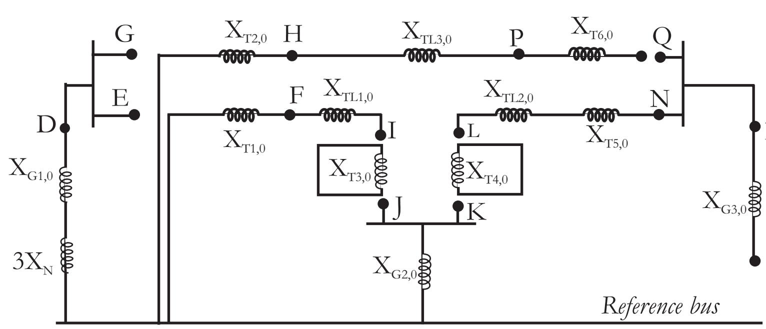


Fig 1.24.4 : *Zero sequence reactance diagram of power system shown in fig 1.24.1*

EXAMPLE 1.25

Determine the positive, negative and zero sequencce networks for the system shown in fig 1.25.1. Assume zero sequence reactances for the generator and synchronous motors as 0.06 p.u. Current limiting reactors of 2.5 Ω are connected in the neutral of the generator and motor No.2. The zero sequence reactance of the transmission line is j300 Ω.

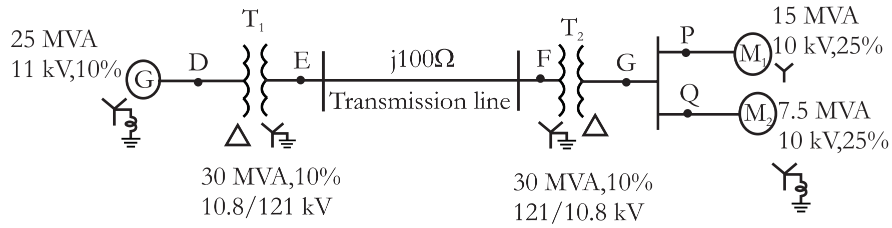


Fig 1.25.1

SOLUTION

Let us choose the generator ratings as new base values for entire system.

Base megavoltampere, $MVA_{b,new} = 25$ MVA

Base kilovolt, $kV_{b,new} = 11$ kV

Sequence reactances of Generator G

Since the generator rating and the new base values are same, the generator p.u. reactances does not change. Also for generator the positive and negative sequence reactances are same.

∴ Positive sequence reactance of generator, $X_{G,1} = 10\% = 10/100 = 0.1$ p.u.

Negative sequence reactance of generator, $X_{G,2} = 0.1$ p.u.

Zero sequence reactance of generator, $X_{G,0} = 0.06$ p.u.

$$\text{Base impedance, } Z_b = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{11^2}{25} = 4.84\ \Omega$$

$$\left.\begin{array}{l}\text{p.u. value of generator}\\ \text{neutral reactance}\end{array}\right\} X_{GN} = \frac{\text{Actual Neutral reactance}}{\text{Base impedance}} = \frac{2.5}{4.84} = 0.517 \text{ p.u.}$$

Sequence reactances of Transformer T_1

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of transformer } T_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

Here, $X_{pu,old} = 10\% = 0.1$, $kV_{b,old} = 10.8$ kV, $MVA_{b,old} = 30$ MVA

$kV_{b,new} = 11$ kV, $MVA_{b,new} = 25$ MVA

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of transformer } T_1\end{array}\right\} = 0.1 \times \left(\frac{10.8}{11}\right)^2 \times \left(\frac{25}{30}\right) = 0.08 \text{ p.u.}$$

In transformer the specified reactance is positive sequence reactance. Also we assume that the positive, negative and zero sequence reactances of the transformer are equal.

∴ Positive sequence reactance of transformer, T_1, $X_{T1,1} = 0.08$ p.u.

Negative sequence reactance of transformer, T_1, $X_{T1,2} = 0.08$ p.u.

Zero sequence reactance of transformer, T_1, $X_{T1,0}$ = 0.08 p.u.

Sequence reactances of Transmission line

$$\left.\begin{array}{l}\text{The base kV on HT side}\\ \text{of transformer } T_1\end{array}\right\} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$= 11 \times \frac{121}{10.8} = 123.24 \text{ kV}$$

Now, $kV_{b,new}$ = 123.24 kV,

$$\text{Base impedance, } Z_b = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{(123.24)^2}{30} = 506.27\ \Omega$$

$$\left.\begin{array}{l}\text{p.u. reactance of}\\ \text{transmission line}\end{array}\right\} = \frac{\text{Actual reactance}}{\text{Base impedance}} = \frac{100}{506.27} = 0.198 \text{ p.u.}$$

The specified reactance in single line diagram is positive sequence reactance. Also the negative sequence reactance of a transmission line is same as that of positive sequence reactance.

$\therefore$ Positive sequence reactance of transmission line, $X_{TL,1}$ = 0.198 p.u.

Negative sequence reactance of transmission line, $X_{TL,2}$ = 0.198 p.u.

$$\left.\begin{array}{l}\text{p.u. value of zero sequence}\\ \text{reactance of transmission line}\end{array}\right\} X_{TL,0} = \frac{\text{Zero sequence reactance in } \Omega}{\text{Base impedance}} = \frac{300}{506.27} = 0.593 \text{ p.u.}$$

Sequence reactances of Transmission line T_2

The ratings and winding connections of transformer T_1 and T_2 are identical and so the sequence reactances of T_1 and T_2 are same.

Positive sequence reactance of transformer, T_1, $X_{T1,1}$ = 0.08 p.u.

Negative sequence reactance of transformer, T_1, $X_{T1,2}$ = 0.08 p.u.

Zero sequence reactance of transformer, T_1, $X_{T1,0}$ = 0.08 p.u.

Sequence reactances of Synchronous motor M_1

$$\left.\begin{array}{l}\text{Base kV on LT side}\\ \text{of transformer } T_2\end{array}\right\} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 123.24 \times \frac{10.8}{121} = 11 \text{ kV}$$

Now, $kV_{b,new}$ = 11 kV,

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_1\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,0ld}}$$

Here, $X_{pu,old}$ = 25% = 0.25, $kV_{b,old}$ = 10 kV, $MVA_{b,old}$ = 15 MVA

$kV_{b,new}$ = 11 kV, $MVA_{b,new}$ = 25 MVA

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_1\end{array}\right\} = 0.25 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{15} = 0.344 \text{ p.u.}$$

The reactance specified in single line diagram is positive sequence reactance. Also the negative sequence reactance of synchronous motor is same as that of positive sequence reactance.

$\therefore$ Positive sequence reactance of motor, M_1, $X_{M1,1}$ = 0.344 p.u.

Negative sequence reactance of motor, M_1, $X_{M1,2}$ = 0.344 p.u.

$$\left.\begin{array}{l}\text{The zero sequence. reactance}\\ \text{of motor } M_1 \text{ on new bases}\end{array}\right\} X_{M1,0} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,0ld}}$$

$$= 0.06 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{15} = 0.083 \text{ p.u.}$$

Sequence reactances of Synchronous motor M_2

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_2\end{array}\right\} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,0ld}}$$

Here, $X_{pu,old}$ = 25% = 0.25, $kV_{b,old}$ = 10 kV, $MVA_{b,old}$ = 7.5 MVA

$kV_{b,new}$ = 11 kV, $MVA_{b,new}$ = 25 MVA

$$\left.\begin{array}{l}\text{New p.u. reactance}\\ \text{of motor } M_2\end{array}\right\} = 0.25 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{7.5} = 0.689 \text{ p.u.}$$

The reactance specified in single line diagram is positive sequence reactance. Also the negative sequence reactance of synchronous motor is same as that of positive sequence reactance.

$\therefore$ Positive sequence reactance of motor, M_2, $X_{M2,1}$ = 0.689 p.u.

Negative sequence reactance of motor, M_2, $X_{M2,2}$ = 0.689 p.u.

$$\left.\begin{array}{l}\text{The zero sequence. reactance}\\ \text{of motor } M_1 \text{ on new bases}\end{array}\right\} X_{M2,0} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,0ld}}$$

$$= 0.06 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{7.5} = 0.165 \text{ p.u.}$$

$$\text{Base impedance, } Z_b = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{11^2}{25} = 4.84\ \Omega$$

$$\left.\begin{array}{l}\text{p.u. value of motor}\\ \text{neutral reactance}\end{array}\right\} X_{MN} = \frac{\text{Actual neutral reactance}}{\text{Base impedance}} = \frac{2.5}{4.84} = 0.517 \text{ p.u.}$$

Positive Sequence network

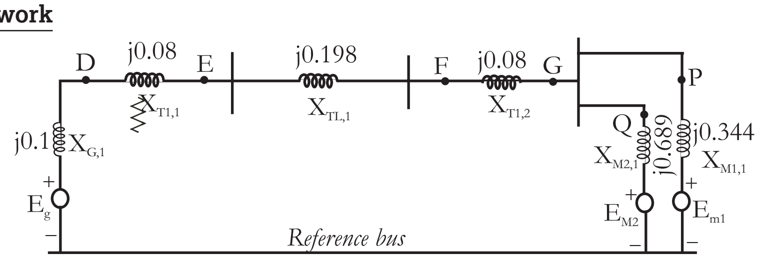


Fig 1.25.2 : *Positive sequence reactance diagramof the power system shown in fig 1.25.1*

Negative Sequence network

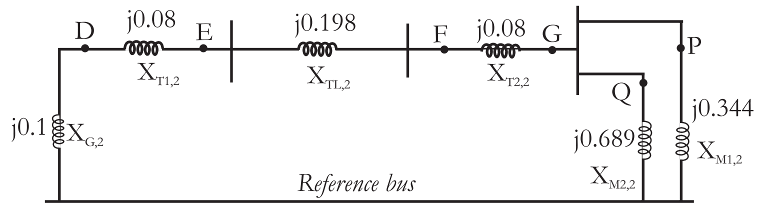


Fig 1.25.3 : *Negative sequence reactance diagram of the power system shown in fig 1.25.1*

Zero Sequence network

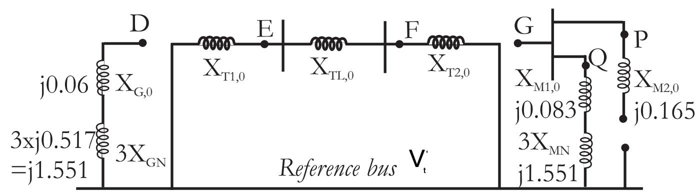


Fig 1.25.4 : *Zero sequence reactance diagram of power system shown in fig 1.25.1*

1.9 SHORT-ANSWER QUESTIONS

Q1.1 What is single line diagram?

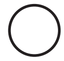
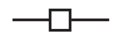
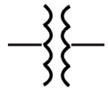
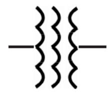
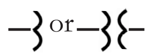
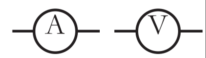
A single line diagram is diagrammatic representation of power system in which the components are represented by their symbols and the interconnection between them are shown by a single straight line (eventhrough the system is 3-phase system). The ratings and the impedances of the components are also marked on the single line diagram.

Q1.2 What are the components of power system?

The components of power system are Generators, Power Transformers, Transmission lines, Substation Transformers, Distribution Transformers and Loads.

Q1.3 Draw the symbols used to represent variuos components in a power system.

The symbols used to represent various components are shown in the following table.

Machine or rotating armature		Power circuit breaker, (oil/gas filled)	
Two-winding power transformer		Air circuit breaker	
Three-winding power transformer		Three-phase, three-wire delta connection	
Fuse		Three-phase star, neutral ungrounded	
Current transformer		Three-phase star, neutral grounded	
Potential transformer		Ammeter and voltmeter	

Q1.4 Define per unit value.

The per unit value of any quantity is defined as the ratio of the actual value of the quantity to the base value expressed as a dcimal. The base value is an arbitary chosen value of the quantity.

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}}$$

Q1.5 What are the quantities whose base values are required to represent the power system by reactance diagram?

The base values of Voltage, Current, Power and impedance are required to represent the power system by reactance diagram. Selection of base values for any two of them determines the base values of the remaining two. Usually the base values of voltage and power are chosen in kilovolt and kVA or MVA respectively. The base values of current and impedance are calculated using the chosen bases.

Q1.6 What is the need for base values?

The components or various sections of power system may operate at different voltage and power levels. It will be convenient for analysis of power system if the voltage, power, current and impedance ratings of components of power system are expressed with reference to a common value called base value. Hence for analysis purpose a base value is chosen for voltage, power, current and impedance. Then all the voltage, power, current and impedance ratings of the components are expressed as a percent or per unit of the base value.

Q1.7 Write the equation for converting the p.u. impedance expressed in one base to another?

$$Z_{pu,new} = Z_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

Q1.8 What are the advantages of per-unit computations?

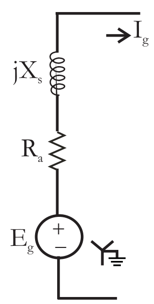
(i) Manufacturers usually specify the impedance of device or machine in per unit on the base of the name plate rating.

(ii) The p.u values of widely different rating machines lie within a narrow range, eventhough the ohmic values has a very large range.

(iii) The p.u. impedance of circuit element connected by transformers expressed on a proper base will be same if it is referred to either side of a transformer.

(iv) The p.u. impedance of a 3-phase transformer is independent of the type of winding connection (**Y** or Δ)

Q1.9 Draw the equivalent circuit of a 3-phase generator.



X_s = Synchonous reactance per phase

R_a = Armature resistance per phase

E_g = Inducted emf per phase

I_g = Current per phase

Q1.10 How the loads are represented in reactance or impedance diagram?

The resistive and reactive loads can be represented by any one of the following representation

(i) Constant power representation

Load power, $S = P + j\,Q$

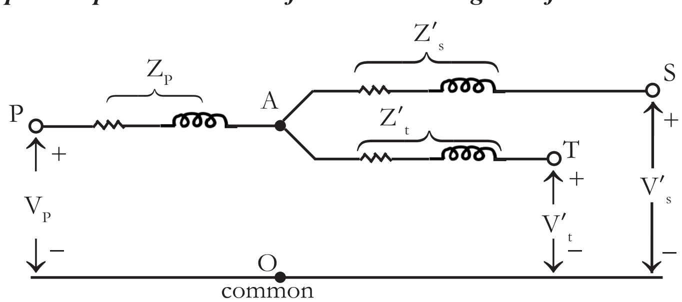
(ii) Constant current representation

Load current, $I = \dfrac{\sqrt{P^2 + Q^2}}{|V|} \angle\delta - \theta$

(iii) Constant impedance representation

Load current, $Z = \dfrac{|V|^2}{P - jQ}$

Q1.11 Draw the single phase equivalent circuit of a three-winding transformer.



Q1.11 : *Single phase equivalnet circuit of three winding transformer*

Q1.12 A Y-Δ, 3-phase transformer bank is formed using three numbers of 1-phase transformers each rated at 300 kVA, 127/13.2 kV. what is the kVA and kV rating of the 3-phase bank?

SOLUTION

kVA rating of 3-phase transformer $= 3 \times 300 = 900$ kVA

Line voltage of Y-connected winding $= \sqrt{3} \times 127 = 220$ kV

Line voltage of Y-connected winding $= 13.2$ kV

$\therefore$ The ratio of line voltage of 3-phase transformer bank $= 220/13.2 \text{ kV } (Y/\Delta)$

Q1.13 If the reactance in ohms is 15 ohms, find the p.u. value forra base of 15 kVA and 10 kVA.

SOLUTION

$$\text{Base impedance, } Z_b = \frac{(\text{kV})^2}{\text{MVA}} = \frac{(\text{kV})^2}{\text{kVA}/1000} = \frac{10^2}{15/1000} = 6666.67\ \Omega$$

$$\text{p.u. value of reactance} = \frac{\text{reactance in ohms}}{\text{Base impedance}} = \frac{15}{6666.67} = 0.0022 \text{ p.u.}$$

Q1.14 A generator rated at 30 MVA, 11 kV has a reactance of 20%. Calculate its p.u. reactances for a base of 50 MVA and 10 kV.

SOLUTION

$$\text{New p.u. reactance of generator} = X_{pu,old} \times \left(\frac{kV_{b,old}}{k\,V_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

Here, $X_{pu,old} = 20\% = 0.2$ p.u. ; $KV_{b,old} = 11$ kV, $MVA_{b,old} = 30$ MVA

$k\,V_{b,new} = 10$ kV ; $MVA_{b,new} = 50$ MVA

$$\therefore \text{ New p.u. reactance of generator} = 0.2 \times \left(\frac{11}{10}\right)^2 \times \frac{50}{30} = 0.403 \text{ p.u.}$$

Q1.15 A Y-connected generator rated at 300 MVA, 33 kV, has a reactance of 1.24 p.u. Find the ohmic value of reactance.

SOLUTION

$$\text{Base impedance, } Z_b = \frac{(\text{kV})^2}{\text{MVA}} = \ = \frac{33^2}{300} = 3.63\ \Omega/\text{phase}$$

$$\text{we know that, p.u. reactance} = \frac{\text{Actual reactance}}{\text{Base impedance}}$$

$\therefore$ Reactance of generator = p.u. reactance × $Z_b = 1.24 \times 3.63 = 4.5012\ \Omega$ / phase

Q1.16 The base kV and base MVA of a 3-phase transmission line is 33 kV and 10 MVA respectively. Calculate the base current and base impedance.

SOLUTION

$$\text{Base current, } I_b = \frac{k\,VA_b}{\sqrt{3}\ kV_b} = \frac{MVA_b \times 1000}{\sqrt{3}\ kV_b} = \frac{10 \times 1000}{\sqrt{3} \times 33} = 174.95\ A$$

$$\text{Base impedance, } Z_b = \frac{(kV_b)^2}{MVA_b} = \frac{33^2}{10} = 108.9\ \Omega / \text{phase}$$

Q1.17 How the induction motor is represented in reactance diagram?

For estimation of steady statee fault current, the induction motor is neglected. But for estimation of fault current immediately after the fault (i.e., to estimate sub-transient fault currents), the induction motor can be represented by a source in series with reactance. The value of the source is the induced emf per phase and the value of reactance is total reactance of induction motor per phase reffered to stator.

Q1.18 What is impedance and reactance diagram?

The impedance diagram is the equivalent circuit of power system in which the various components of power system are represented by their approximate or simplified equivalent circuits. The impedance diagram is used for load flow studies. The reactance diagram is the simplified equivalent circuit of power system in which the various components are represented by their reactances. The reactance diagram can be obtained from impedance diagram if all the resistive components are neglected. The reactance diagram is used for fault calculations.

Q1.19 What are the approximations made in impedance diagram?

The following approximations are made while forming impedance diagram.

(i) The neutral reactances are neglected.

(ii) The shunt branches in equivalent circuit of induction motor are neglected.

Q1.20 What are the factors that need to be omitted for an impedance diagram to reduce it to a reactance diagram? (or)

what are the approximations made in reactance diagram?

The following approximations are made in reactance diagram

(i) The neutral reactances are neglected.

(ii) Shunt branches in the eguivalent circuits of transformer are neglected.

(iii) The resistnces are neglected.

(iv) All static loads and induction motors are neglected.

(v) The capacitance of the transmission lines are neglected.

Q1.21 Give equations for transforming base kV on LV side to HV side of transformer and vice-versa.

$$\text{Base kV on HT side} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$\text{Base kV on HT side} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

Q1.22 For the power system shown in fig Q22, by taking generator rating as base values, specify the base values of the transmission line and motor circuit.

The transmission line and motor circuit

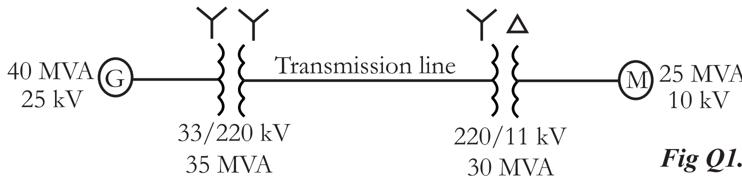


Fig Q1.22

SOLUTION

Generator circuit (selected base)

Base kilovolt = 25 kV

Base megavoltampere = 40 MVA

Transmission line

$$\text{Base kilovolt} = 25 \times \frac{220}{33} = 166.67\ \text{kV}$$

Base megavoltampere = 40 MVA

Motor Circuit

$$\text{Base kilovolt} = 166.67 \times \frac{11}{220} = 8.33\ \text{kV}$$

Base megavoltampere = 40 MVA

Q1.23 What is a bus?

The meeting point of various components in a power system is called a bus. The bus is a conductor made of copper or aluminium having negligible resistance. The buses are considered as points of constant voltage in power system.

Q1.24 What is bus admittance matrix?

The matrix consisting of the self and mutual admittances of the network of a power system is called bus admittance matrix. It is given by the admittance matrix **Y** in the node basis equation of a power system and it is denoted as $\mathbf{Y_{bu}s}$, The bus admittance matrix is symmetrical.

Q1.25 Name the diagonal and off-diagonal elements of bus admittance matrix.

The diagonal elements of bus admittance matrix are called self admittance of the buses and off-diagonal elements are called mutual admittances of the buses.

Q1.26 Write the equation to find the elements of new bus admittance matrix after eliminating n^th row and column in n × n bus admittance matrix.

The elements Y_{jk} of new bus admittance matrix is given by

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn}\, Y_{nk}}{Y_{nn}} \text{ for } j = 1,2,3\,......,(n-1) \text{ and } k = 1,2,3,......,(n-1)$$

where Y_{jk}, Y_{jn}, Y_{nk} and Y_{nn} are elements of original or given bus admittance matrix of order (n × n).

Q1.27 From the bus admittance matrix for the system whose reactance diagram is shown in fig Q27.

SOLUTION

$$Y_{bus} = \begin{bmatrix} \frac{1}{j0.1}+\frac{1}{j0.25}+\frac{1}{j0.5} & -\left(\frac{1}{j0.25}+\frac{1}{j0.5}\right) \\ -\left(\frac{1}{j0.25}+\frac{1}{j0.5}\right) & \frac{1}{j0.25}+\frac{1}{j0.5}+\frac{1}{j0.2} \end{bmatrix}$$

$$= \begin{bmatrix} -j16 & j6 \\ j6 & -j11 \end{bmatrix}$$

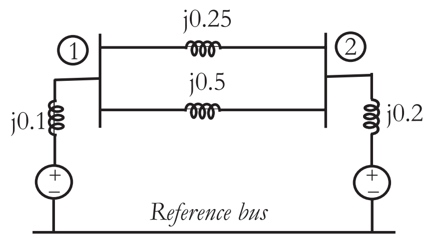


Fig Q1.27

Q1.28 What is bus impedance matrix?

The matrix consisting of driving point impedances and transfer impedances of the network of a power system is called bus impedance matrix. It is given by the inverse of bus admidance matrix and it is denoted as $\mathbf{Z}_{bus}$. The bus impedance matrix is symmetrical.

Q1.29 Name the diagonal elements and of diagonal elements of bus impedance matrix.

The diagonal elements of bus impedance matrix are called driving point impedances of the buses and off-diagonal elements of bus impedance matrix are called transfer impedances of the buses.

Q1.30 What are the methods available for forming bus impedance matrix?

The following two methods are available for forming bus impedance matrix.

Method 1 : From the buis admittance matrix and then take its inverse to get bus impedance matrix

Method 2 : Directly form the bus impedance matrix from the reactance diagram. This method utilizes the techniques of modifications of exixting bus impedance matrix due to addition of new bus.

Q1.31 Write the four ways of adding an impedance to an existing system so as to modify bus impedance matrix.

To modify a bus impedance matrix, a branch of impedance Zb can be added to origial system in the following four different ways.

Case 1 : Adding a branch of impedance Z_b from a new bus-p to the reference bus.

Case 2 : Adding a branch of impedance Z_b from a new bus-p to an reference bus.

Case 3 : Adding a branch of impedance Z_b from a new bus-q to the reference bus.

Case 4 : Adding a branch of impedance Z_b between two existing buses h and q.

Q1.32 How the Zbus is modified when a branch of impedance Z_b is added from a new bus-p to the reference bus?

When a branch of impedance Z_b is added from a new bus-p to the reference bus, the order of the bus impedance matrix increases by one.

Let the original bus impedance matrix have an order of n and so the new bus impedance matrix have an order of (n + 1). The first n × n submatrix of new bus impedance matrix. The elements of $(n + 1)^{th}$ column and row are all zeros except the diagonal. The $(n + 1)^{th}$ diagonal elementsis the added branch impedance Z_b.

Q1.33 How the Zbus is modified when a branch of impedance Zb is added from a new bus-p to the existing bus-q?

When a branch of impedance Zb is added from a new bus-p to the existing bus-q the order of the bus impedance matrix increases by one.

Let the original bus impedance matrix have an order of n and so the new bus impedance matrix have an order of (n + 1). The first n × n submatrix of new bus impedance matrix. The elements of $(n+1)^{th}$ row are the elements of qth row. The $(n+1)^{th}$ diagonal element is given by sum of Z_{qq} and Z_b.

Q1.34 Determine Z_{bus} when $Y_{bus} = \begin{bmatrix} -j1 & j2 \\ j2 & -j5 \end{bmatrix}$

SOLUTION

$$\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1} = \frac{\text{Adjoint of } Y_{bus}}{\text{Determine of } Y_{bus}}$$

$$\text{Determinant of } Y_{bus} = \begin{vmatrix} -j1 & j2 \\ j2 & -j1 \end{vmatrix} = (-j1)(-j5) - (j2)^2 = -5 + 4 = -1$$

$$\text{Adjoint of } \mathbf{Y}_{bus} = \begin{bmatrix} -j5 & -j2 \\ -j2 & -j1 \end{bmatrix}^T = \begin{bmatrix} -j5 & -j2 \\ -j2 & -j1 \end{bmatrix}$$

$$Z_{bus} = -1 \begin{bmatrix} -j5 & -j2 \\ -j2 & -j1 \end{bmatrix} = \begin{bmatrix} j5 & j2 \\ j2 & j1 \end{bmatrix}$$

Q1.35 What are symmetrical components?

An unbalanced system of N related vectors can be resolved into N systems of balanced vectors. The N-sets of balanced vectors are called symmetrical components. Each set consist of N-vectoprs which are equal in length and having equal phase.

Q1.36 Write the symmetrical components of three phase system.

In a 3-phase system, the three unbalanced vectors (either current or voltage vectors) can be resolved into three balanced system of vectors. They are

1. Positive sequence components.
2. Negative sequence components.
3. Zero sequence components.

Q1.37 What are positive sequence components?

The positive sequence components of a 3-phase unbalanced vectors consists of three vectors of equal magnitude, displaced fron each other by 120° in phase and having the same phase sequence as the original vectors.

Q1.38 What are the negative sequence components?

The negative sequence components of a 3-phase unbalanced vectors consists of three vectors of equal magnitude displaced from each other by 120° in phase and having the phase sequence opposite to that of the original vectors.

Q1.39 What are zero sequence components?

The zero sequence components of a 3-phase unbalanced vectors consists of 3-vectors of equal magnitude and with zero phase displacement from each other.

Q1.40 Express the value of the operator "a" and "a^2" in both polar and rectangular form.

$$a = 1 < 120^o - \text{polar form}$$
$$= -0.5 + j0.866 - \text{Rectangular form}$$
$$a^2 = 1 < 240^o - \text{polar form}$$
$$= -0.5 - j0.866 - \text{Rectangular form}$$

Note : a^2 is conjugate of a.

Q1.41 Prove that $1 + a + a^2 = 0$.

SOLUTION

$$a = 1\angle 120^o = -0.5 + j0.866$$
$$a^2 = 1\angle 240^o = -0.5 - j0.866$$
$$\therefore\ 1 + a + a^2 = 1 - 0.5 + j0.866 - 0.5 - j0.866 = 0$$

Q1.42 Express the unbalanced voltages V_a, V_b and V_c in terms of semmetrical components V_{a1}, V_{a2}, and V_{a0}.
Ans :

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + a^2V_{a1} + aV_{a2} \qquad \text{or} \qquad \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a^2 \\ 1 & a & a \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$
$$V_c = V_{a0} + a\ V_{a1} + a^2\ V_{a2}$$

Q1.43 Express the unbalanced voltages V_{a0}, V_{a1} and V_{a2} in terms of semmetrical components V_a, V_a, and V_c.

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$
$$V_{a1} = \frac{1}{3}(V_a + a\ V_b + a^2\ V_c) \qquad \text{or} \qquad \begin{bmatrix} V_{a0} \\ V_{b0} \\ V_{c0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$
$$V_{a2} = \frac{1}{3}(V_a + a^2\ V_b + a\ V_c)$$

Q1.44 If $I_a = 12\angle 0^o A$, $I_b = 10\angle 90^o A$ and $I_c = 10\angle -90^o A$. ***Find the zero sequence current.***

SOLUTION

The zero sequence current, $I_{a0} = \frac{1}{3}(I_a + I_b + I_c)$

$$= \frac{1}{3}(12\angle 0^o + 10\angle 90^o + 10\angle -90^o)$$
$$= \frac{1}{3}(12 + j10 - j10) = \frac{12}{3} = 4$$
$$\therefore\ I_{a0} = 4\angle 0^o A$$

Q1.45 If $I_a = 18\angle 0^o A$, $I_b = 10\angle -30^o A$ and $I_c = 10\angle 30^o A$. ***Find the positive sequence current.***

SOLUTION

The positive sequence current, $I_{a1} = \frac{1}{3}(I_a + a\ I_b + a^2\ I_c)$

$$= \frac{1}{3}\left[18\angle 0^o + (1\angle 120^o \times 10\angle -30^o) + (1\angle 240^o \times 10\angle 30^o)\right]$$
$$= \frac{1}{3}(18 + j10 - j10) = \frac{18}{3} = 6$$
$$\therefore\ I_{a0} = 6\angle 0^o$$

Q1.46 What are the sequence impedance and sequence networks?

The sequence impedances are the impedances offered by the devices or components for the like sequence components of the current.

The single phase equivalent of a circuit of a power system consisting of impedances to current of any one sequence only is called sequence network.

Q1.47 What is meant by positive, negative and zero sequence reactance diagram?

The reactance diagram of power system, when formed using positive, negative and zero sequence component currents are called positive, negative and zero sequence impedances respectively.

Q1.48 What is meant by positive, negative and zero sequence reactance diagram?

The reactance diagram of a power system, when formed using positive, negative or zero sequence reactances are called positive, negative and zero sequence reactance diagram respectively.

Q1.49 Draw the zero sequence network of a generator when the neutralo is grounded and when it is ungrounded?

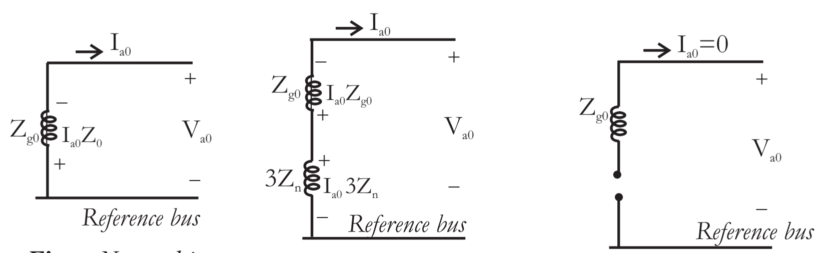


Fig a: *Neutral is solidly grounded*

Fig b: *Neutral is grounded through reactance*

Fig c: *Neutral is ungrounded*

Fig Q1.49 : *Zero sequence networks of generator*

Q1.50 Draw the zero sequence network of Y/Δ with neutral of star grounded and △/△

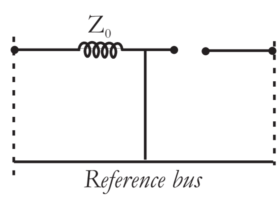


Fig a : *Zero sequence network of Y/Δ transformer with neutral of Y grounded*

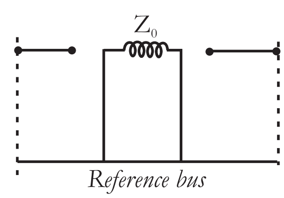


Fig b : *Zero sequence network of Δ/Δ transformer*

1.10 EXRCISES

I. State whether the following statements are TRUE / FALSE

1. A balanced 3-phase system is always analysed on per phase basis.
2. The base values for power, voltage, current and impedance can be selected independently.
3. The impedance of a device or component is usually specified in p.u. on the base of name plate rating.
4. In 3-phase system the base kV is phase value and base kVA is per phase kVA.
5. In forming reactance/ impedance diagram the base kVA is same for every section but the base kV depends on through it.
6. The p.u. impedance of a transformer depends on the Y or Δ connection of the winding.
7. In three winding transformer transformer, the three windings may have different kVA rating.
8. In impedance and reactance diagram the neutral impedance has to be included because the return current flows through it.
9. In reactance diagram the induction motors and reactance are neglected.
10. If $\mathbf{Y}_{bus}$ is symmetrical then corresponding $\mathbf{Z}_{bus}$ is also symmetrical.
11. In matrix partitioning method of bus elimination any bus can be eliminated.
12. The positive sequence components hase same phase sequence as that of original vectors.
13. The Negative sequence components hase same phase sequence as that of original vectors.
14. The induced emfs of synchronous machines are positive sequence voltages.
15. The impedance per phase of transmission line for balanced currents is independent of phase sequence.
16. For all types of transformers the series impedances of all sequences are equal.
17. When neutral is grounded there is path for zero sequence current in transformer.
18. Zero sequence current can flow in one winding of a transformer only when there is a path for it in the other winding.
19. In delta connected winding there is no zero sequence currents.
20. In loads, the zero sequence currents will flow in the network only if a return path exists for it.

Answers

1. True	6. False	11.False	16. True
2. False	7. True	12. True	17. False
3. True	8. False	13. False	18. True
4. False	9. True	14. True	19. False
5. True	10. True	15. True	20. True

II. Fill in the blanks with appropriate words

1. In a .... the components of power system are represented by symbols and the interconnections between them are shown by straight lines.
2. For a balanced 3-phase system the .... is same as positive sequence reactance diagram.
3. The impedance diagrams are used fo .... and reactance diagrams are used for ....
4. The meeting point of various components in a power system is called ....
5. The diagonal point $\mathbf{Y}_{bus}$ are called .... and off-diagonal elements are called ....
6. The diagonal point $\mathbf{Z}_{bus}$ are called .... and off-diagonal elements are called ....
7. An unbalanced system of N related vectors can be resolved into N system of balanced vectors called ....
8. The .... components consiusts of three vactors equal in magnitude and phase
9. In transmission line the ....reactance is .... times the positive sequence reactance.
10. The .... and .... sequence network of generator will not have any sources.

Answers

1. Single line diagram	6. driving point impedance, transfer impedances
2. reactance diagram	7. symmetrical components
3. load flow studies, fault calculations	8. zero sequence
4. bus	9. zero sequence, 2 to 3.5
5. self admittances, mutual admittances	10. negative, zero

iii. Unsolved problems

E1. 1 (a) A generator is rated 30 MVA, 10.5 kV. Its Y-connected winding has a reactance of 1 p.u. Find the ohmic value of the reactance of winding.

(b) If the generator is working in a circuit for which the bases are specified as 10 MVA, kV. Then find the p.u. value of generator working on the specified base.

E1.2 A 15 MVA, 10.5 kV, 3ϕ generator has a synchronous reactance of 0.2 p.u. and it is connected to a transmission line through a transformer rated 15 MVA, 33/11kV with X = 0.15 p.u.

(i) Calculate the p.u. reactances by taking generator rating as base values.

(ii) Calculate the p.u. reactances by taking transformer rating as base values.

E1.3 A single line diagram of an unloaded power system shown in fig E3.1. The generators are rated as follows. Draw the reactance diagram.

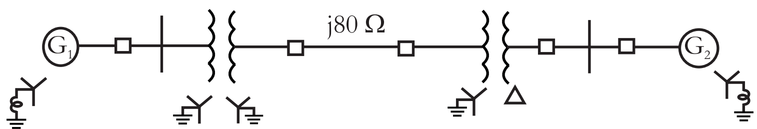


Fig E 1.3

Generator 1	50 MVA, 13.8 kV, $X'' = 0.15$ p.u.
Generator 2	40 MVA, 33 kV, $X'' = 0.20$ p.u.
Y-Y transformer	60 MVA, 16 kV/110 kV, $X = 0.1$ p.u.
Y-△ transformer	40 MVA, 33 kV/110 kV, $X = 0.15$ p.u.

E1.4 A 40 MVA, 25 kV, 3ϕ generator has a subtransient reactance of 25%. It is connected through a △-Y transformer to a high voltage transmission line having totral series reactance of 50 Ω. At the load end of the lines is Y-Y step down transformer. Both transformer banks are composed of 1ϕ transforer connected for 3ϕ operation. Each of the 3 transformer composing each bank is rated 16.67 MVA, 13.8 kV/100 kV with a lekage reactance of 20%. The load represented as impedance, is drawing 330 MVA at 24 kV, 0.9 pf lag. Draw the single line diagram of power network. Choose a base of 30 MVA, 24 kV in the load cicuit. Determine also the coltage at the terminals of the generator.

E1.5 Solve the node voltages.

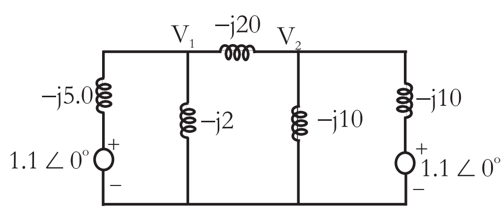


Fig E 1.5

E1.6 For the network shown in fig E6.1. Give the total number of elements, nodes, buses and branches. Write the element of Y_{bus} matrix directly by inspection.

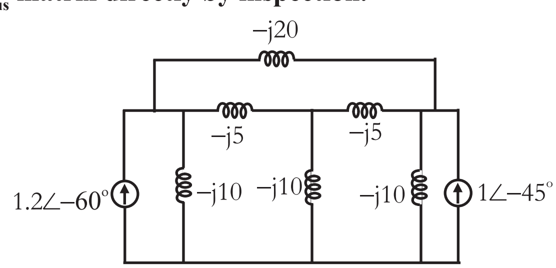


Fig E1.6

E1.7 Determine the reduced admittance matrix by eliminating nodes (3) and (4). Values marked in the E7.1 are p.u. admittances and currents.

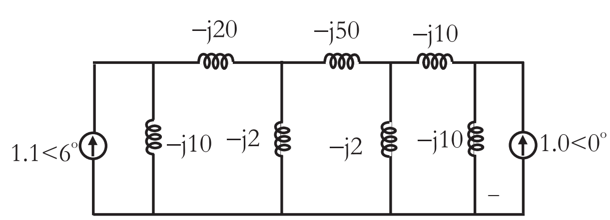


Fig E 1.7

E1.8 For the system shown in fig E8.1., determine Z_{bus} .

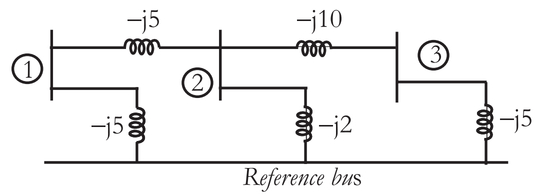


Fig E 1.8

E1.9 Determine Z_{bu} for the network shown in fig E9.1., where the impedances are given in p.u. Preserve all the 3 nodes.

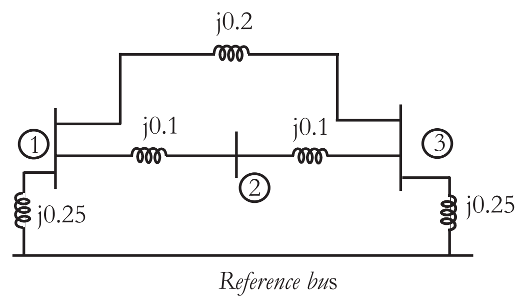


Fig E 1.9

E1.10 The voltage across a 3ϕ unbalanced load are $V_a = 200V$, $V_b = 200 < 180°V$ and $V_c = 600 < 145.2$ V **respectively. Determine the symmetrical components of voltages. Phase sequence is abc.**

E1.11 Draw the positive, negative and zero sequence impedance networks for the power system shown in fig E11.1.

Choose a base 50 MVA, 220 kV in the 50 Ω transmission line and mark all reactances in p.u. The ratings of the generators and transformers are :

G_1 : 25 MVA, 11 kV, $X'' = 20\%$

G_2 : 25 MVA, 11 kV, $X'' = 20\%$

The negative sequence reactance of each synchronous machine is equal to its subtransient reactance. The zero sequence reactance of each machine is 8%. Assume that the zero sequence reactancesof transmission lines are 250% of their positive sequence reactances.

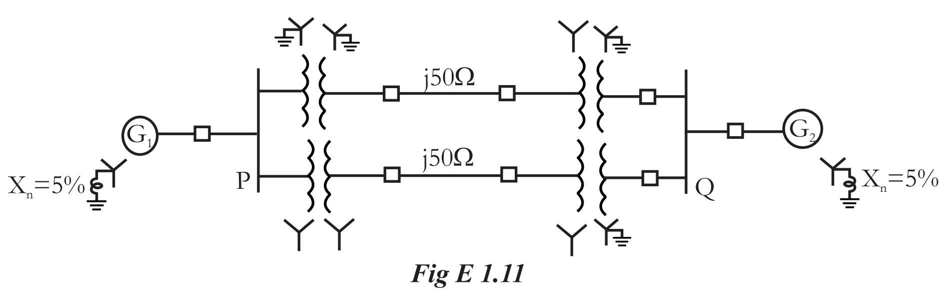


Fig E 1.11