1

SIMPLE STRESSES AND STRAINS

1.1 INTRODUCTION

At the outset let us discuss what is strength of materials. We know that matter is made up of small particles called molecule. Molecules in solid are attached to each other by the force of cohesion. When we apply external force, the distance between the molecules tend to increase which is resisted by the force of cohesion.

As the force (applied) is increased, the resisting force (due to cohesion) also increases simultaneously till the maximum resistance is reached. **This maximum resistance offered by the material is called it's strength.** For example wall of gunbarrel must be able to resist pressure developed as a result of firing; piston cylinder must be able to resist pressure developed as a result of combustion; the wall of water pipe, steam pipe must be able to sustain pressure of fluid flowing through the pipe. **Strength of materials involves the analytical method of determining-strength, stiffness and stability of load carrying component of machine.**

In strength of material we consider that

— material is elastic
— deformation is small
— when machine member/structure is loaded these should neither break nor deform excessively

Many structural member/material like steel, Aluminium are perfectly elastic within limits *i.e.* if the load is not exceeding much.

The strength of materials depends on —

— the type of loading
— temperature
— internal structure of the material

Let us start with the term load.

External force applied on the body is called load denoted by 'P'. It's SI unit is Newton.

Since Newton is small, force is expressed in Kilo-Newton and Mega Newton.

$$1\ kN = 10^3\ N$$

1 Mega Newton = 10^6 N
1 Giga Newton = 10^9 N

1.2 CLASSIFICATION OF LOAD

Load may be in the form of
— Axial load (P)
(a) Tensile (P_t) (b) Compressive (P_c)
— Shear (τ)
— Torque (T)
— Moment (M)
Or Combination of these.

Load which is applied to most components in engineering is a combination (superposition) of these. For example, a rotating shaft which transmits power is subjected not only to torsion but also to bending and to direct shear. We will discuss these in details later.

Loads may also be classified on the basis of their variation with a respect to time as static, quasi-static and dynamic. Static loads are those which do not change their magnitude or direction or point of application with respect to time. Those loads which vary at a very small rate so that inertial effects may be neglected are called quasi-static. Loads of dynamic character change with time at a fast rate. The action of such loads set up vibration, fatigue etc. in the body.

As an engineer, we study the various types of load and its effect on the elastic bodies. We also analyze and design various machines and load bearing structure. Analysis and design of a given structure involve determination of stresses and deformations.
It has been observed that
(i) load may develop 'stress' and 'strain' in a body simultaneously.
(ii) load may develop 'deflection' in a body i.e. in beam; cantilever etc.
(iii) load may develop 'buckling' in the body i.e. the case of column.
We will discuss the effect of load on the elastic bodies in various chapters of this book.

1.3 STRESS OR ENGINEERING STRESS

When load is applied on an elastic body, internal resisting force is developed inside the material to counter the effect of the external force.

The internal resisting force per unit area of the body is called stress or enginneering stress and is denoted by the letter σ.

Difference between Stress and Pressure:

Stress is not same as pressure. Pressure is reserved for a specific stress state in which there is no shear components and all the normal components are equal.

Stress $(\sigma) = \dfrac{P}{A}$. It represents the **average value** of stress over the cross-section rather than the stress at a specific point of the cross-section.

So, if a body is loaded externally then internal forces are distributed throughout the region of the body and stresses will be caused. For the defination of stress, we intersect the body by a imaginary plane like shown in the Fig. 1.1 (a).

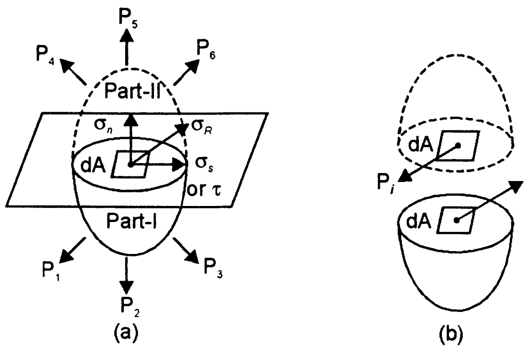

Fig. 1.1

Both parts of the body then interact through forces of equal magnitude and opposite in direction Fig. 1.1(b).

Let dP be the elementary load acting on an area dA of the cross-section then $\left(\frac{dP}{dA}\right)$ is the average stress on this element.

The limit $\sigma = \underset{\Delta A \to 0}{\text{limit}} \frac{\Delta P}{\Delta A} = \frac{dP}{dA}$ is called the stress* at a point of the cross-section.

The Standard Unit (SI unit) for stress is N/m² 'or' Pascal. Typical level of stress are several million Pascals, so the most covenient unit for stress is the megapascal or MPa. The stress magnitude 1 MPa is equal to the pressure (p) at a depth (z) of about 100 m in water or 37 in rock using the relationship $p = \rho g z$, where

ρ = density of material

g = acceleration due to gravity

1 N/mm^2 = 1 MPa (mega pascal); 1 Pa = 1 N/m^2

Mathematically, Stress** $(\sigma) = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} \frac{\text{N}}{\text{m}^2}$.

Stress is a tensor* entity. The stress tensor characterizes the stress state at a point in the body. Stress is measured by photoelasticity technique** (in this book we will compute the stress and check with the permissible stress or design stress for proper functioning of the machine member or structural member).

The stress at any point on a plane can be resolved into two components — one along normal and the other perpendicular to the normal *i.e.* tangent to the plane.

The component of stress along the normal is called the normal stress (σ_n) or direct stress (σ) and the component tangential to the plane is called the tangential stress (σ_s) or shear stress (τ).

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*Six components are required to describe the state of stress at a point in a body under the most general loading condition which we will discuss in 3-D stress.

**Stress developed on the body due to application of load, should never exceed the ultimate tensile strength for brittle materials and yield strength for ductile materials.

***A physical quantity is said to be tensor if it is neither a scalar nor a vector.

(The complete set of stress components in a solid or fluid medium are written as τ_{ij} which will be discussed in chapter 3-D stress).

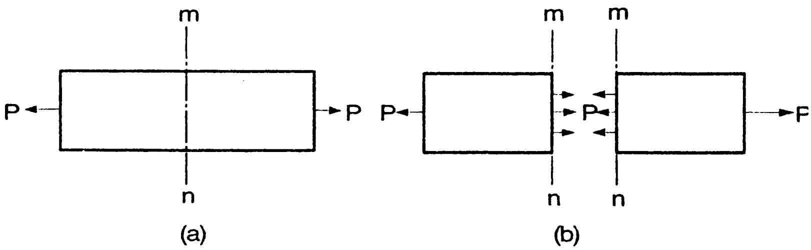


Fig. 1.2

Let us consider a prismatic bar having cross-sectional area A subjected to load P, then normal stress at any point of the bar $= \frac{P}{A}$.

The component of stress along x-axis and y-axis are written as σ_x and σ_y respectively. When an elastic body is subjected to stresses σ_x and σ_y along x and y-axes respectively then we call it biaxial stress system. Biaxial stress arises in the analysis of pressure vessel (thin); beams; shaft and many other structural parts.

When the normal stress acts outwards from the plane then it is called *tensile stress*.

Tensile stress is considered positive (+).

Similarly, normal stress acting towards the plane is called 'compressive stress' and it is considered negative (–).

A tensile stress tends to stretch the member (of machine part or structural part) apart, whereas, a compressive stress tends to compress the material of the load carrying member and shortens the member itself.

Shear Stress

When equal and opposite transverse loads P and P' of magnitude P are applied to a member AB, shearing stress τ are created over any section located between the points of application of two loads as shown in Fig. 1.3.

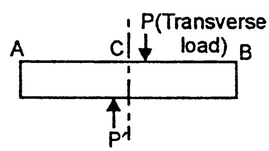


Fig. 1.3

Fig 1.3

In practice shearing stress act simultaneously with normal stress. However, there are cases where the shear stresses are considerably larger than the normal stresses and as such the normal streses are ignored in the analysis.

Riveted, bolted and welded joints are examples of simple shear.

Let us consider a rectangular block fixed at the lower surface and subjected to load P parallel to its surface as in Fig. 1.3(a).

By application of load P (which is tangenital), the plane gets distorted.

If A = Area of top or bottom of the block.

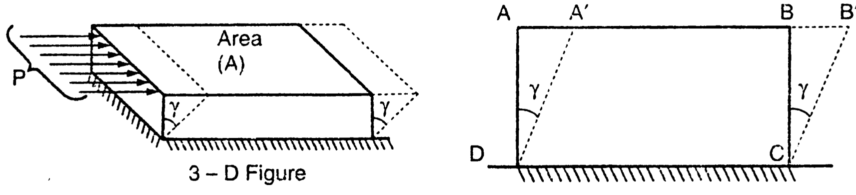


Fig 1.3(a)

Then shear stress $(\tau) = \dfrac{P}{A}$. The value obtained is an average value of shearing stress over the entire section.

Practical application shear stress are given below:

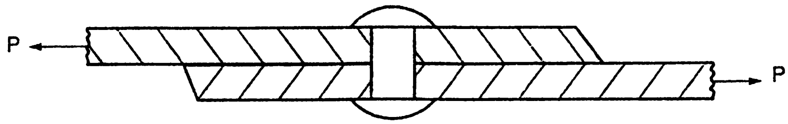


Fig 1.4

In this figure there is only one critical section of the rivet where the failure by shear can take place. This is called a case of single shear (Fig. 1.4).

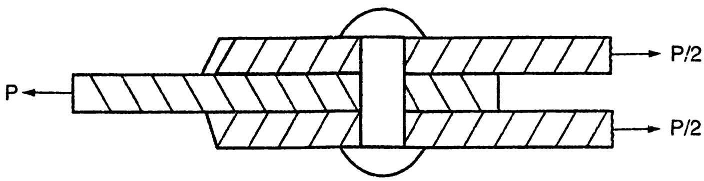


Fig 1.5

In the same manner, there is a case of double shear where the tendency of failure occurs at two sections as shown in Fig. 1.5.

$$\text{Here, average shear stress } (\tau) = \left[\frac{P/2}{A}\right] = \left[\frac{P}{2A}\right]$$

The other example of shear is punching operation by which small hole or slot can be made on the metal plates.

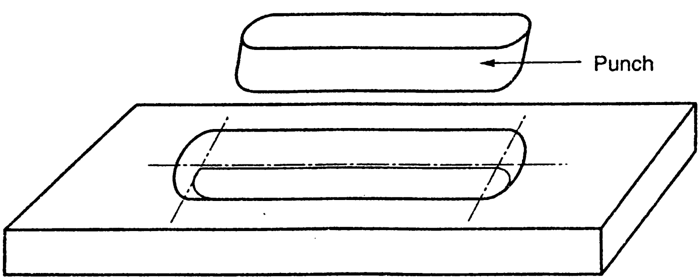


Fig. 1.6

Principle of Shear Stress

It states that a shear stress across a plane is always accompanied by a balancing or complementary shear stress across the plane.

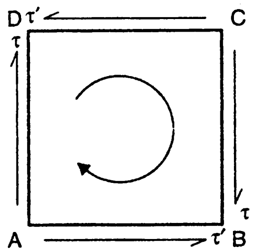


Fig. 1.7

Let us consider a rectangular block *ABCD* subjected to shear stress τ on the face *AD* and *BC*. Further, consider a unit thickness of the block. Therefore, force acting on the face *AD* and *CB*,

$$P = \tau \cdot AD = \tau \cdot CB$$

Since these are equal and opposite forces so they will form a couple whose moment is equal to $\tau \times AD \times AB$.

If the block is in equilibrium then there must be a restoring force where ($\tau\uparrow$ and $\tau\downarrow$ will try to rotate clockwise) moment must be equal to this couple.

Let the shear stress on the face *DC* and *AB* be τ'. Hence, force acting on the face *AB* and *CD* are

$$P = \tau' AB = \tau' CD$$

These two forces ($\tau' AB$ and $\tau' CD$) will also form a couple (CCW).

Equating these two Moments

$$\tau \cdot AD \cdot AB = \tau' AB \cdot AD$$

$$\therefore \qquad \tau = \tau'$$

As a result of shear forces which forms a couple, the diagonal *BD* will be subjected to tension whereas diagonal *AC* will be subjected to compression.

If the material of the machine under study is poor is tension, it will fail due to excessive tensile stress across diagonal *BD*. Similarly, if the material is poor in compression, it will fail due to excessive compressive force across the diagonal *AC*.

EXAMPLE 1: *What force is required to punch a 20 mm diameter hole is a plat that is 25 mm thick? The shear strength is 350 MN/m².*

SOLUTION Given

$$d = 20 \text{ mm}$$

$$t = 25 \text{ mm}$$

$$\tau = 350 \text{ MN/m}^2 = 350 \times 10^3 \text{ kN/m}^2$$

The resisting area is the shaded area along the perimeter and shearing force is equal to the punching force P.

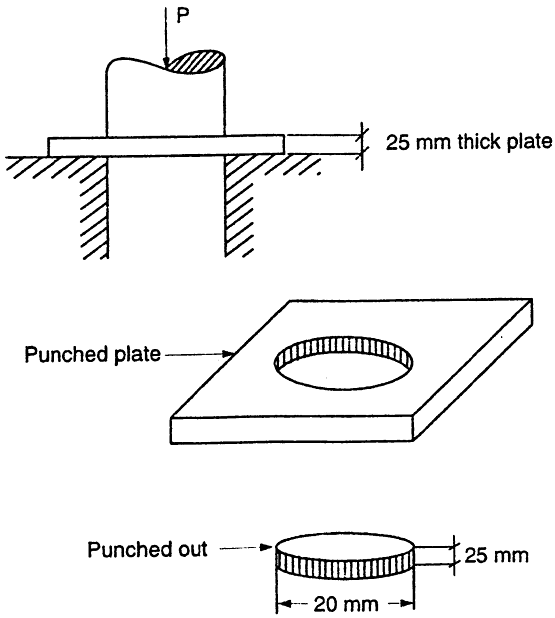


Fig. 1.8

$$\text{Shearing force (P)} = \text{Shearing stress} \times \text{Resisting area}$$

$$= \tau \times \pi \times d \times t$$

$$= 350 \times 10^3 \times \left(\pi \times \frac{20}{10^3} \times \frac{25}{10^3}\right) \text{kN}$$

$$= 350 \times \frac{22}{7} \times 20 \times \frac{20}{10^3} \text{ kN}$$

$$= 550 \text{ kN}$$ **Ans.**

EXAMPLE 2 *Two blocks of wood width 'b' and thickness 't' are glued together along the joint inclined at the angle θ as shown in the figure. Using the free body diagram show that the shearing stress on the glued joint is $\tau = \dfrac{P \sin 2\theta}{2A}$ where A is the cross sectional area.*

SOLUTION Shear force (F_S) = $\tau \cdot$ Area of oblique plane

$$= \tau \cdot A \operatorname{cosec} \theta \qquad \ldots(\text{I})$$

Further $$F_S = P \cos \theta \qquad \ldots(\text{II})$$

From I and II we have

$$\tau \cdot A \operatorname{cosec} \theta = P \cdot \cos \theta$$

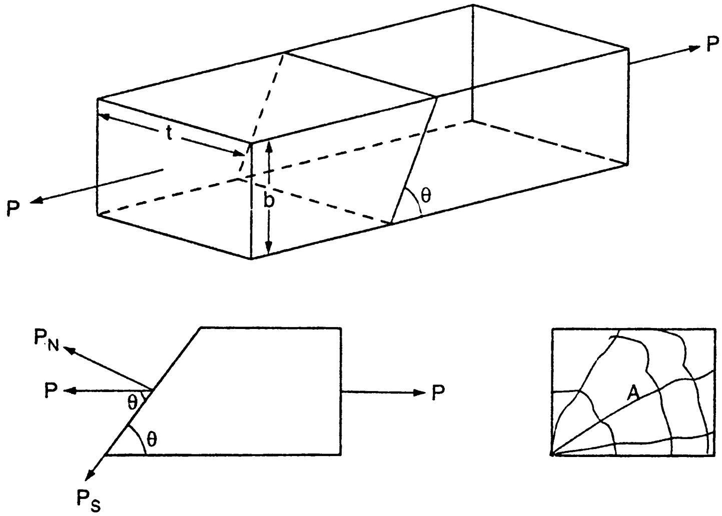


Fig. 1.9

$$\therefore \qquad \tau = \frac{P\cos\theta}{A\operatorname{cosec}\theta} = \frac{P\cos\cdot\sin\theta}{A}$$

$$= \frac{2P\sin\theta\cos\theta}{2A}$$

$$= \frac{P}{2A}\sin 2\theta$$

Proved

Bearing Stress (σ_{br})

When one body rests on another and transfers a load normal to it then such form of stress is called bearing stress.

Bearing stress is developed **at the surface in contact.**

$$\text{Bearing stress } (\sigma_{br}) = \frac{\text{Applied load}}{\text{Bearing Area}}$$

$$= \frac{P}{A_{br}}$$

where A_{br} = Bearing Area.

For flat surfaces in contact, the bearing area is the area over which the load is transferred from one member to other.

If two parts are having different area, then the smaller area is used.

Bearing stress is very important in design and in Building Construction.

EXAMPLE 1 *A rivet joint shown in figure has 110 mm wide plate and 20 mm rivet diameter. The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of the rivet. Determine*

(a) *the minimum thickness of each plate and*
(b) *The largest average tensile stress in the plates.*

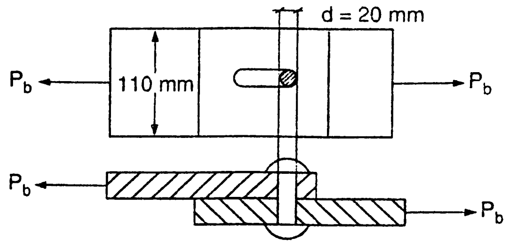


Fig. 1.10

SOLUTION Given

$$d = 20 \text{ mm}$$

$$\sigma_{bearing} = 120 \text{ MPa} = 120 \text{ N/mm}^2$$

$$\tau = 60 \text{ MPa} = 60 \text{ N/mm}^2$$

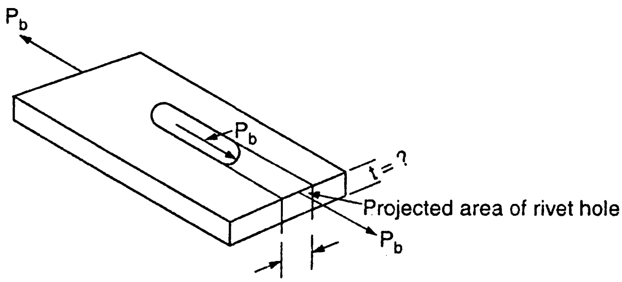


Fig. 1.11

(a) From shearing of rivet

$$P = \tau \cdot A_{rivet}$$

$$= 60 \times \frac{\pi}{4}(20)^2$$

$$= 6000\,\pi \text{ Newton}$$

From bearing of plate material

$$P = \sigma_{bearing} \times A_{bearing}$$

$$6000\pi = 120 \times (20t)$$

$$\therefore \quad t = \left(\frac{6000\,\pi}{120 \times 20}\right) \text{mm} = 7.849 \simeq 7.85 \text{ mm} \qquad \textbf{Ans.}$$

(b) Largest average tensile stress in the plate

$$P = \sigma \cdot A$$

$$6000\,\pi = [7.85 \times (110 - 20)]$$

$$\sigma = \left(\frac{6000\,\pi}{7.85 \times 90}\right) \text{N/mm}^2 \text{ or MPa}$$

$$= 26.69 \text{ MPa} \simeq 26.7 \text{ MPa}$$ **Ans.**

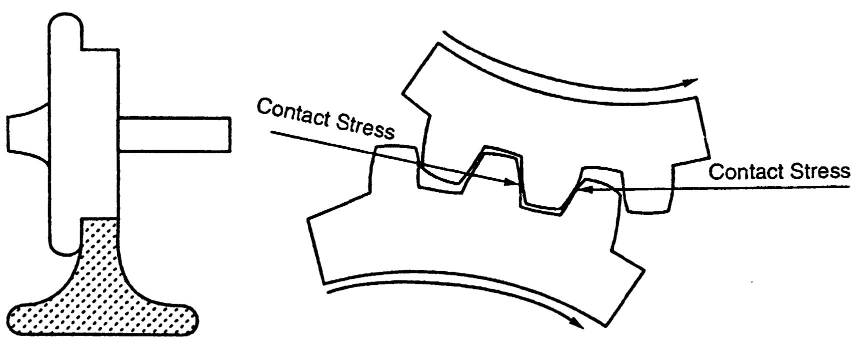
Contact Stress

In the case of bearing stress, applied load is distributed over a relatively large area. **But when the load applied is over a very small area, contact stress comes in the picture.**

Examples of contact stress are:

(i) steel rail wheel on a rail

(ii) two convex curved surfaces such as gear teeth in contact.



Rail wheel on rail

Fig. 1.12

Hydrostatic Stress*

This stress arises when body is subjected to equal external pressure at all points on the body *e.g.* when the body is immersed in a fluid at a large depth. Hydrostatic stress is compressive in nature and is equal to the pressure intensity to which the body is subjected. It is denoted by letter p.

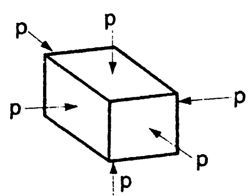


Fig. 1.12 (a)

True Stress

True stress is equal to the load divided by the instantaneous cross-sectional area through which it acts. True stress occurs due to the change in cross-section that occurs with changing load

$$\sigma = \frac{P}{A_i} \quad \text{where } A_i = \text{instantaneous Cross-sectional Area}$$

Besides these stresses discussed so far there are other types of stresses also. These are:

Residual Stress: Residual stresses are due to manufacturing process that leave a stress in a material. Welding leaves residual stresses in the metal welded.

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*In metal forming process also hydrostatic stress arises.

Structural Stress: These stresses are produced in structural member because of the weights they support. These stresses are found in building foundation and framework as well as in machine parts.

Pressure Stress: Pressure stresses are induced in vessel containing pressurized fluid.

Flow Stress: Flow stress occurs when a mass of flowing fluid induce a dynamic pressure on a conduit wall.

Fatigue Stress: Fatigue stress arises due to cyclic loading. Rotation of shaft continuously develops a fatigue stress.

*These stresses are used in various engineering field.

1.4 STRAIN

The study of strain in a elastic body is the study of the displacement of points in the body relative to another when the body is deformed.

A load carrying member deforms under the influence of load applied.

Common forms of deformation or distortion** are —

(i) elongation (ii) twisting
(iii) bending (iv) shearing

Strain is also called unit deformation and is found by dividing the total deformation by the original length of bar.

Strain is denoted by letter epsilon (ε)

$$\text{Strain } (\varepsilon) = \frac{\text{Total deformation}}{\text{Original length}}$$

Strain may be longitudinal denoted by (ε_l)
Strain may be shear denoted by (γ or ϕ)
Strain may be volumetric denoted by (ε_v)

$\varepsilon_l = \dfrac{\Delta l}{l_i}$, where Δl = total elongation; l_i = original length, l_f = final length

$\therefore \quad \Delta l = (l_f - l_i)$

Similarly, $\varepsilon_v = \dfrac{\Delta V}{V}$, where ΔV = change in volume; V = original volume.

$\Delta V = (V_f - V_i)$

where V_f = final volume

V_i = initial volume

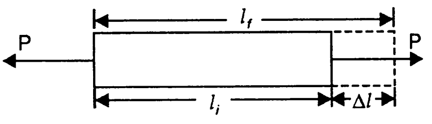

Fig. 1.13

Longitudinal strain (ε_l)

Longitudinal strain causes extension 'or' contraction in the body.

*Whether a machine member or structural member, stresses develop on these when loads are applied but as a designer or engineer, our aim should be to limit the stress by proper selection of material, factor of safety (FOS) and by suitable design in such a manner that machine member or structural member should not fail on application of loads. Unwanted stress leads to failure of the machine or structure.

**If the distortion disappears and the metal returns to its dimension upon removal of the load, the strain is called elastic strain. If the distortion disappears and metal remains distorted, the strain is called 'plastic strain'.

Shear strain (γ or ϕ)

Shear strain is the strain produced under shear stress.

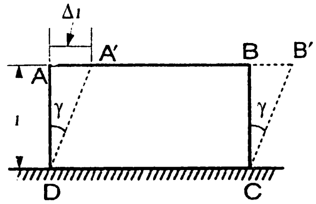

Fig.1.14

In the above figure

$$\text{Shear strain} = \tan\gamma = \frac{\Delta l}{l}$$

Shear strain causes angular distortion on the body.

Volumetric strain (ε_v) 'OR' Elastic dialation:

When a body is subjected to hydrostatic stress, it causes a change in the volume.

$$\varepsilon_v = \frac{\Delta V}{V}$$

Let a cuboid of sides $x, y,$ and z are subjected to hydrostaic stress and as a result of stress its sides change by dx, dy and dz respectively.

$$\therefore \quad \text{Change in volume} = (x + dx)(y + dy)(z + dz) - xyz$$

$$= (xyz + xydz + yzdx + zxdy) - xyz$$

$$\begin{bmatrix} \text{Neglecting smaller forms} \\ \text{viz } dx \cdot dy, dy \cdot dz \text{ etc.} \end{bmatrix}$$

$$= yzdx + zxdy + xydz$$

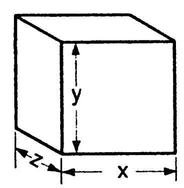

Fig.1.15

$$\therefore \quad \text{Volumetric strain } (\varepsilon_v) = \frac{yzdx + zxdy + xydz}{xyz}$$

$$= \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$$

$$= \varepsilon_x + \varepsilon_y + \varepsilon_z$$

This shows that volumetric strain is equal to the sum of linear strains in x, y and z direction.

1.5 MEASUREMENT OF STRAIN

The instrument by which strain is measured is called **extensiometer.** The most commonly used extensiometer are the resistance wire **electrical strain gauge** which is made of fine wire filament cemented on the surface of the body so that the wire of the strain gauge deforms as the surface of the body deforms. The operation of the strain gauge is based on the principle that as the wire elongates or shrinks, it's electrical resistance changes in proportion to the change of its length.

1.6 POISSON'S RATIO (ν)

When a body is subjected to a load, strain occurs in the lateral direction also which is opposite in sign to that of the direction of load applied.

It has also been observed that lateral strain is always proportional to the longitudinal strain.

If you take a rubber piece having dia. '*d*' and length *l* and if you apply a tensile load you will find that the length of rubber piece increases whereas the diameter decreases.

We define Poisson's ratio as the ratio of lateral strain to axial strain or longitudinal strain. Poisson's ratio is denoted by letter ν (nu).

i.e.

$$\text{Poisson's ratio } (\nu) = \frac{\text{Lateral strain}}{\text{Axial strain}} = \frac{-\varepsilon_l}{\varepsilon_a}$$

$$= \frac{\text{Change in dia./original dia.}}{\text{Change in length/original length}} = -\frac{\Delta d/d}{\Delta l/l}$$

when the sample piece has circular section**.

The negative sign on the lateral strain is introduced to ensure that Poisson's Ratio (ν) is a positive number.

Most commonly used metallic materials have a Poisson's ratio value between 0.25 and 0.35. Poisson's ratio is an elastic property of the material. For steel, it's value is approximately 0.3.

Elastomers and rubber may have Poisson's ratios approaching 0.50.

Approximate value of Poisson's Ratio (ν) of various material are given below:

Material	*Poisson's Ratio (ν)*
Aluminium	0.33
Brass	0.33
Cast Iron	0.27
*Concrete	0.10-0.25 (depending upon grade)
Copper	0.33
Phospher Bronze	0.35
Carbon and Alloy steel	0.29
Stainless steel (18-8)	0.30
Titanium	0.30

1.7 HOOK'S LAW

Hook's law states that within elastic limit stress is proportional to strain.

Mathematically $\sigma \propto \varepsilon$

or, $\sigma = E \cdot \varepsilon$ (when strain occurs in only one direction)

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*Concrete is graded according to it's compressive strength which varies from 14 MPa to 48 MPa. Tensile strength of concrete is extremly low.

**Section may be rectangular square elliptical or any shape

In that case,

$$\text{Poisson's ratio } (\nu) = \frac{\text{Change in Cross Section Area/Original Cross Section Area}}{\text{Change in length/original length}}$$

This constant of proportionality (E) is called modulus of elasticity of the material. E is a measure of the stiffness of a material. E is determined by the slope of straight line portion of the stress–strain curve. A material having a steeper slope on the strain–strain curve will be stiffer and will deform less under load than a material having less steep slope (Fig. 1.16).

Fig. 1.16 illustrates the concept by showing the straight line portion of the stress-strain curve for steel, titanium, aluminium and magnesium.

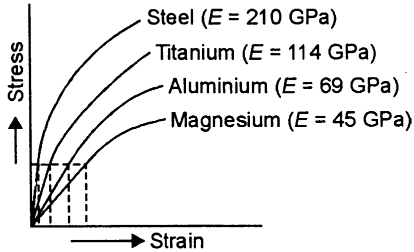

Fig. 1.16

Further, shear stress (τ) $\propto$ shear strain (γ)

or, $\tau = G \cdot \gamma$

$\therefore$ $G = \dfrac{\tau}{\gamma}$

The constant of proportionality (G) is called the **'modulus of rigidity'.**

Hook's law is obeyed within certain limit by most ferrous alloys and other engineering materials such as concrete; timber and non-ferrous alloys with reasonable accuracy.

Poisson's ratio permits us to extend Hook's law of uniaxial stress to the biaxial and triaxial stresses also. Body subjected to σ_x and σ_y or σ_y and σ_z or σ_z and σ_x only are called biaxial stress and a body subjected to 3 stresses *i.e.* σ_x; σ_y and σ_z are called 3-D stresses. Biaxial stresses and triaxial stresses arise in the analysis of pressure vessels, beams, shaft and any other structure parts.

If an element is subjected to tensile stresses in the x and y direction then the strain in the x direction due to tensile stress σ_x is $\dfrac{\sigma_x}{E}$. Similtaneously, the tensile stress σ_y will produce lateral contraction in the x-direction of amout $(-)\nu \cdot \dfrac{\sigma_y}{E}$. So, the resultant strain in the x-direction will be

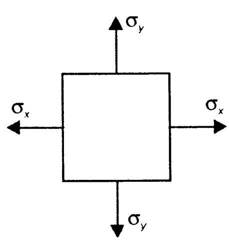

Fig. 1.17

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \cdot \frac{\sigma_y}{E}$$

Similarly, total strain in the y-direction will be

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \cdot \frac{\sigma_x}{E}$$

Poisson's ratio (ν) are 0.25 to 0.30 for steel, approximately 0.33 for most other metals, 0.20 and 0.5 for concrete and rubber, respectively.

1.8 GENERALISED HOOK'S LAW (for Linear Elastic and Isotropic Materials)

Let an element of an elastic body is subjected to three dimensional stress σ_x, σ_y and σ_z along x, y and z axes, respectively.

Strain due to σ_x is $\varepsilon_x = \dfrac{\sigma_x}{E}$ (longitudinal)

and lateral strain $\varepsilon_y = \varepsilon_z = -\nu \cdot \dfrac{\sigma_x}{E}$ (lateral)

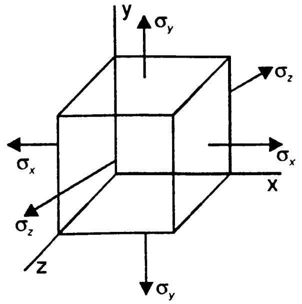

Fig. 1.18

Similarly, stress σ_y applied along y-axis produces longitudinal strain $\varepsilon_y = \frac{\sigma_y}{E}$ and lateral strain $\varepsilon_x = \varepsilon_z = -\nu \cdot \frac{\sigma_y}{E}$ and stress σ_z applied along z-axis produces longitudinal strain $\varepsilon_z = \frac{\sigma_z}{E}$ and lateral strain $\varepsilon_x = \varepsilon_y = -\nu \cdot \frac{\sigma_z}{E}$

Therefore, total strain

along x-axis $\quad \varepsilon_x = \frac{\sigma_x}{E} - \nu \cdot \frac{\sigma_y}{E} - \nu \cdot \frac{\sigma_z}{E}$

along y-axis $\quad \varepsilon_y = \frac{\sigma_y}{E} - \nu \cdot \frac{\sigma_z}{E} - \nu \cdot \frac{\sigma_x}{E}$

along z-axis $\quad \varepsilon_z = \frac{\sigma_z}{E} - \nu \cdot \frac{\sigma_x}{E} - \nu \cdot \frac{\sigma_y}{E}$

$$\therefore \quad \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\sigma_x}{E}(1 - 2\nu) + \frac{\sigma_z}{E}(1 - 2\nu)$$

$$\therefore \quad \text{Volumetric strain } (\varepsilon_v) = (\varepsilon_x + \varepsilon_y + \varepsilon_z) = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)(1 - 2\nu)$$

also known as Generalised Hook's law.

1.9 STRESS–STRAIN CURVE

The behaviour of ductile materials such as steel* which is subjected to tensile load is studied by a testing specimen in a tensile testing machine. Steel refers to alloy of iron and carbon and in many cases, other elements. Steels are classified as carbon steels, alloy steel and stainless steel.

The result of tensile test are expressed by means of stress–strain relationship and plotted in the form of a graph as shown in Fig. 1.18(a).

Stress–strain curve generated from tensile test result help engineers to know constitutive relationship between stress and strain for a particular material.

The typical region in stress–strain curve are—

(a) elastic region (b) yielding (c) strain hardening (d) necking and failure

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*According to American Iron and Steel Institute (AISI), a steel is considered to be carbon steel when carbon percentage is 0.16–0.29% (also called mild steel).

Density of mild steel = 7.85 g/cm^3, $E = 210 \times 10^3$ MPa

Carbon steel is also called as plain carbon steel.

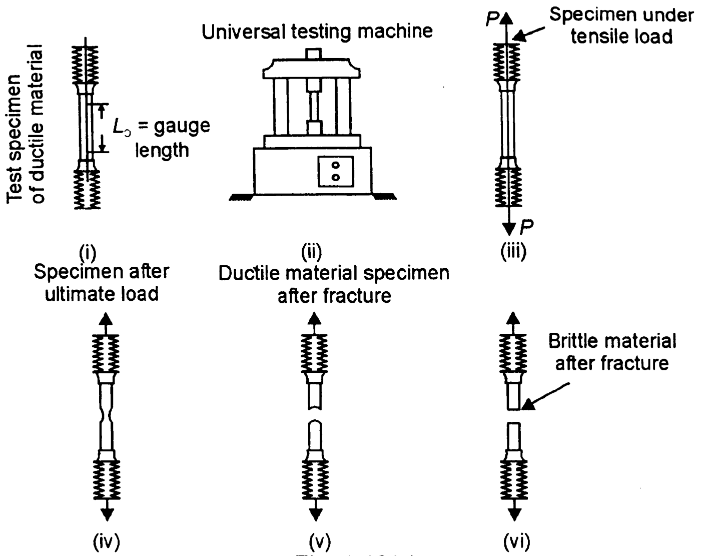
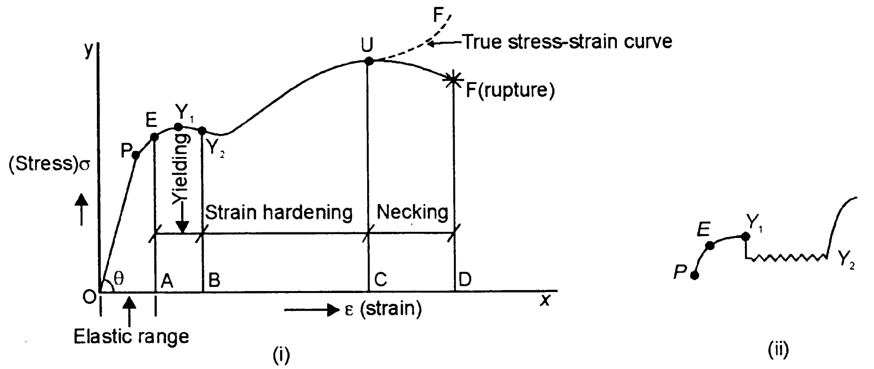


Fig. 1.18(a)

In the following graph:

P denotes proportional limit	E denotes elastic limit
Y_1 denotes upper yield point	Y_2 denotes lower yield point
U denotes ultimate strength	F denotes fracture point



Stress–Strain Curve for Mild Steel

Fig. 1.18(b)

When the loads are increased gradually, the strain (ε) is proportional to load or stress up to point (P). So, P is called proportional limit. Hook's law is applicable in this region.

N.B.: Yield criterion often expressed as yield surface or yield locus, is an hypothesis concerning the limit of elasticity under any combination of stresses.

Lower carbon steels suffer from yield point runout where the material has two yield points. The first yield point (Y_1) is higher than second yield (Y_2) and yield drops dramatically (Y_2) after upper yield power (Y_1). If a low carbon steel is only stressed to some point between the upper and lower yield point then the surface may develop LUDER BANDS. A luder band is a localized band of plastic deformation which occurs in certain materials before fracture.

Even if the specimen is beyond P and up to E it will regain its initial size and shape when the load is removed. This indicates that the material is in elastic range upto point E. Therefore, E is called the elastic limit. **Elastic limit of the material is defined as the maximum stress without any permanent deformation.**

The proportional limit and elastic limit* are very close to each other and it is difficult to distinguish between points P and E on the stress-strain diagram.

Many a times these two are taken to be equal. When the specimen is stressed beyond E, plastic deformation occurs and material starts yielding. During this stage, it is not possible to recover the initial size and shape of the specimen on the removal of load.

Beyond E, the strain increases at a faster rate up to point Y_1 *i.e.*, there is an appreciable increase in strain without much increase in stress.

In the case of mild stress (M.S.) there is small reduction in load and the curve drops down to point Y_2 immediately after yielding starts.

Y_1 is the upper yield point

and Y_2 is the lower yield point

upper and lower yield points can be obtained only in a carefully controlled test, otherwise the region from E and Y_1 to Y_2 merges in to one curve nearly horizontal apparently an increase of strain from 0.12% to 2% at a constant rate from Y_1 to Y_2, is an unstable region and the curve may show many peaks and valley as shown in Figure 1.18 (b). The onset of plastic deformation is called yielding of material. The lower value is called yield stress.

The yield strength is defined as the maximum stress at which a marked increase in elongation occurs without increases in the load.

After the yield point, if the load is increased further then the stress curve rises upto point U. The stress corresponding to point U is called ultimate stress of the specimen.

Increasing stress after yield point shows that material is becoming stronger as it deforms. This is known as ****work hardening or strain hardening.**

After point U, necking of specimen starts and at point F, fracture of test specimen occurs so F is called fracture point. The stress corresponding to point F is called 'Breaking stress'.

Dotted line in the curve shows the actual or true stress-strain diagram.

The structural steel that contains 0.2% carbon as an alloy is classified as low carbon steel. **With increase in carbon content, steel becomes less ductile but has higher yield stress and higher ultimate stress.**

For many ductile materials other than mild steel *e.g.*, aluminium, copper etc. no definite yield point is obtained. For such materials the shape of curve on stress-strain diagram is shown in Figure 1.18 (c).

In this case, an arbitrary yield stress may be determined by offset method.

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*Most of engineering structure are designed to function within their elastic range only.

To remove strain hardening we use heat treatment process commonly known as **annealing. After proper annealing the material returns to its original state.

In this method a line MN is drawn parallel to initial slope line OA on the stress-strain curve but is offset by some standard amount of strain such as 0.002 (or 0.2%). The intersection of line and the stress-strain curve defines the yield stress (Y). The yield stress determined in this way is called **Proof stress.**

High tensile steels, aluminium alloys, copper fall in this category.

The actual or true stress-strain curve is shown by dotted line in Fig. 1.18 (a) for mild steel and in Fig 1.18(c) for aluminium/copper.

Steel and aluminium alloy specimen exhibit ductile behaviour and a fracture occurs only after a considerable amount of deformation.

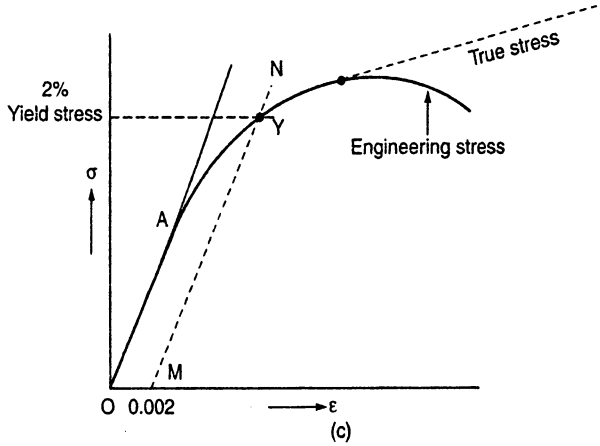


Fig. 1.18 (c)

The failure of steel and aluminium occur primarily due to shear strain along the plane forming 45° angle with the axis of the rod.

A typical 'cup and cone' fracture may be detected to steel and aluminium specimen. But the failure of cast iron occurs suddenly exhibiting a square fracture across the cross-section (See Figs. 1.18 (e) and (f)).

In the above discussion load was applied gradually. But under rapid or **impact loading*** two additional material parameters have relevance and these are—resilience and toughness.

Resilence defines the ability of material to absorb energy without suffering plastic strain.

Toughness defines the ability of material to absorb energy prior to fracture.

While using equation $\sigma = \varepsilon \cdot E$; it does not matter if we are loading or unloading the specimen, the stress-strain response is a straight line of slope E.

A material having proportional limit close to elastic limit such as steel is termed a linear, elastic material. For stress value below the elastic limit, the loading and unloading response follow the same path. All materials do not behave as that of steel.

*We will discuss 'impact loading' in the chapter—strain energy and its application.

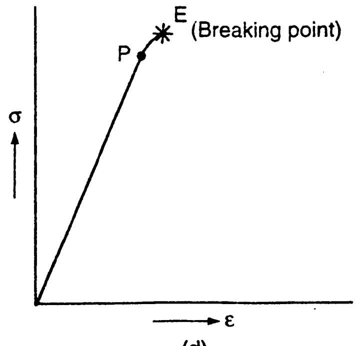


Fig. 1.18 (d)

Brittle materials do not exhibit the yield point. Materials that fail in tension at relatively low values of strains are classified as brittle materials. Examples are concrete, stone, cast iron, glass, ceramic material and many common alloys.

N.B. Stress-strain diagrams of various materials vary widely and different tensile tests conducted on the same material may yield different result depending upon the temperature of the specimen and the speed of the loading.

Stress-Strain Diagram for Concrete*

For a given steel, the **yield strength** is the same in both **tension** and **compression.**

For most 'brittle materials', ultimate strength in compression is much larger than the ultimate strength in tension. This is due to the presence of flaws such as microscopic cracks or Cavities which tend to weaken the material in tension. Properties in tension and compression of concrete is shown in Fig. 1.18(e).

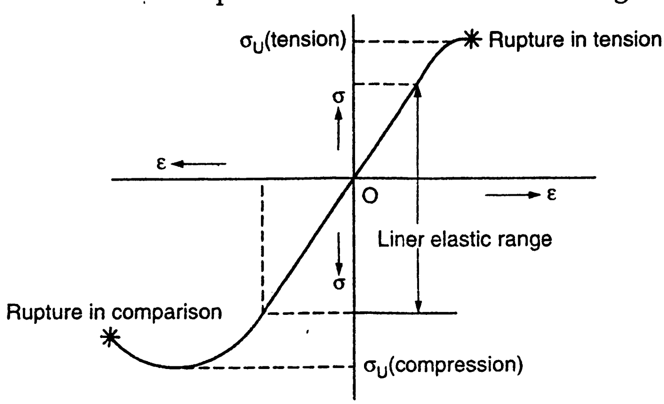


Fig. 1.18 (e) Stress-strain diagram for concrete

Value of E for common materials:

Material	*E (GPa)*
Steel	200-210
Copper	120
Aluminium	70
Bronze	83
Cast iron	100

*Concrete is an anisotropic material.

1.10 DIFFERENCE BETWEEN ENGINEERING STRESS AND TRUE STRESS

[MTU-2011-12 (2nd Semester)]

Particular	*Engineering stress*	*True stress*
1. Stress formula	Engg. Stress $(\sigma) = \frac{P}{A}$ Here, A is the original area of cross-section.	True stress $(\sigma_t) = \frac{P}{A}$ Here, A is the deformed area of cross-section after the load application.
2. Behaviour of material on application of load	Engineering stress (σ) is directly proportional to the load P, decreases with P during necking phase up to fracture point or breaking point of the specimen of ductile material.	True stress (σ_t) is proportional to P but also inversely proportional to A. True stress increases until rupture of the specimen occurs (See Fig. 1.18(a)).
3. Uses	Elastic behaviour of a deformable body is studied by Engineering stress.	Plastic behaviour of a material is studied by true stress.

1.11 THERMAL STRESS AND STRAIN

Owing to temperature variation in the material, stresses and strains are setup. Metal expand on heating and contract on cooling.

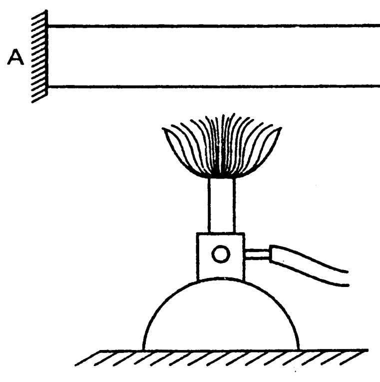


Fig. 1.19

If their expansion or contraction is prevented as shown in Fig. 1.19, then thermal stresses and strains are produced.

The increased length of the rod is given by

$$l_t = l_o (1 + \alpha \Delta T)$$

l_o = initial length of the rod

where l_t = final length of the rod at increased temperature

α = coefficient of thermal expansion

ΔT = rise in temperature

$= (T_f - T_i)$

where T_f = final temperature and T_i = initial temperature

$$\text{Thermal strain } (\varepsilon_l) = \frac{l_t - l_o}{l_o}$$

$$= \frac{l_o(1 + \alpha\Delta T) - l_o}{l_o}$$

$$= \alpha\Delta T$$

and Thermal stress $(\sigma_{\text{thermal}}) = E \cdot \alpha \cdot \Delta T$.

1.12 THERMAL STRESSES IN COMPOSITE BAR (Fig. 1.20)

Let us consider a composite bar (rod-1 and tube 2) as shown in Fig. 1.20.

If the rod '1' and tube '2' were not connected with each other then their expansion would be $L \cdot \alpha_1 \Delta T$ and $L \cdot \alpha_2 \Delta T$ respectively.

However, as the two are connected at the ends so thermal stresses and strains will be produced.

Coefficient of expansion (α) is different for different materials. Here α_1 is more than α_2, so rod 1 will expand more than tube 2.

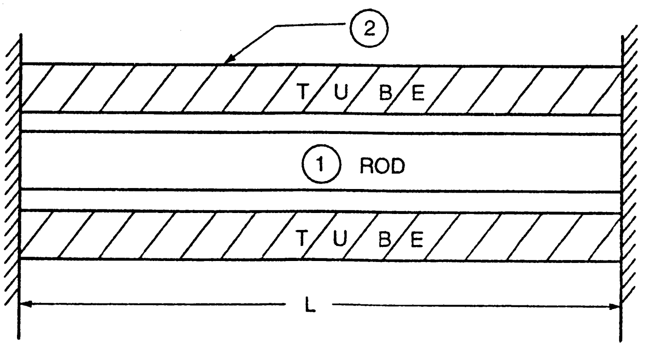


Fig. 1.20

Since there is no external force applied to the above composite bar, the sum of forces due to thermal stresses will be zero.

i.e. $P_2 - P_1 = 0$

or, $P_2 = P_1 = P$ (Assume)

Now, $\Delta L_1 + \Delta L_2 = L \cdot \alpha_1 \cdot \Delta T - L \cdot \alpha_2 \cdot \Delta T$

$$= L\Delta T(\alpha_1 - \alpha_2)$$

Also, $\Delta L_1 = \dfrac{PL}{A_1 E_1}$

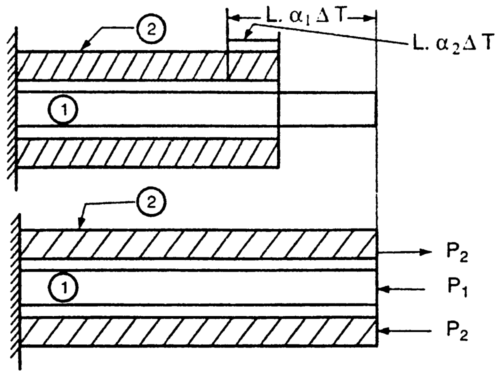


Fig. 1.21

and, $\Delta L_2 = \dfrac{PL}{A_2 E_2}$

$$\therefore \quad \Delta L_1 + \Delta L_2 = PL\left(\frac{1}{A_1E_1} + \frac{1}{A_2E_2}\right)$$

or, $$L\Delta T\,(\alpha_1 - \alpha_2) = PL\left(\frac{1}{A_1\,E_1} + \frac{1}{A_2\,E_2}\right)$$

$$\therefore \quad P = \frac{(\alpha_1 - \alpha_2)\,\Delta T}{\left(\dfrac{1}{A_1E_1} + \dfrac{1}{A_2E_2}\right)}$$

Therefore, thermal stress in the rod

$$\sigma_1 = \frac{P}{A_1}$$

$$= \frac{(\alpha_1 - \alpha_2)\,\Delta T}{\left(\dfrac{1}{E_1} + \dfrac{A_1}{A_2}\cdot\dfrac{1}{E_2}\right)}$$

and thermal stress in the tube

$$\sigma_2 = \frac{P}{A_2} = \frac{(\alpha_1 - \alpha_2)\,\Delta T}{\left(\dfrac{A_2}{A_1}\cdot\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)}$$

EXAMPLE 1 *A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at – 20°C, what is the minimum diameter of the rod? Assume* $\alpha = 11.7$ *μ m/m–°C and* $E = 200$ *GPa.*

SOLUTION Actual expansing of steel rod = Free expansion of steel rod + Expansion due to tensile load.

i.e. $\delta = \delta_{\text{thermal}} + \delta_{\text{tensile load}}$

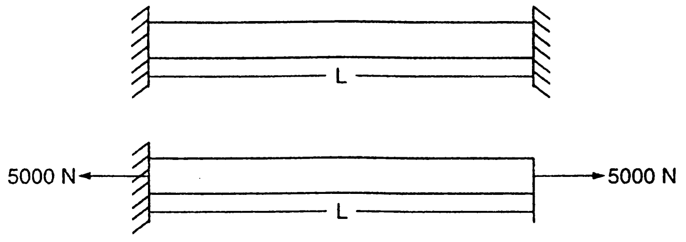


Fig. 1.22

$$\frac{\sigma \cdot L}{E} = L \cdot \alpha \cdot \Delta T + \frac{P \cdot L}{A \cdot E}$$

$$\sigma = L\,\alpha\,\Delta T \times \frac{E}{L} + \frac{PL}{AE} \times \frac{E}{L}$$

$$\sigma = \alpha E \Delta T + \frac{P}{A}$$

$$130 = 11.7 \times 10^{-6} \times 2 \times 10^{5} \times 40 + \frac{5000}{A}$$

$\Rightarrow$ $$A = \left(\frac{5000}{36.4}\right)$$

or, $$\frac{\pi}{4} d^2 = \frac{5000}{36.4}$$

$\therefore$ $$d = \sqrt{\frac{5000 \times 4}{36.4 \times \pi}}$$

$$= 13.22 \text{ mm}$$ **Ans.**

EXAMPLE 2 *Steel railroad rails 10 m long are laid with a clearance of 3 mm at a temperature at 15°C. At what temperature will the rails just touch? What stress would be induced in the rails? At that temperature if there were no initial clearance? Assemed $\alpha = 11.7$ μm/m°C and E = 200 GPa.*

SOLUTION $T_1 = ?$ and $T_2 = 15°C$

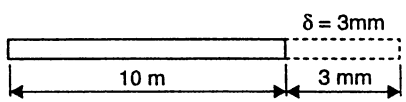


Fig. 1.23

$$\delta_T = L \cdot \alpha \cdot \Delta T$$

$$= \alpha L (T_1 - T_2)$$

$$3 = 11.7 \times 10^{-6} \times 1000 \times (T_1 - 15)$$

$\Rightarrow$ $$T_1 = 40.64°$$

Required stress $\sigma_{th} = \alpha \cdot E \cdot (T_1 - T_2)$

$$= 11.7 \times 10^{-6} \times 200 \times 10^{3}\,(40.64 - 15)$$

$$= 11.7 \times 10^{-6} \times 25.64$$
$$= 59.99$$
$$\simeq 60 \text{ MPa}$$ **Ans.**

EXAMPLE 3 *A bronze bar 3 m long with a cross-sectional area of 320 mm² is placed between two rigid walls as shown in figure. At a temperature of – 20°C, the gap Δ = 2.5 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. (Use α = 18 × 10⁻⁶ m/m °C and E = 80 GPa)*

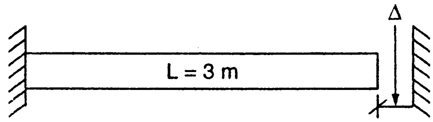


Fig. 1.24

SOLUTION Here $\delta_{\text{thermal}} = \delta + \Delta$

$$\alpha \cdot L \cdot \Delta T = \frac{\sigma L}{E} + 2.5$$

$$18 \times 10^{-6} \times 3000 \times \Delta T = \frac{35 \times 3000}{80 \times 10^3} + 2.5$$

$$\Delta T = \frac{3.8125}{18 \times 10^{-6} \times 3000} = 70.60°\text{C}$$

$$T = 70.60 - 20 = 50.60°\text{C}$$ **Ans.**

EXAMPLE 4 *A rigid bar of negligible weight is supported as shown in Fig. 1.25. If W = 80 kN. Compute the temperature change that will cause the stress in the steel rod to be 55 MPa.*

Assume the coefficient of linear expansion are 11.7 μm/m°C for steel and 18.9 μm/m°C for bronze.

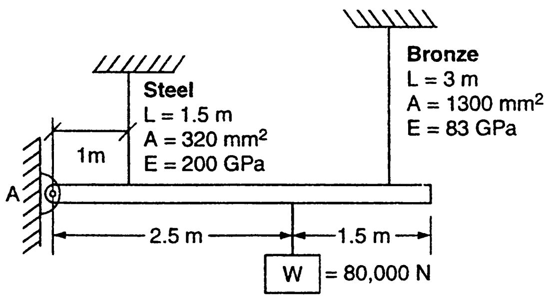


Fig. 1.25

SOLUTION $\Sigma M_A = 0$ gives

$$4P_{br} + P_{st} = 2.5 \times 80{,}000$$

$$4\sigma_{br} \times 1300 + 55 \times 320 = 2.5 \times 80{,}000$$

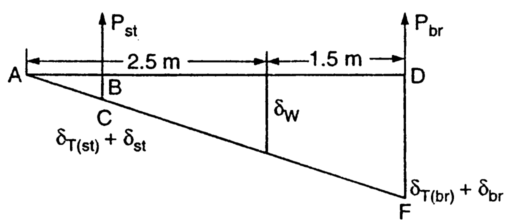


Fig. 1.26

$$\sigma_{br} = \frac{2.5 \times 80{,}000 - 55 \times 320}{4 \times 1300} = 35.08 \text{ MPa}$$

From similar triangles, ABC and ADF

$$\frac{\delta_T(st) + \delta_{ST}}{1} = \frac{\delta_T(br) + \delta_{br}}{4}$$

$$\delta_T(st) + \delta_{ST} = 0.25\,[\delta_T(br) + \delta_{br}]$$

$$(\alpha\, L\Delta T)_{st} + \left(\frac{\sigma \cdot L}{E}\right)_{st} = 0.25\left[(\alpha L\Delta T)_{br}\left(\frac{\sigma \cdot L}{E}\right)_{br}\right]$$

$$11.7 \times 10^{-6} \times 1500\,\Delta T + \frac{55 \times 1500}{2000}$$

$$= 0.25\left[18.9 \times 10^{-6} \times 3000\,\Delta T + \frac{35.08 \times 3000}{83000}\right]$$

$$\Delta T = 28.3°\text{C}$$

A temperature drop of 28.3°C is needed to stress the steel to 55 MPa.

EXAMPLE 5 *At a temperature of 80°C, a steel tyre 12 mm thick and 90 mm wide that is to be shrunk on to a locomotive driving wheel 22 m in diameter just fits over the wheel, which is at a temperature of 25°C.*

Determine the contact pressure between the tyre and wheel after the assembly cools to 25°C. Neglect the deformation of the wheel caused by the pressure of the tyre.

Assume $\alpha = 11.7$ μm/m°C and $E = 200$ GPa.

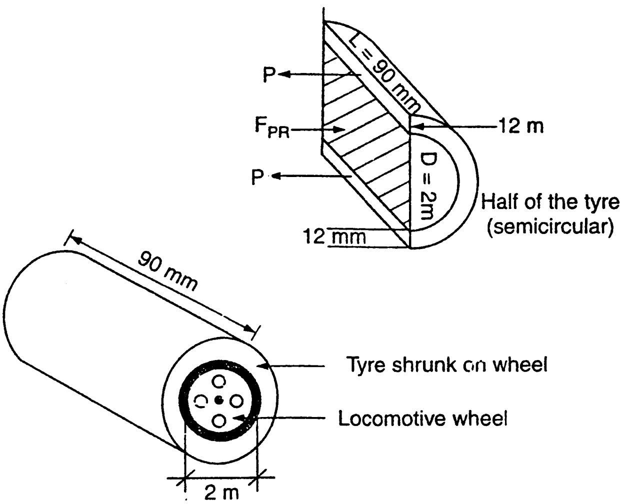


Fig. 1.27

SOLUTION We have

$$\delta = \delta_T$$

$$\frac{P \cdot L}{A \cdot E} = L \cdot \alpha \cdot \Delta T$$

$$\Rightarrow \qquad P = AE\,\alpha\,\Delta T$$
$$= (90 \times 12) \times 200 \times 10^3 \times 11.7 \times 10^{-6} \times (80 - 25)$$
$$= 1080 \times 200 \times 10^3 \times 11.7 \times 55$$
$$= 128996 \text{ Newton}$$

Now $\quad F_{\text{projected}} = 2P$

$$p \cdot D \cdot L = 2P$$
$$p\,(2000)\,(90) = 2 \times 138996$$
$$\therefore \qquad p = 1.544 \text{ MPa}$$

Ans.

1.13 VOLUMETRIC STRAIN OF A RECTANGULAR BAR WHEN SUBJECTED TO UNIAXIAL LOAD *P*

Let a rectangular bar is subjected to load P along x-axis.

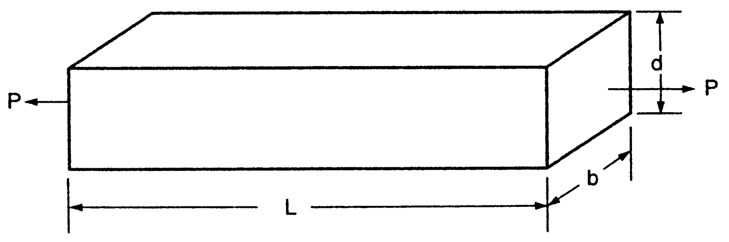


Fig. 1.28

Let $\quad L$ = length of bar
$\quad b$ = width of bar
and $\quad d$ = depth of bar.

Let due to application of load,

Change in length $= \delta L$
Change in width $= \delta b$
and Change in depth $= \delta d$

$\therefore$ Final dimensional of the bar will be

$$(L \times \delta L) \times (b + \delta b) \times (d + \delta d)$$

$\therefore$ Final volume $= (L + \delta L)(b + \delta b)(d + \delta d)$

(neglecting product of small quantities)

$\therefore$ Change in volume $(\delta_v) = (V_f - V_i)$

$$= (Lbd + bd\delta L + Lb\delta d + Ld\delta d) - lbd$$
$$= bd\delta L + Lb\delta d + Ld\delta b$$

$\therefore$ Volumetric strain $(\varepsilon_v) = \dfrac{\delta V}{V}$

$$\varepsilon_v = \frac{\delta L}{L} + \frac{\delta d}{d} + \frac{\delta b}{b}$$

$\frac{\delta d}{d}$ and $\frac{\delta b}{b}$ are lateral strain.

$\therefore$ ε_v = longitudinal strain + 2 × lateral strain

= longitudinal strain – 2 × ν longitudinal strain

$$= \frac{\delta L}{L} - 2 \times \nu \times \frac{\delta L}{L}$$

$\therefore$ Volumetric strain of rectangular bar subjected to uniaxial load P

$$= \frac{\delta L}{L}(1 - 2\nu)$$

1.14 VOLUMETRIC STRAIN OF A CYLINDERICAL ROD

Let the initial length of rod = L and dia. of rod = d

After uniaxial load P, let its length increases to $(L + \delta L)$ and diameter decrease to $(d - \delta d)$

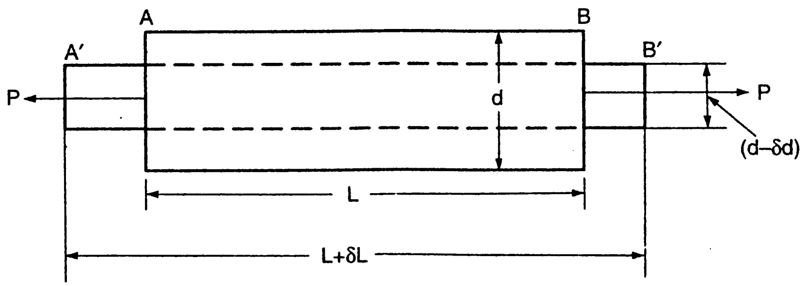


Fig. 1.29

$\therefore$ Change in volume $(\delta_v) = (V_f - V_i)$

$$= \frac{\pi}{4}\left\{(d - \delta d)^2 \cdot (L + \delta L)\right\} - \left\{\frac{\pi}{4} d^2 L\right\}$$

$$= \frac{\pi}{4}\left\{\cancel{d^2 L} - 2dL\delta d + d^2\delta L - \cancel{d^2 L}\right\}$$

$$= \frac{\pi}{4}\left\{d^2\delta L - 2dL\delta d\right\}$$

$\therefore$ Volumetric strain $(\varepsilon_v) = \frac{\delta_V}{V}$

$$= \frac{\cancel{\frac{\pi}{4}}(d^2\delta L - 2dL \cdot \delta d)}{\cancel{\frac{\pi}{4}}\, d^2 L}$$

$\therefore \quad \varepsilon_v$ of cylinder rod $= \dfrac{\delta L}{L} - \dfrac{2\delta d}{d}$

$= \{\text{longitudinal strain} - 2 \times \text{diametral strain}\}$

1.15 VOLUMETRIC STRAIN OF A RECTANGULAR BLOCK SUBJECTED TO NORMAL STRESSES ON ALL ITS FACE

Let us consider a rectangular block having dimensions $x \times y \times z$ so that

$AB = x$

$BF = z$ and $BC = y$

Let the block is subjected to stress σ_x, σ_y and σ_z along x, y and z axes respectively. As a result of stresses the block will be strained along x, y and z axes. Let strain along x, y and z axes be $\varepsilon_x, \varepsilon_y$ and ε_z respectively.

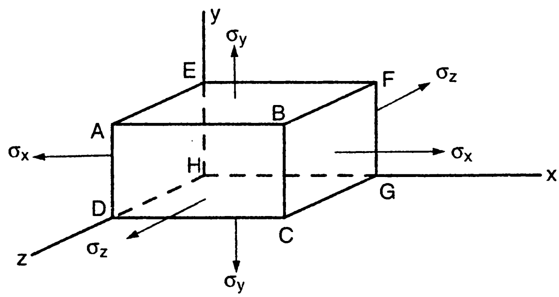


Fig. 1.30

Hence, $\varepsilon_x = \dfrac{\sigma_x}{E} - \nu \cdot \dfrac{\sigma_y}{E} - \nu \cdot \dfrac{\sigma_z}{E}$

$\varepsilon_y = \dfrac{\sigma_y}{E} - \nu \cdot \dfrac{\sigma_x}{E} - \nu \cdot \dfrac{\sigma_z}{E}$

$\varepsilon_z = \dfrac{\sigma_z}{E} - \nu \cdot \dfrac{\sigma_x}{E} - \nu \cdot \dfrac{\sigma_y}{E}$

Now, volumetric strain $(\varepsilon_v) = \varepsilon_x + \varepsilon_y + \varepsilon_z = \dfrac{\sigma_x}{E}(1-2\nu) + \dfrac{\sigma_y}{E}(1-2\nu) + \dfrac{\sigma_z}{E}(1-2\nu)$

$$= \left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right) - \frac{2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E} - \frac{2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{(\sigma_x + \sigma_y + \sigma_z)}{E}[1 - 2\nu]$$

If a body is subjected to uniform hydrostatic pressure p, then each of the stress components is equal to $-p$, therefore volumetric strain

$$(\varepsilon_v) = \frac{-p-p-p}{E}(1-2\nu) = (-)\frac{3p}{E}(1-2\nu)$$

EXAMPLE 1 *A specimen of any given material is subjected to a uniform triaxial stress. Determine the theoretical maximum value of Poisson's ratio.*

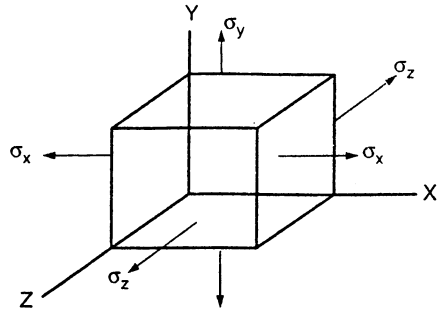


Fig. 1.31

SOLUTION Let the specimen (as shown in Figure) is subjected to σ_x, σ_y and σ_z along x, y and z axes respectively.

Then $$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \quad \text{...(I)}$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \quad \text{...(II)}$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \quad \text{...(III)}$$

Adding I, II, and III

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z) - \frac{2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\nu) \quad \text{...(IV)}$$

For uniform triaxial stress we have $\varepsilon_x = \varepsilon_y = \varepsilon_z$ and $\sigma_x = \sigma_y = \sigma_z$

Hence, eq. (IV) reduces to

$$3\varepsilon = \frac{3\sigma}{E}\{1 - 2\nu\}$$

$$\varepsilon = \frac{\sigma}{E}\{1 - 2\nu\}$$

Since ε and σ must be of the same sign if follows that $(1 - 2\nu)$ must be positive *i.e.* $1 - 2\nu \geq 0$

$$\Rightarrow \quad \nu \leq \left(\frac{1}{2}\right)$$ **Ans.**

EXAMPLE 2 *A solid cylinder of diameter d has an axial load P. Show that it's change in diameter is* $\left(\frac{4P\nu}{\pi E d}\right)$.

SOLUTION The load can be considered tensile or compressive. Here, let us consider tensile load P.

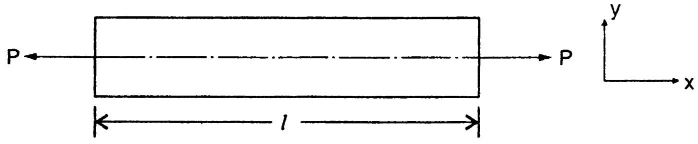


Fig. 1.32

$$\text{Poisson ratio } (\nu) = (-)\frac{\text{lateral strain}}{\text{longitudinal strain}} = (-)\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\nu \cdot \varepsilon_x$$

where ε_y = diametral strain along y-axis

ε_x = longitudinal strain along x-axis

$$\left(\frac{\delta d}{d}\right) = -\nu \cdot \varepsilon_x$$

$$\delta d = -\nu \cdot d \cdot \varepsilon_x = -\nu \cdot d \cdot \frac{\Delta l}{l}$$

$$= -\nu \cdot d \cdot \frac{P}{AE}$$

$$= -\nu \cdot d \cdot \frac{P}{\frac{\pi}{4} d^2 E}$$

$$= (-)\frac{4P\nu}{\pi E d}$$

(–) sign, since there is decrease in diameter when the cylinder is subjected to tensile load along x-axis.

EXAMPLE 3 *A rectantular block 1 m long, 0.4 m wide and 20 mm thick is subjected to biaxial stresses σ_x and σ_y acting along length and width respectively. If the increase in length is 0.6 mm and increases in width is 0.09 mm find*

(a) σ_x and σ_y

(b) change is thickness of the plate

(c) change is volume of the plate

SOLUTION Given $\delta_l = 0.6\text{mm}$; $\delta_b = 0.09\text{mm}$; $\delta_z = 0$

$l = 1\text{m} = 1000\text{mm}$

$E = 200 \times 10^3$ MPa or N/mm²

$b = 0.4\text{m} = 400$ mm

$t = 20\text{mm}$

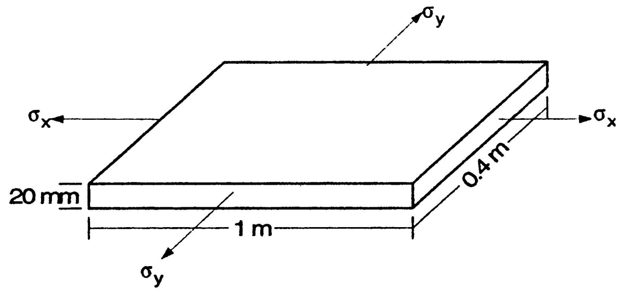


Fig. 1.33

(a) For biaxial system:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu \cdot \sigma_y)$$

$$\frac{\delta l}{l} = \frac{1}{E}(\sigma_x - \nu \cdot \sigma_y)$$

$$\frac{0.6}{1000} = \frac{1}{200 \times 10^3}(\sigma_x - 0.25\sigma_y)$$

$$\therefore \qquad \sigma_x - 0.25\,\sigma_y = 120 \qquad \ldots(\mathrm{I})$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu \cdot \sigma_x)$$

$$\frac{\delta b}{b} = \frac{1}{E}(\sigma_y - \nu \cdot \sigma_x)$$

$$\frac{0.9}{400} = \frac{1}{200 \times 10^3}(\sigma_y - 0.25\sigma_x)$$

$$\therefore \qquad -0.25\,\sigma_x + \sigma_y = 45 \qquad \ldots(\mathrm{II})$$

Solving (I) and (II) $\sigma_x = 140$ MPa and $\sigma_y = 80$ MPa **Ans.**

(b) Change in thickness

$$\varepsilon_t = \frac{\delta t}{t} = \frac{1}{E}(\sigma_z - \nu \cdot \sigma_x - \nu\sigma_y)$$

$$\frac{\delta t}{20} = \frac{1}{200 \times 10^3}(0 - 0.25 \times 140 - 0.25 \times 80)$$

$$\Rightarrow \qquad \delta t = -0.0055 \text{ mm}$$ **Ans.**

(c) Volumetric strain

$$\varepsilon_V = (\varepsilon_x + \varepsilon_y + \varepsilon_z)$$

$$= \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta t}{t}$$

$$= \frac{0.6}{1000} + \frac{0.09}{400} + \frac{-0.0055}{20}$$

$$= 0.55 \times 10^{-3}$$

So, change in volume $(dV) = \varepsilon_V \cdot V$

$$= 0.55 \times 10^{-3} \times (1000 \times 400 \times 20)$$

$$= 4400 \text{ mm}^3$$

Since value of dV is positive so there is increase in volume of the plate. **Ans.**

EXAMPLE 4 *A circle of diameter $d = 220$ mm is made on a unstressed aluminium plate 400 mm × 400 mm and of thickness $t = 20$ mm. Forces acting on the plate cause normal stresses $\sigma_x = 80$MPa and $\sigma_y = 140$ MPa. For $E = 70$ GPa and $\nu = \frac{1}{3}$ determine:*

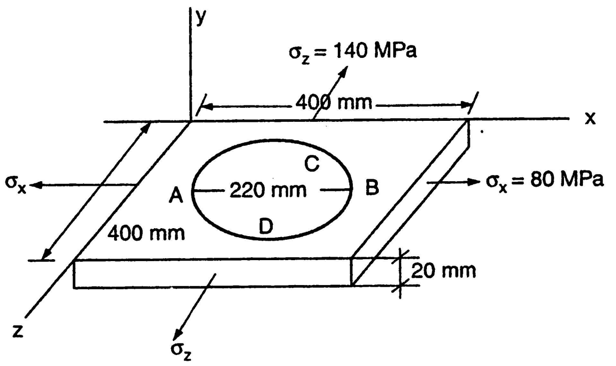


Fig. 1.34

(a) change in length of diameter AB
(b) change in length of diameter CD
(c) change in thickness of the plate
(d) change in volume of the plate

SOLUTION Given

$$\sigma_x = 80 \text{ MPa} = 80 \times 10^6 \text{ N/m}^2$$

$$\sigma_z = 140 \text{ MPa} = 140 \times 10^6 \text{ N/m}^2$$

$$E = 70 \times 10^9 \text{ N/m}^2$$

We know that by application of load stress and strain occurs in the material.

Let ε_x, ε_y and ε_z are the strain along x, y and z axes.

Then, $\varepsilon_x = \frac{\sigma_x}{E} - \nu \cdot \frac{\sigma_y}{E} - \nu \cdot \frac{\sigma_z}{E}$

$$= \frac{1}{E}[\sigma_x - \nu \cdot \sigma_y - \nu \cdot \sigma_z]$$

$$= \frac{10^6}{70 \times 10^9} = \left[80 - 0 - \frac{140}{3}\right]$$

$$= 0.476 \times 10^{-3}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \cdot \frac{\sigma_x}{E} - \nu \cdot \frac{\sigma_z}{E}$$

$$= 0 - \frac{1}{3} \times \frac{80}{70 \times 10^3} - \frac{1}{3} \times \frac{140}{70 \times 10^3}$$

$$= -\frac{1}{3 \times 70 \times 10^3}[80 + 140]$$

$$= -1.047 \times 10^{-3}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu \cdot \frac{\sigma_x}{E} - \nu \cdot \frac{\sigma_y}{E}$$

$$= \frac{140}{70 \times 10^3} - \frac{1}{3} \times \frac{80}{70 \times 10^3} - 0$$

$$= \frac{1}{70 \times 10^3}\left[140 - \frac{80}{3}\right]$$

$$= 1.62 \times 10^{-3}$$

Now,

(*a*) Change in length in diameter *AB i.e.*

$$\delta_{AB} = \varepsilon_x \times d$$

$$= 0.476 \times 10^{-3} \times 220$$

$$= 0.1047 \text{ mm}$$

(*b*) Change in length in diameter *CD i.e.*

$$\delta_{CD} = \varepsilon_z \times d$$

$$= 1.62 \times 10^{-3} \times 220$$

$$= 0.356 \text{ mm}$$

(*c*) Change in thickness of plate *i.e.*

$$\delta_y = \varepsilon_y \times t$$

$$= -1.047 \times 10^{-3} \times 20 \text{ mm}$$

$$= -0.0209 \text{ mm (decreases in thickness)}$$

(*d*) Change in volume (δ_V)

$= \varepsilon_V \times$ original volume

$=$ volumetric strain $\times$ original volume

Volumetric strain of aluminium plate is given by

$$\varepsilon_V = \frac{(\sigma_x + \sigma_y + \sigma_z)(1 - 2\nu)}{E}$$

$$= \frac{(80 + 0 + 140)\left(1 - 2 \times \frac{1}{3}\right)}{70 \times 10^9}$$

$$= 220 \times \frac{1}{3} \times \frac{1}{70 \times 10^9}$$

$$= \frac{220}{210} \times 10^{-9}$$

$\therefore$ Change in Volume $(\delta_V) = \varepsilon_V \times V$

$$= \frac{22}{21} \times 10^{-9} \times (400 \times 400 \times 20)\,\text{mm}^3$$

$$= \frac{22}{21} \times 10^{-9} \times 32 \times 10^5\ \text{mm}^3$$

$$= \frac{22 \times 32}{21} \times 10^{-4}\ \text{mm}^3$$

$$= 33.523 \times 10^{-4}\ \text{mm}^3$$

$$= 3.352 \times 10^{-3}\ \text{mm}^3$$ **Ans.**

1.16 RELATION BETWEEN ELASTIC CONSTANTS (*E*, *K* AND *G*)

(*a*) *Relation between E and K:*

$L =$ side of the cube

and $dL =$ change in length of cube

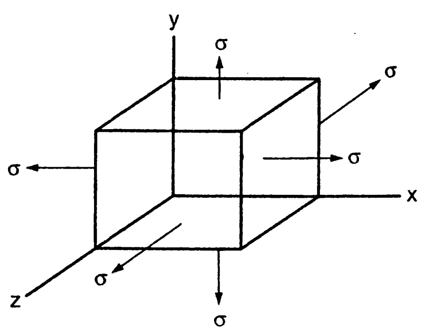


Fig. 1.36

Let the element of elastic body in the form of cube is subjected to equal tensile stress (σ) along x, y and z axes then total strain along x-axis.

$$\frac{dL}{L} = \frac{\sigma}{E} - \nu \cdot \frac{\sigma}{E} - \nu \cdot \frac{\sigma}{E}$$

$$= \frac{\sigma}{E}(1 - 2\nu) \quad \text{...(i)}$$

Initial volume of the cube

$$(V) = L^3 \quad \text{...(ii)}$$

Now, differentiating both sides, $dV = 3L^2 dL$

$$\therefore \quad \left(\frac{dV}{V}\right) = \frac{3L^2 \times dL}{L^3} = \frac{3dL}{L}$$

$$\therefore \quad \left(\frac{dL}{L}\right) = \left(\frac{dV}{3V}\right) \quad \text{...(iii)}$$

From equation (i) and (iii)

$$\frac{dV}{3V} = \frac{\sigma}{E}(1 - 2\nu)$$

$$\frac{dV}{V} = \frac{3\sigma}{E}(1 - 2\nu)$$

$$\text{Bulk Modulus } (K) = \frac{\text{Stress}}{\text{Volumetric strain}}$$

$$= \frac{\sigma}{\frac{3\sigma}{E}(1 - 2\nu)}$$

$$K = \frac{E}{3(1 - 2\nu)}$$

$$\therefore \quad E = 3K(1 - 2\nu) \quad \text{...(iv)}$$

(b) *Relation between E and G:* Let the block of side 'L' and thickness unity is subjected to shear stresses τ, at faces AB and CD, hence complementary shear stresses (τ) will be developed on faces AD and BC, as a result the cube is distorted to $ABC'D'$.

$$\text{Now, strain in } AC = \frac{AC' - AC}{AC}$$

$$= \frac{C'H}{AC}$$

$$= \frac{CC' \cos 45^\circ}{AB\sqrt{2}}$$

$$= \frac{CC'}{2AB}$$

$$= \frac{1}{2} \times \left(\frac{CC'}{BC} \right)$$

$$= \frac{1}{2} \gamma$$

$$= \frac{\gamma}{2}$$

$$= \frac{\tau}{2G} \qquad \ldots(1)$$

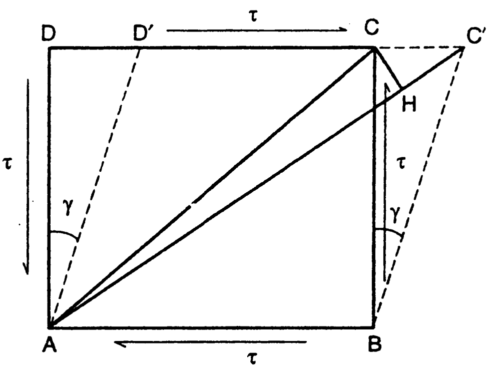


Fig. 1.37

Strain along $AC = \frac{\tau}{E}$ and strain along $BD = (-)\,\nu \cdot \frac{\tau}{E}$ $\left[\text{Shear strain } (\gamma) = \frac{\tau}{G}\right]$

Further Strain along diagonal AC = strain due to tensile stress in AC

− strain due to compressive stress along BD

$$= \left[\frac{\tau}{E} - \left(-\nu \frac{\tau}{E}\right)\right] = \left[\frac{\tau}{E} + \nu \cdot \frac{\tau}{E}\right]$$

So combined strain along $AC = \frac{\tau}{E}[1+\nu]$ ...(2)

∴ From equation (1) and (2)

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\nu)$$

$$\therefore \quad G = \frac{E}{2(1+\nu)} \qquad \ldots(3)$$

where G = modulus of rigidity

E = modulus of elasticity.

Eliminating ν from equ. (3) we get the relation between three elastic constants which is $E = \frac{9GK}{3K+G}$

EXAMPLE 1 *A bar of 30mm diameter is subjected to a axial pull of 60kN. The measured extension on a guauge length of 200mm is 0.09 mm and the change is diameter is 0.0039mm. Calculate the Poisson's ratio and the values of the three moduli.*

SOLUTION Diameter of the bar = 30mm

$$\text{Area of the bar } (A) = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (30)^2 = 706.86 \text{ mm}^2$$

N.B.: If the direction of shear stress τ is reversed then also the same result will occur. In that case diagonal BD will be elongated and AC will be compressed simultaneously.

Tensile load = 60 kN = 6000 N

$\therefore$ Tensile stress $(\sigma) = \dfrac{6000}{706.86}\text{N/mm}^2 = 84.88\text{ N/mm}^2$

Change in length (δ) = 0.09 mm

Longitudinal strain $(\varepsilon_l) = \dfrac{\delta}{l} = \dfrac{0.09}{200} = 0.00045$

Young's Modulus $(E) = \dfrac{\sigma}{\varepsilon_l} = \left(\dfrac{84.88}{0.00045}\right) = 18862\text{ N/mm}^2$

Lateral strain $= \dfrac{\Delta d}{d} = \dfrac{0.0039}{30} = 0.00013$

$$\text{Poisson's ratio } (\nu) = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$= \frac{0.00013}{0.00045} = \frac{13}{45}$$

Let G be modulus of rigidity

We know that $E = 2G(1 + \nu)$

$$188622 = 2G\left(1 + \frac{13}{45}\right)$$

$$\therefore \quad G = \frac{188622 \times 45}{2 \times 58}$$

$$= 73172.327\text{ N/mm}^2$$

Let K be the bulk modulus

We know that $E = 3K(1 - 2\nu)$

$$188622 = 3K\left(1 - \frac{26}{45}\right)$$

$$K = \frac{188622 \times 45}{3 \times 19}$$

$$= 148912.1\text{ N/mm}^2$$

Hence,

$$\left.\begin{aligned} \nu &= \frac{13}{45} = 0.28 \\ E &= 188622\text{ N/mm}^2 \\ G &= 73172.327\text{ N/mm}^2 \\ K &= 148912.1\text{ N/mm}^2 \end{aligned}\right\} \quad \textbf{Ans.}$$

1.17 EXPRESSION FOR THE ELONGATION *D* OF A VERTICAL BAR OF LENGTH *L*, UNIFORM CROSS-SECTIONAL AREA *A*, SUPPORTED AT ITS UPPER AND UNDER ITS OWN WEIGHT *W*

Let us consider an element of thickness dx at a distance x from the lower end of the bar. Let ρ is the mass density of bar material.

For length X, $W_X = \rho \cdot g \cdot A \cdot x$

As a result of this load portion of length dx will suffer small elongation $d\delta$ such that

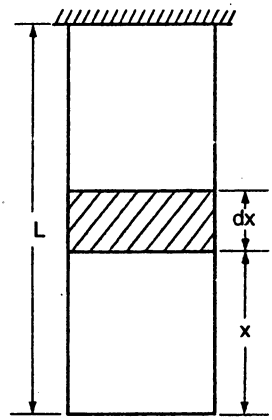


Fig. 1.38

$$d\delta = \frac{W \cdot dx}{A \cdot E}$$

$$= \frac{\rho \cdot g \cdot A \cdot x \cdot dx}{A \cdot E}$$

Total elongation $\delta = \int_0^L d\delta$

$$= \int_0^L \frac{\rho \cdot g \cdot \cancel{A} \cdot x\, dx}{\cancel{A} \cdot E}$$

$$= \frac{\rho g}{E} \int_0^L x\, dx$$

$$= \frac{\rho g L^2}{2E} \quad ...(1)$$

Weight of bar $(W) = \rho \cdot g \cdot A \cdot L$

$$\therefore \quad \rho = \frac{W}{A \cdot L \cdot g}$$

Substituting the value of P in the above equation we get,

$$\delta = \frac{W}{A \cancel{L} \cancel{g}} \times \frac{\cancel{g} \cancel{L^2} L}{2E}$$

$$\therefore \quad {}^{*}\boxed{\delta = \frac{WL}{2AE}}$$

1.18 PRINCIPLE OF SUPERPOSITION

When a number of loads are acting on a body then the resulting strain will be algebraic sum of the strain caused by individual loads.

While solving the problem by superposition principle, first the free body diagram is drawn, afterwards the deformation (increase or decrease) of the each section is obtained.

* δ and any computed value of dimensions viz d; l etc of the specimen should be expressed in mm while solving the numerical problem.

The total deformation (Increase or decrease) of the body will then be equal to the algebraic sum of the deformation of individual section.

Let us analyse bars of the varying sections:

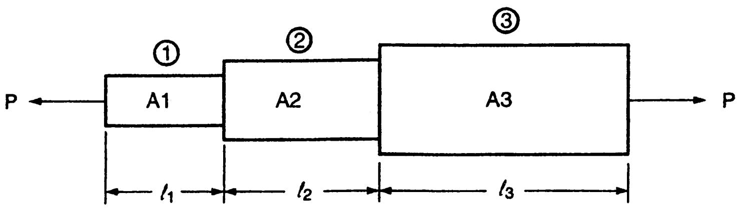


Fig. 1.39

In the figure there are three stepped bars of different cross-section A_1, A_2, A_3 and different length l_1, l_2 and l_3 respectively.

Let P = axial load acting on the bar

and E = Young's modulus of the bar.

Although each section is subjected to the same axial load P, yet the stresses, strain and change in lengths will be different.

The stress in section 1, $\sigma_1 = \dfrac{P}{A_1}$

The stress in section 2, $\sigma_2 = \dfrac{P}{A_2}$

The stress in section 3, $\sigma_3 = \dfrac{P}{A_3}$

The strain in section 1, $\varepsilon_1 = \dfrac{\sigma_1}{E} = \dfrac{P}{A_1 E}$

Similarly, strain in section 2, $\varepsilon_2 = \dfrac{P}{A_2 E}$

and, $\varepsilon_3 = \dfrac{P}{A_3 E}$

Also, strain in portion 1 = $\dfrac{\text{Change in length of portion 1}}{\text{Original length of portion 1}}$

$$\varepsilon_1 = \frac{\delta l_1}{l_1}$$

$\therefore$ Change in length of portion 1, $\delta l_1 = \varepsilon_1 l_1$

$$= \frac{P l_1}{A_1 E}$$

Similarly, change in length of portion 2, $\delta l_2 = \dfrac{P \cdot l_2}{A_2 \cdot E}$

and change in length of portion 3, $\delta l_3 = \dfrac{P \cdot l_3}{A_3 \cdot E}$

$\therefore$ Total change in length of the bar

$$\sum_{n=1}^{n=3} \delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \left[\frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E}\right]$$

$$= \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$

This expression is used in solving the problem when a round bar/stepped bar/rectangular bar is subjected to axial load and the total change in length of the bar is needed.

In general total change in length of bar

$$\sum_{n=1}^{n=n} dl = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \cdots + \frac{l_n}{A_n}\right]$$

EXAMPLE 1 *The loads acting on a 3 mm diameter has 6 meter total length divided into three segments as shown in figure. Find the total elongation of the bar (E = 210 GPa).*

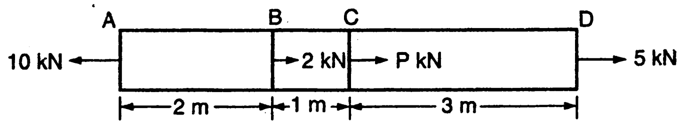


Fig.1.40

SOLUTION For static equilibrium $P + 2 + 5 = 10$

$\therefore$ $P = 3\text{ kN}.$

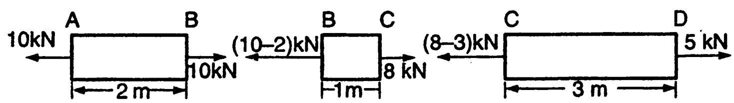


Fig. 1.41

Now, drawing *FBD*

$$A = \frac{\pi}{4}(d)^2 = 7.0714 \times 10^{-6}\,\text{m}^2$$

$\therefore$ Total elongation of bar

$$= \frac{1}{EA}[P_{AB} \times L_{AB} + P_{BC} \times L_{BC} + P_{CD} \times L_{CD}]$$

$$= \frac{1}{210 \times 10^9 \times 7.0714 \times 10^{-6}}$$

$$\left[(10 \times 10^3 \times 2) + (8 \times 10^3 \times 1) + (5 \times 10^3 \times 3)\right] \text{m}$$

$$= \frac{10^3}{210 \times 7.0714 \times 10^3}[20 + 8 + 15]$$

$$= \frac{43}{210 \times 7.0714}$$

$= 0.0289$ m $= 28.95$ mm **Ans.**

EXAMPLE 2 *A homogenous rod of constant cross-section is attached to unyielding supports. It carries an axial load P applied as shown in figure. Determine the reaction at A and B.*

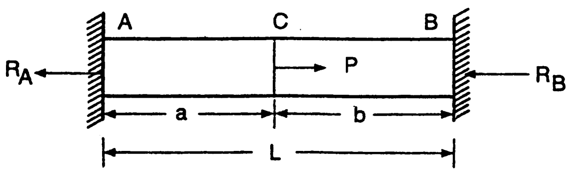


Fig. 1.42

SOLUTION Applying the condition of static equilibrium.

$$R_A + R_B = P \quad \ldots(1)$$

As the bar is fixed rigidly between supports.
Therefore, extension in portion AC = Contraction in portion CB.
Let us draw F.B.D. for portion AC and CB separately.

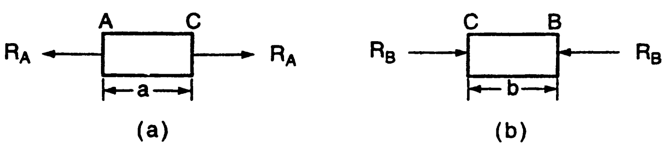


Fig. 1.43

$$\delta_{AC} = \frac{R_A \cdot a}{A \cdot E}; \quad \delta_{CB} = \frac{R_B \cdot b}{A \cdot E}$$

Given $\delta_{AC} = \delta_{CB}$

$$\therefore \quad \frac{R_A \cdot a}{A \cdot E} = \frac{R_B \cdot b}{A \cdot E}$$

$$\frac{R_A}{R_B} = \frac{b}{a}$$

$$R_A = \left(\frac{b}{a}\right) \cdot R_B \qquad \text{...(2)}$$

Substituting the value of R_A in eqn. (2)

$$\frac{b}{a} \cdot R_B + R_B = P$$

$$R_B\left(\frac{b}{a} + 1\right) = P$$

$$\boxed{R_B = \frac{P \cdot a}{a + b}} \qquad \textbf{Ans.}$$

and, $$R_A = \left(\frac{b}{\cancel{a}}\right) \cdot \frac{P \cdot \cancel{a}}{(a + b)}$$

$$\boxed{R_A = \frac{P \cdot b}{(a + b)}} \qquad \textbf{Ans.}$$

EXAMPLE 3 *A 550 mm long round bar of copper has a diameter of 30 mm over a length of 200 mm, diameter of 20 mm over a length of 200 mm and a diameter of 10 mm over its remaining length. Determine the stresses in each section and elongation of the rod when it is subjected to a pull of 30 kN. Assume E = 100 kN/mm².*

SOLUTION Axial pull (P) = 30 kN

= 30000 Newton

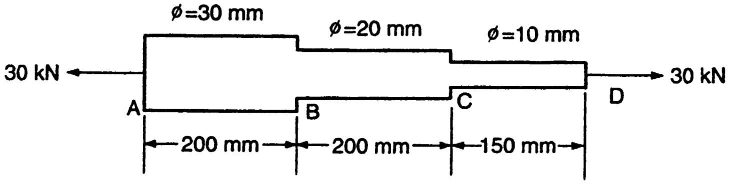


Fig. 1.44

Young's Modulus (E) = 100 kN/mm^2

$= 100 \times 10^3 \text{ N/mm}^2$

$= 10^5 \text{ N/mm}^2$

Stress in portion AB, $\sigma_{AB} = \dfrac{P_{AB}}{A_{AB}}$

where, P_{AB} = load (Axial) in the portion AB

A_{AB} = cross-sectional area in the portion AB

$$\therefore \quad \sigma_{AB} = \frac{300 \times 10^3}{\frac{\pi}{4}(30)^2} = 42.463 \text{ N/mm}^2$$

$$A_{AB} = 706.5 \text{ mm}^2$$

$$A_{BC} = \frac{\pi}{4}(d)^2 = \frac{\pi}{4} \times (20)^2$$

$$A_{BC} = 314.28 \text{ mm}^2$$

Similarly, $\sigma_{BC} = \dfrac{P_{BC}}{A_{BC}}$

$$= \frac{30,0000}{\frac{\pi}{4} \times (20)^2} = 95.54 \text{ N/mm}^2 \quad \textbf{Ans.}$$

$$A_{CD} = \frac{\pi}{4} \times (10)^2 \text{ mm}^2$$

$$= 78.57 \text{ mm}^2$$

$$\sigma_{CD} = \frac{30,000}{\frac{\pi}{4}(10)^2}$$

$$= 381.82 \text{ N/mm}^2$$

$\therefore$ Total elongation of the rod

$$(\delta) = \frac{P}{E} \cdot \left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right)$$

$$= \frac{30000}{(10)^5} \left(\frac{200}{706.85} + \frac{200}{314.28} + \frac{150}{78.57} \right)$$

$$\delta = 0.84 \text{ mm} \quad \textbf{Ans.}$$

EXAMPLE 4 *A steel bar of 25 mm diameter is loaded as shown in Fig. 1.45. Calculate the stresses in each portion and the total elongation. Assume E_{steel} = 200 GPa.*

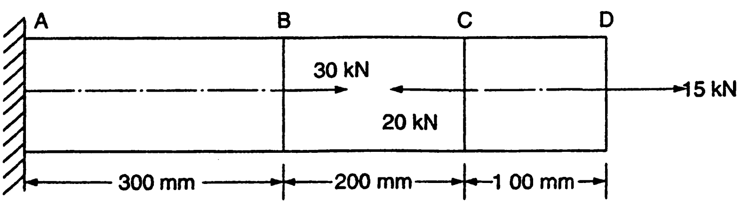


Fig. 1.45

SOLUTION Let us draw *FBD* of various portion of bar (starting from *D* for convenience).

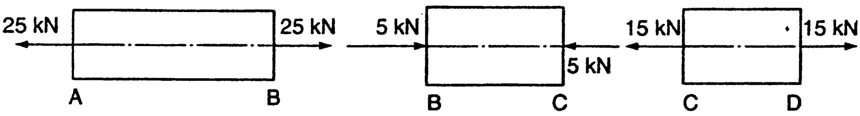


Fig. 1.46

$$\text{C.S. area of the bar} = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4} \times (25)^2$$

Hence, $A = 490.873 \text{ mm}^2$

$$\sigma_{AB} = \frac{P_{AB}}{A} = \frac{25 \times 10^3}{490.873}$$

$$= 50.93 \text{ MPa}$$

$$\sigma_{BC} = (-)\frac{P_{BC}}{A} \text{ (since load is compressive so negative sign used)}$$

$$= (-)\frac{5 \times 10^3}{490.873} = (-)\,10.186 \text{ MPa}$$

Total elongation

$$= \frac{1}{AE}\Sigma P_n \cdot L_n$$

$$= \frac{1}{AE}[P_{AB} \cdot L_{AB} - P_{BC} \cdot L_{BC} + P_{CD} \cdot L_{CD}]$$

$$= \frac{1}{490.873 \times 200 \times 10^3}\,[25 \times 10^3 \times 300 - 5 \times 10^3 \times 200 + 15 \times 10^3 \times 100]$$

$$= \frac{10^3}{490.873 \times 2 \times 10^5}[25 \times 300 - 5 \times 200 + 15 \times 100]$$

$$= \frac{10^3}{490.873 \times 2 \times 10^5}[7500 - 1000 + 1500]$$

$$= \frac{10^3 \times 8000}{490.873 \times 2 \times 10^5}$$

$$= \frac{80}{490.873 \times 2}$$

$= 0.08148 \text{ mm} \simeq 0.0815 \text{ mm}$ **Ans.**

EXAMPLE 5 *A single axial load P = 50 kN is applied at end C of the brass road ABC. Knowing that E = 105 GPa, determine the diameter d of the portion BC for which the deflection at point C will be 3 mm.*

SOLUTION Applying formula

$$\delta_{\text{total}} = \delta_{AB} + \delta_{BC}$$

$$= \frac{P \cdot L_{AB}}{A_{AB} \cdot E_{AB}} + \frac{P \cdot L_{BC}}{A_{BC} \cdot E_{BC}}$$

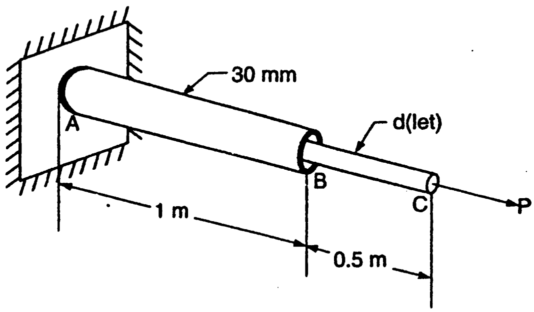


Fig. 1.47

$$E_{AB} = E_{BC} = E_{\text{brass}} = 105 \text{ GPa (given)}$$

$$= 105 \times 10^3 \text{ MPa}$$

$$3 = \frac{50 \times 10^3 \times 1 \times 10^3}{\frac{\pi}{4}(30)^2 \times 105 \times 10^3} + \frac{50 \times 10^3 \times 0.5 \times 10^3}{\frac{\pi}{4}(d)^2 \times 105 \times 10^3}$$

or $$3 = \frac{50 \times 10^3 \times \cancel{10^3}}{\frac{\pi}{4} \times 105 \times \cancel{10^3}}\left[\frac{1}{(30)^2} + \frac{0.5}{(d)^2}\right]$$

or, $$3 = \frac{4 \times 50 \times 10^3}{\pi \times 105}\left[\frac{1}{900} + \frac{0.5}{(d)^2}\right]$$

or, $$\left(\frac{3 \times \pi \times 105}{4 \times 105 \times 10^3}\right) - \frac{1}{900} = \frac{0.5}{d^2}$$

or, $$4.9455 \times 10^{-3} - 1.111 \times 10^{-3} = \frac{0.5}{d^2}$$

or, $$\frac{3.8345}{10^3} = \frac{0.5}{d^2}$$

$\Rightarrow$ $d = 11.42$ mm **Ans.**

EXAMPLE 6 *In the given figure AB and BC are made of an aluminium for which E = 70 GPa. Knowing that the magnitude of P is 4 kN. Determine*

(a) the value of Q so that the deflection at A is zero

(b) the corresponding deflection at B.

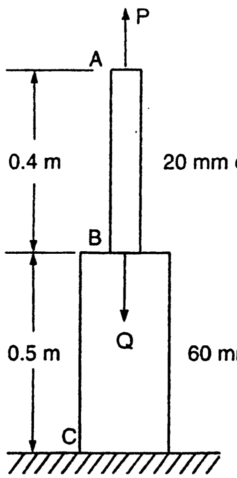


Fig. 1.48

SOLUTION (a) Given deflection at $A = 0$ and $P = 4$ kN

$$\Rightarrow \quad 0 = \frac{P \cdot L}{A \cdot E} - \frac{(Q-P) \cdot L}{A \cdot E}$$

$$\Rightarrow \quad \frac{P \cdot L}{A \cdot E} = \frac{(Q-P) \cdot L}{A \cdot E}$$

$$\Rightarrow \quad \frac{P \times 9.4 \times 10^3}{\frac{\pi}{4}(20)^2 \times E} = \frac{(Q-P) \times 0.5 \times 10^3}{\frac{\pi}{4}(60)^2 \times E}$$

$$\Rightarrow \quad \frac{P \times 0.4}{400} = \frac{(Q-P) \times 5}{9}$$

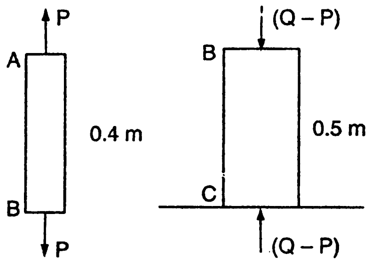


Fig. 1.49

$$\Rightarrow \quad 36P = 5Q - 5P$$

$$\Rightarrow \quad 41P = 5Q$$

$$\Rightarrow \quad Q = \left(\frac{41}{5}\right)P = \frac{41}{5} \times 4 = \frac{164}{5}$$

$$\therefore \quad \boxed{Q = 32.8 \text{ kN}}$$ **Ans.**

(b)
$$\delta_{BC} = \frac{(32.8 - 4) \times 10^3 \times 500 \times 28}{22 \times 3600 \times 70 \times 10^3}$$

$$= \frac{14.4}{198}$$

$$= 0.0727 \text{ mm}$$

EXAMPLE 7 *The specimen shown in Figure is made from a 25 mm diameter cylindrical steel rod with two 38 mm sleeves bonded to the rod as shown in figure. Knowing that E = 200 GPa, determine* *(a) the load P so that the total deformtion is 0.05 mm.*

(b) the corresponding deformation of the central part BC.

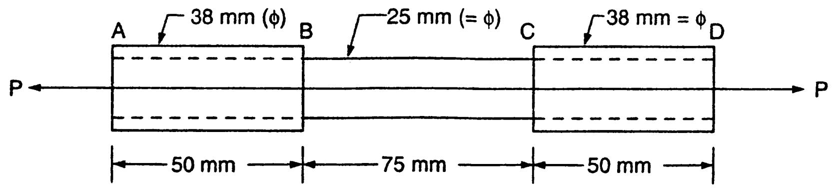


Fig. 1.50

SOLUTION Given $\delta_{total} = 0.05$ mm; $E = 200$ GPa $= 200 \times 10^3$ MPa.

(a) $\delta_{total} = \delta_{AB} + \delta_{BC} + \delta_{CD}$, by superposition theorem

$$0.05 = \frac{P \cdot L_{AB}}{A_{AB} \cdot E} + \frac{P \cdot L_{BC}}{A_{BC} \cdot E} + \frac{P \cdot L_{CD}}{A_{CD} \cdot E}$$

$$0.05 = \frac{P}{E}\left[\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} + \frac{L_{CD}}{A_{CD}}\right]$$

$$0.05 = \frac{P}{200 \times 10^3}\left[\frac{50}{\frac{\pi}{4}(38)^2} + \frac{75}{\frac{\pi}{4}(25)^2} + \frac{50}{\frac{\pi}{4}(38)^2}\right]$$

$$\left(\frac{0.05}{1}\right) = \frac{P}{2 \times 10^5 \times \frac{\pi}{4}}\left[\frac{2 \times 50}{(38)^2} + \frac{75}{(25)^2}\right]$$

$$\frac{0.05}{1} = \frac{4P}{2\pi \times 10^5}\left[\frac{100}{(38)^2} + \frac{75}{(25)^2}\right]$$

$$\frac{0.05 \times 2\pi \times 10^5}{4} = P \times \left[\frac{100}{(38)^2} + \frac{75}{(25)^2}\right]$$

$$\frac{5 \times 2\pi \times 10^3}{4} = P \times (0.069252 + 0.12)$$

$$P = 41479 \text{ N}$$

$$= 41.479 \text{ kN}$$

$$\boxed{P = 41.5 \text{ kN}}$$ **Ans.**

(b) Corresponding to $P = 41.5$ kN, the central part BC

$$\text{Deformation } i.e.\ \delta_{BC} = \frac{41.5 \times 10^3 \times 75}{\frac{\pi}{4}(25)^2 \times 200 \times 10^3}$$

$$= \frac{41.5 \times 10^3 \times 75 \times 4 \times 7}{22 \times 625 \times 2 \times 10^5}$$

$$= 0.03169 \text{ mm}$$

$\therefore$ $\boxed{\delta_{BC} = 0.0317 \text{ mm}}$ **Ans.**

1.19 TENSION IN WIRES OF A HINGED BAR SUBJECTED TO AXIAL PULL (P)

EXAMPLE 1 *In the figure OABC is a rigid bar which is hinged at 0 and supported in a horizontal position by two identical steel wires. A vertical load P is applied at C. Find the tension T_1 and T_2 in the steel wires.*

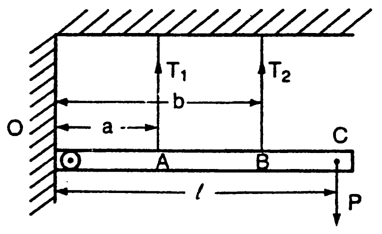


Fig. 1.51

SOLUTION The $T_1 + T_2 + F_y = P$

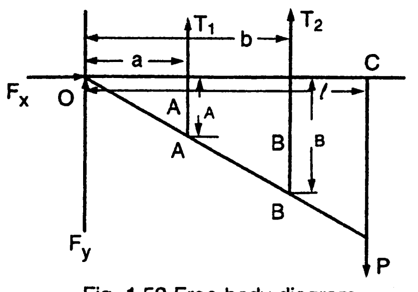


Fig. 1.52 Free body diagram

From Δs $OA'A$ and $OB'B$

$$\frac{\delta_A}{a} = \frac{\delta_B}{b} = \frac{T_1}{T_2} \quad \ldots(1)$$

$$\frac{\delta_A}{\delta_B} = \frac{a}{b} = \frac{T_1}{T_2} \quad \ldots(2)$$

Taking moment about O and equating it to zero

$$T_1 a + T_2 \cdot b = P \cdot l \quad \ldots(3)$$

Given $\frac{a}{b} = \frac{T_1}{T_2}$

or $$\frac{a^2}{b^2} = \frac{aT_1}{bT_2}$$

or $$\frac{a^2}{b^2} + 1 = \frac{aT_1 + bT_2}{bT_2}$$ (Adding 1 both sides)

or $$\frac{a^2 + b^2}{b^2} = \frac{aT_1 + bT_2}{bT_2}$$

$$\therefore \quad aT_1 + bT_2 = \frac{T_2(a^2 + b^2)}{b}$$

Equation (3) reduces to

$$\frac{T_2(a^2 + b^2)}{b} = P \cdot l$$

$$\boxed{\begin{aligned} &\therefore \frac{P \cdot b \cdot l}{(a^2 + b^2)} = T_2 \\ &\text{Similarly } T_1 = \frac{P \cdot a \cdot l}{(a^2 + b^2)} \end{aligned}}$$

N.B: This can be used as a formula when P. a, b, l are given.

Let us confirm by the following example.

EXAMPLE 2 *A rigid bar OABC is hinged at A and supported in a horizontal position by two identical steel wires as shown in figure. A vertical load of 30 kN is applied at C. Find the tensile forces T_1 and T_2 induced in these wires by the vertical load.*

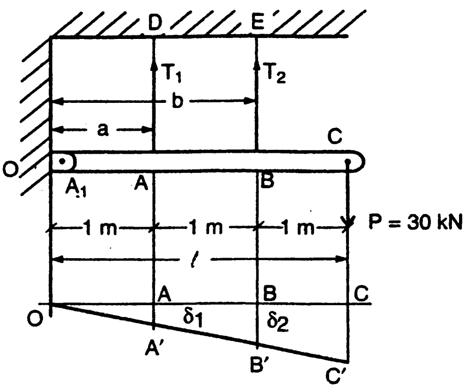


Fig. 1.53

SOLUTION By formula

$$T_1 = \frac{P \cdot a \cdot l}{(a^2 + b^2)}$$

$$= \frac{30 \times 10^3 \times 1 \times 3}{(1^2 + 2^2)}$$

$$= \frac{\cancel{90{,}000}}{\cancel{5}} = 18{,}000 \text{ Newton}$$

$$= 18 \text{ kN}$$

$$T_2 = \frac{P \cdot b \cdot l}{(a^2 + b^2)}$$

$$= \frac{{}^{6}\cancel{30} \times 10^3 \times 2 \times 3}{\cancel{5}_1}$$

$$= 36 \text{ kN} \quad \textbf{Ans.}$$

EXAMPLE 3 *The rigid bar AB is attached to two vertical rods as shown in Fig. 1.54 is horizontal before the load P is apoplied. Determine the vertical movement of P if it's magnitude is 50 kN.*

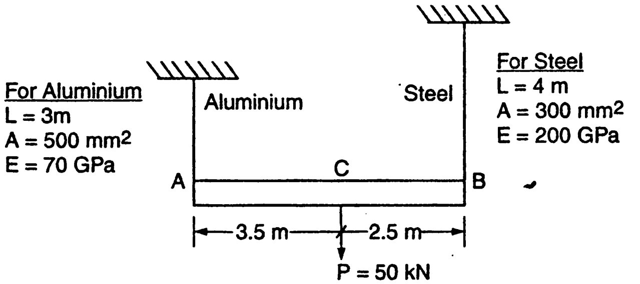


Fig. 1.54

SOLUTION **For Aluminium**

$\Sigma M_B = 0$ gives

$6P_{Al} = 2.5 \times 50$

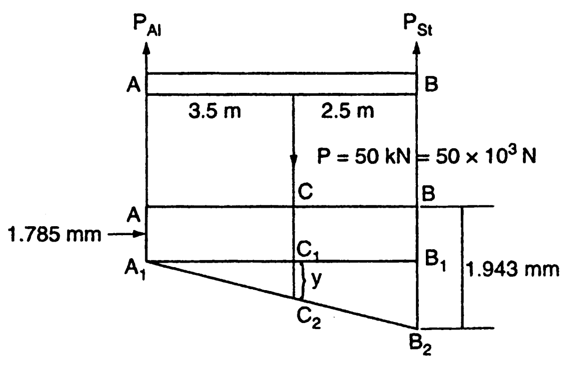


Fig. 1.55

$$\therefore \qquad P_{Al} = \frac{2.5 \times 50}{6}$$

$$= 20.83 \text{ kN} = 20.83 \times 10^3 \text{ Newton}$$

$$\delta_{Al} = \frac{P \cdot L}{A \cdot E} = \frac{20.83 \times 3 \times (1000)^2}{500 \times (70 \times 10^3)} \text{mm}$$

$$= \frac{20.83 \times 3000}{500 \times 70} \text{mm} = 1.785 \text{ mm}$$

For Steel

$$\Sigma M_A = 0 \text{ gives}$$

$$\sigma P_{st} = 3.5 \times 50$$

$$P_{st} = \frac{3.5 \times 50}{6} = 29.16 \text{kN}$$

$$= 19.16 \times 10^3 \text{ N}$$

$$\sigma_{\text{steel}} = \frac{P \cdot L}{A \cdot E}$$

$$= \frac{29.16 \times 10^3 \times (4 \times 1000)}{300 \times 200 \times 10^3}$$

$$= \frac{29.16 \times 4000}{300 \times 200} = 1.943 \text{ mm}$$

Now, from similar triangles $A_1C_1C_2$ and $A_1B_1B_2$

$$\frac{y}{A_1C_1} = \frac{B_1B_2}{A_1B_1}$$

$$\frac{y}{3.5} = \frac{1.943 - 1.785}{6}$$

$$\Rightarrow \qquad y = \frac{3.5 \times (1.943 - 1.785)}{6} = 0.092 \text{ mm}$$

Now, vertical movement of P

$$= CC_2$$

$$= CC_1 + y$$

$$= 1.785 + 0.092$$

$$= 1.877 \text{ mm} \qquad \textbf{Ans.}$$

EXAMPLE 4 *The rigid bar AB and CD is shown in Fig. 1.56 are supported by pins A and C and the two rods. Determine the maximum force P that can be applied if its vertical movement is limited to 5 mm. Neglect the weights of all members.*

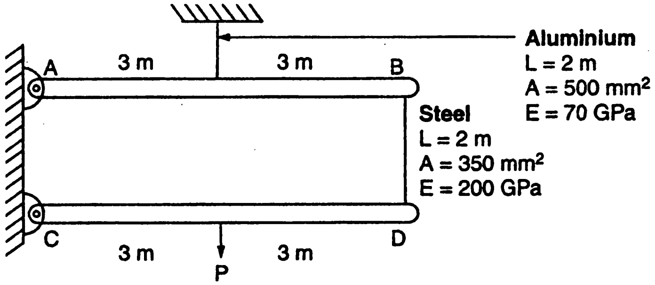


Fig. 1.56

SOLUTION Let us start from part AB.

$$\Sigma M_A = 0 \text{ gives}$$

$$3P_{al} = 6P_{st}$$

$$\therefore \quad P_{al} = 2P_{st}$$

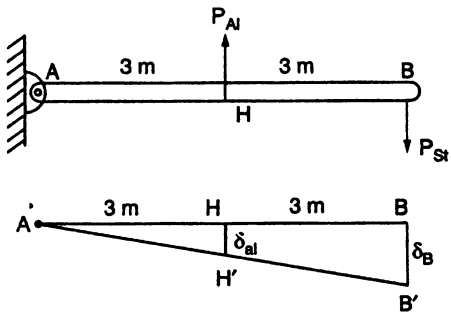


Fig. 1.57

Now, from similar triangles, ABB' and AHH' we get

$$\frac{BB'}{6} = \frac{HH'}{3}$$

$$\frac{\delta_B}{6} = \frac{\delta_{al}}{3}$$

$$\therefore \quad \delta_B = 2\delta_{al}$$

$$= 2\left[\frac{P_{al} \cdot L}{A \cdot E}\right]$$

$$= 2\left[\frac{P_{al} \times 2000}{500 \times 70 \times 10^3}\right] = \frac{1}{8750} P_{al}$$

$$= \frac{1}{8750} \times 2P_{st}$$

$$= \frac{1}{4375} P_{st} \text{ is the movement of } B$$

Part *CD*

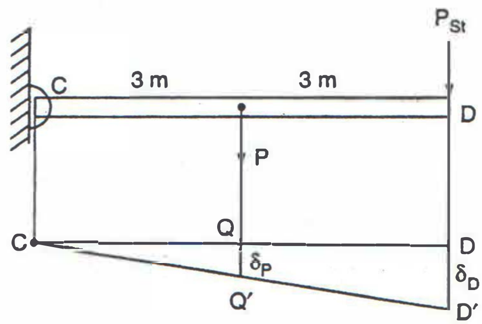


Fig. 1.58

Movement of *D*

$$\delta_D = \delta_{st} + \delta_B$$

$$= \frac{P \cdot L}{A \cdot E} + \frac{1}{4375} P_{st}$$

$$= \frac{P_{st} \times 2000}{300 \times 2 \times 10^5} + \frac{1}{4375} P_{st}$$

$$= 3.333 \times 10^{-5} P_{st} + 2.285 \times 10^{-4} P_{st}$$

$$= 2.618 \times 10^{-4} P_{st}$$

$\Sigma M_C = 0$ for static equilibrium

$$6P_{st} = 3P$$

$$P_{st} = \frac{1}{2} P$$

By similar triangles CQQ' and CDD'

$$\frac{\delta_P}{3} = \frac{\delta_D}{6}$$

$$\delta_P = \frac{1}{2} \delta_D$$

$$\left(\frac{5}{1000}\right) = \frac{1}{2} \times 2.618\, P_{st} \times 10^{-4} \qquad [\because \delta_P = 5 \text{ mm}]$$

$$\therefore \quad P_{st} = \left[\frac{0.005 \times 2}{2.618 \times 10^{-4}}\right]$$

or, $\frac{1}{2}P = 38.19$

$\therefore$ $P = 76.38$ kN **Ans.**

EXAMPLE 5 *Two vertical rods attached to the light bar in Fig. 1.59 are identical except for length. Before the load W was attached, the bar was horizontal and the rods were stress free. Determine the load in each rod if W = 6600 N.*

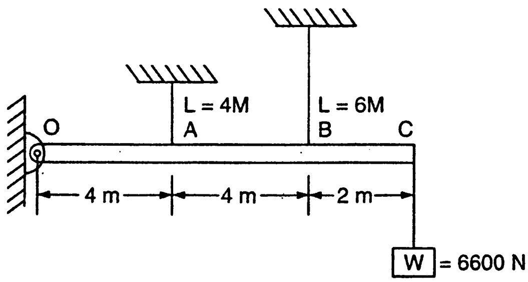


Fig. 1.59

SOLUTION $\Sigma M_O = 0$ gives

$$4P_A + 8P_B = 10 \times 6600$$

$$P_A + 2P_B = 16500 \quad \ldots(I)$$

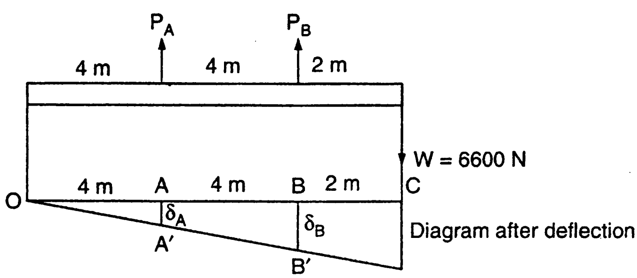


Fig. 1.60

Now, from similar triangle OAA' and OBB'

$$\frac{\delta_A}{4} = \frac{\delta_B}{8}$$

$$\frac{P_A \cdot L_A}{A_A \cdot E_A} \times \frac{1}{4} = \frac{P_B \cdot L_B}{A_B \cdot E_B} \times \frac{1}{8}$$

Since rods are identical except length

So $A_A = A_B = A$ (let)

$E_A = E_B = E$ (let)

$$\frac{P_A \cdot 4}{A \cdot E} \times \frac{1}{4} = \frac{P_B \cdot 6}{A \cdot E} \times \frac{1}{8}$$

$$P_A = \frac{3P_B}{4} \qquad \ldots(\text{II})$$

Substituting the value of P_A in equation (I)

$$\frac{3P_B}{4} + 2P_B = 16500$$

$$3P_B + 8P_B = 16500 \times 4$$

$$11P_B = 16500 \times 4$$

$$\therefore \quad P_B = \left(\frac{66000}{11}\right) = 6000 \text{ Newton}$$

$$P_A = 16500 - 2P_B$$

$$= 165000 - 12000$$

$$= 4500 \text{ Newton}$$

EXAMPLE 6 *The block of weight W hangs from the point at A. The bars AB and AC are pinned to the support at B and C. The C.S. area are 800 mm² for AB and 400 mm² for AC. Neglecting the weight of the bars, determine the maximum safe value of W if the stress in AB is limited to 110 MPa and in AC to 120 MPa.*

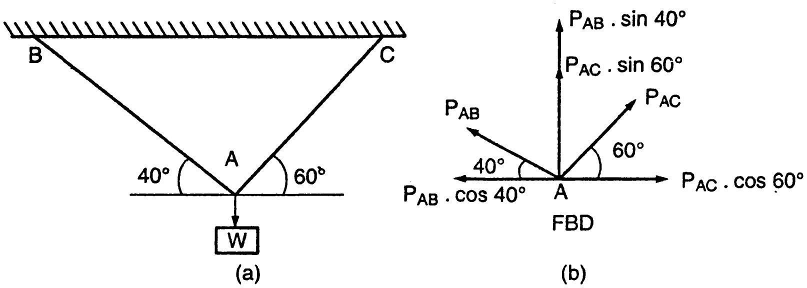


Fig. 1.61

SOLUTION Drawing *F.B.D* as in Fig. 1.50 (b) of joint *A*, we get

$\Sigma F_x = 0$ gives

$$P_{AC} \times 0.5 - P_{AB} \times 0.766 = 0 \qquad \ldots(1)$$

$\Sigma F_y = 0$ gives

$$P_{AC} \times 0.866 + P_{AB} \times 0.642 - W = 0 \qquad \ldots(2)$$

Solving (1) and (2) we get

$$P_{AB} = 0.508 \text{ W} \qquad \left.\begin{aligned}\sin 40 &= 0.642\\ \cos 40 &= 0.766\end{aligned}\right]$$

and, $\quad P_{AC} = 0.778 \text{ W}$

The value of W that will cause the stress in each bar to equal its maximum safe magnitude is determined as follows:

For AB:

$\because \quad P = \sigma \cdot A$

$$0.508\,W = 110 \times 10^6 \text{ N/m}^2 \times 800 \times 10^{-6} \text{ m}^2$$

$\therefore \quad W = 173 \times 10^3$ Newton

$= 173$ kN

For AC:

$\because \quad P = \sigma \cdot A$

$$0.778\,W = (120 \times 10^6 \text{ N/m}^2) \times (400 \times 10^{-6} \text{ m}^2)$$

$$W = 61.7 \times 10^3 \text{ N}$$

$= 61.7$ kN

The maximum safe value of W is the smaller one.

$\therefore \quad$ 61.7 kN **Ans.**

1.20 EXPRESSION FOR THE TOTAL EXTENSION OF A UNIFORMLY TAPERING RECTANGULAR BAR WHEN SUBJECTED TO AXIAL LOAD *P*.

Let P = axial load on the bar, L = length of the bar

a = width of bigger end, b = width of smaller end

E = Young's Modulus, t = thickness of bar

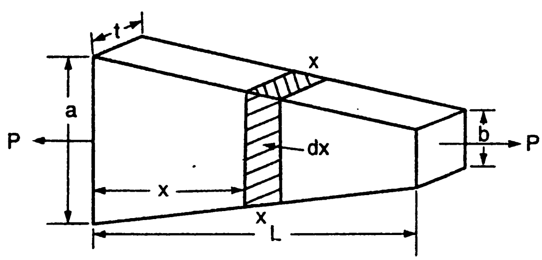


Fig. 1.52

Width of the bar of section $XX = a - \left(\dfrac{a-b}{L}\right)x$

$$= a - Kx \text{ where } \left(\frac{a-b}{L}\right) = K \quad \text{(Constant)}$$

Area at section XX = width × thickness

$$= (a - Kx)\,t$$

Stress at section XX,

$$\sigma_{XX} = \frac{P}{(a - Kx)\,t}$$

Extension of the small elemental length dx

$$= \text{Strain} \times \text{length } dx$$

$$= \frac{\text{Stress}}{E} \times dx$$

$$= \frac{\left[\dfrac{P}{(a-Kx)\,t}\right]}{E} \times dx$$

$$= \frac{P}{E\,(a-Kx)\,t} \times dx$$

$$\text{Total extension of the bar } (dL) = \int_0^L \frac{P}{E\,(a-Kx)\,t}\,dx$$

$$= \frac{P}{Et}\int_0^L \frac{dx}{(a-Kx)}$$

$$= \frac{P}{Et}\cdot \log_e\,[a-kx]_0^L \times \left(-\frac{1}{K}\right)$$

$$= -\frac{P}{Et}\,[\log_e\,(a-KL) - \log_e a]$$

$$= \frac{P}{Et}\left[\log_e\left(\frac{a}{a-KL}\right)\right]$$

$$= \frac{P}{Et\left(\dfrac{a-b}{L}\right)}\left[\log_e\left\{\frac{a}{a-\left(\dfrac{a-b}{L}\right)\cdot L}\right\}\right]$$

$$= \frac{PL}{Et\,(a-b)}\cdot \log_e\left(\frac{a}{b}\right) \qquad \textbf{Ans.}$$

N.B. This may be treated as a formula.

EXAMPLE 1 *Find out the total elongation caused by an axial load of 100 kN applied to a flat bar 20 mm thick, tapering from a width of 120 mm to 40 mm in a length of 10 m as shown in Fig. 1.63. (Given E = 200 GPa.)*

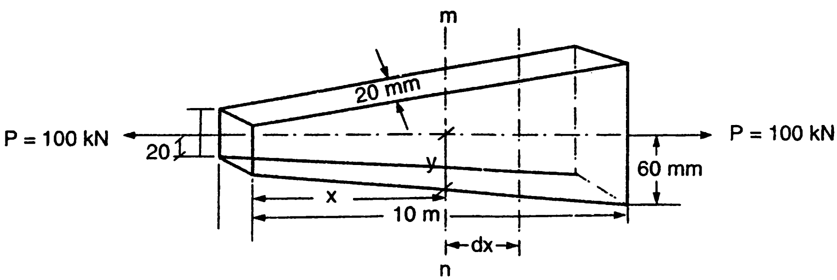


Fig. 1.63

SOLUTION From the question we find that cross-section is not constant so the equation $\sigma = \dfrac{P}{A}$ does not hold good in this problem.

However, it may be used to find the elongation on a differential length for which cross-section is constant.

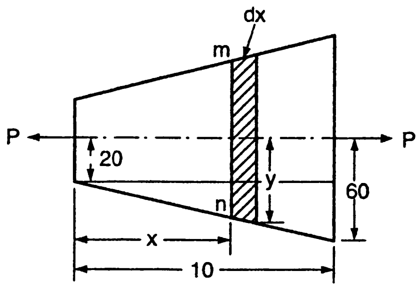


Fig. 1.64

At mn-section, $\dfrac{y-20}{x} = \dfrac{60-20}{10}$

$\therefore \quad y = (4x + 20)$ mm

Area of the section $(A) = 20 \times 2y = (160x + 800)\ \text{mm}^2$

At mn, in a differential length dx, the elongation

$$\because \quad \delta = \frac{P \cdot L}{AE}$$

$$\therefore \quad d\delta = \frac{(100 \times 10^3)\, dx}{(160x + 800)\,(10^{-6})\,(200 \times 10^9)}$$

$$= \left[\frac{0.500\, dx}{160x + 800}\right]$$

$$\therefore \quad \text{Total elongation } (\delta) = 0.500 \int_0^{10} \frac{dx}{160x + 800}$$

$$= \frac{0.500}{160}\left[\ln(160x + 800)\right]_0^{10}$$

$\delta = 3.4$ mm **Ans.**

EXAMPLE 2 *A solid truncated conical bar of circular cross-section tapers uniformly form a diameter D_1 at large end to D_2 at small end. The length of bar is L. Determine the elongation due to axial force P applied at each end.*

SOLUTION Let us consider an elemental length dx at a distance x from the bigger end.

$\therefore$ Diameter of the bar at x distance from bigger end

$$D_X = D_1 - (D_1 - D_2) \cdot \frac{x}{L}$$

$$= D_1 - mx, \text{ where} \left(\frac{D_1 - D_2}{L}\right) = \text{m}$$

$\therefore$ Sectional area at this section, $A_x = \frac{\pi}{4}(D_1 - mx)^2$

Axial strain at this section,

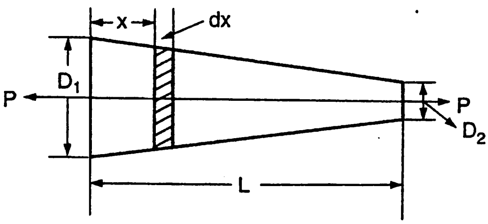


Fig. 1.65

$$\varepsilon_x = \frac{\text{Axial stress}}{E}$$

$$= \frac{P/A_x}{E}$$

$$= \frac{P}{\frac{\pi}{4}(D_1 - mx)^2 \cdot E}$$

Extension of the elemental length dx

$$= \varepsilon_x \times dx$$

$$= \frac{4P}{\pi (D_1 - mx)^2 E} dx$$

$\therefore$ Total extension of the bar

$$= \int_0^L \frac{4P\,dx}{\pi (D_1 - mx)^2 E}$$

$$= \frac{4P}{\pi E}\int_0^L (D_1 - mx)^{-2} dx$$

$$= \frac{4P}{\pi E}\left[\frac{1}{m(D_1 - mx)}\right]_0^L$$

$$= \frac{4P}{\pi E m}\left[\frac{1}{(D_1 - mL)} - \frac{1}{D_1 - \text{m} \times 0}\right]$$

$$= \frac{4P}{\pi E m}\left[\frac{1}{D_1(D_1 - D_2)} - \frac{1}{D_1}\right] \qquad \left[\because m = \frac{D_1 - D_2}{L}\right]$$

$$= \frac{4P}{\pi E m}\left[\frac{1}{D_2} - \frac{1}{D_1}\right]$$

$$= \frac{4P(D_1 - D_2)}{\pi E m D_1 D_2}$$

$$= \frac{4P(D_1 - D_2)}{\pi E D_1 D_2 \left(\frac{D_1 - D_2}{L}\right)}$$

$$\boxed{\therefore \delta = \frac{4PL}{\pi E D_1 D_2}} \quad \textbf{Ans.}$$

EXAMPLE 3 *If a tension bar is found to taper uniformly from (D – a) cm to diameter (D + a) cm. Prove that the error involved in using the mean diameter to calculate Young's modulus is* $\left(\frac{10a}{D}\right)^2$ *percent.*

SOLUTION By formula extension $(\delta) = \dfrac{4PL}{\pi(D-a)(D+a)E}$

$$\therefore \qquad E = \frac{4PL}{\pi(D^2 - a^2)\delta}$$

In case we are to calculate E on the basis of mean diameter D, extension δ being the same

$$\therefore \qquad E = \frac{4PL}{\pi D^2 x}$$

$$\therefore \qquad \text{Error in calculating } E = \frac{4PL}{\pi(D^2 - a^2)x} - \frac{4PL}{\pi D^2 x}$$

$$\text{and,} \qquad \text{Error in calculating } E = \frac{\left[\frac{4PL}{\pi(D^2 - a^2)x} - \frac{4PL}{\pi D^2 x}\right]100}{\frac{4PL}{\pi(D^2 - a^2)x}}$$

$$= \frac{\left[\frac{1}{D^2 - a^2} - \frac{1}{D^2}\right]100}{\frac{1}{D^2 - a^2}}$$

$$= 100\left(\frac{a^2}{D^2}\right) = \left(\frac{10a}{D}\right)^2 \qquad \textbf{Proved.}$$

1.21 STRESSES IN COMPOSITE MEMBERS

A composite member is composed of two or more different materials joined together in such a way that the system is elongated or compressed as single unit.

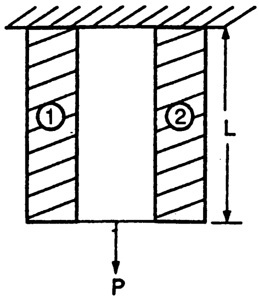


Fig. 1.66

In the figure composite bar of different materials have been shown.

Let P = total load on composite bar

L = length of each bar.

For

Bar 1	Bar 2
A_1 = Area of cross-section	A_2 = Area of cross-section
P_1 = load carried by it	P_2 = load carried by it
E_1 = Young's Modulus	E_2 = Young's Modulus
σ_1 = Stress induced	σ_2 = Stress induced

Total load on composite bar,

$$P = P_1 + P_2 \qquad \ldots(1)$$

The stress in bar 1, $\sigma_1 = \dfrac{P_1}{A_1}$

$$\therefore \quad P_1 = \sigma_1 \cdot A_1 \qquad \ldots(2)$$

Similarly, $P_2 = \sigma_2 \cdot A_2$ ...(3)

Substituting the value of P_1 and P_2 in equation (1)

$$P = \sigma_1 A_1 + \sigma_2 A_2 \qquad \ldots(4)$$

Strain in bar (1), $\varepsilon_1 = \dfrac{\sigma_1}{E_1}$

Similarly, strain in bar (2), $\varepsilon_2 = \dfrac{\sigma_2}{E_2}$

We know, strain in bar 1 = Strain in bar 2

$$\boxed{\therefore \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}} \qquad \textbf{Ans.}$$

1.22 STATICALLY INDETERMINATE MEMBERS

There are certain loaded members in which the equations of static equilibrium are not sufficient for a solution. This condition exists in structure where the number of unknown forces exceeds the number of equilibrium equations. Such cases are called statically indeterminate and require the use of addition relations that depend on the elastic deformation in the member and are known as **compatability equation.**

A compound bar is a case of an indeterminate system which is discussed below:

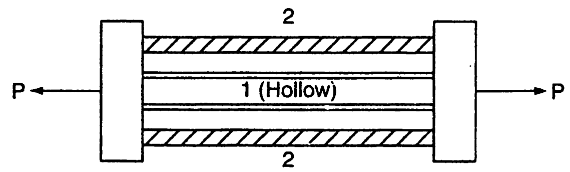


Fig. 1.67

Two bars of different materials (1 and 2) here make a composite bar or compound bar is subjected to load P.

Our objective is to find the load shared by hollow tube 1 and solid bar 2.

Applying condition of static equilibrium

$$P = P_1 + P_2 \qquad \text{...(I)}$$

Applying compatibility equation

i.e. $\delta_1 = \delta_2$ (i.e. deformation of bar is equal to tube)

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2} \quad \text{(As the length of 1 and 2 are equal } i.e.\ L_1 = L_2 = L)$$

or, $$P_1 = \frac{P_2 A_1 E_1}{A_2 E_2} \qquad \text{...(II)}$$

Substituting P_1 in (I) we get

$$P = \frac{P_2 A_1 E_1}{A_2 E_2} + P_2$$

$$= \frac{P_2 A_1 E_1 + P_2 A_2 E_2}{A_2 E_2}$$

$$= \frac{P_2(A_1 E_1 + A_2 E_2)}{A_2 E_2}$$

$\therefore$ $$P_2 = \frac{P \cdot A_2 E_2}{A_1 E_1 + A_2 E_2}$$

Similarly, $$P_1 = \frac{P \cdot A_1 E_1}{A_1 E_1 + A_2 E_2}$$

EXAMPLE 1 *A compound tube is made by shrinking a thin sheet tube on a thin brass tube. A_s and A_b are the sectional areas of the steel and brass tubes and E_s and E_b are the corresponding values of Young's Modulus. Show that for any tensile load the extension of the compound tube is equal to that of a single tube of the same length and total cross sectional area but having a Young's modulus of* $\left(\dfrac{E_sA_s + E_bA_b}{A_s + A_b}\right)$.

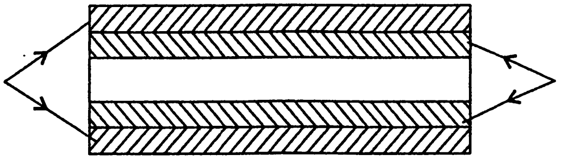


Fig. 1.68

SOLUTION Let the load on the compound tube be P.

Area of steel tube $= A_s$

Area of brass tube $= A_b$

Let the stresses in the steel tube and brass tube be σ_s and σ_b respectively therefore

$$\varepsilon_s = \varepsilon_b$$

or,
$$\left(\frac{\sigma_s}{E_s}\right) = \left(\frac{\sigma_b}{E_b}\right)$$

$\therefore$
$$\sigma_s = \frac{E_s}{E_b}\cdot\sigma_b$$

Load on steel + load on brass = total load on compound tube

$$\sigma_s\cdot A_s + \sigma_b\cdot A_b = P$$

$\therefore$
$$\frac{E_s}{E_b}\cdot\sigma_b\cdot A_s + \sigma_b\cdot A_b = P$$

$$\sigma_b\left[\frac{E_s}{E_b}\cdot A_s + A_b\right] = P$$

$\therefore$
$$\sigma_b = \left[\frac{E_b}{E_sA_s + E_bA_b}\right]\cdot P$$

$\therefore$ Extension of the compound tube $= dl =$ extension of steel or brass tube

$$dl = \frac{\sigma_b}{E_b}\cdot l$$

$$= \left[\frac{P}{E_sA_s + E_bA_b}\right]\cdot l \qquad \ldots(I)$$

Let E be the Young's Modulus of the tube of area (A_s) carrying the same load and undergoing the same extension.

$$dl = \left[\frac{P}{(A_s + A_b)\,E}\right] \qquad \ldots(\text{II})$$

From (I) and (II), we get

$$\frac{P \cdot l}{(A_s + A_b)\,E} = \frac{P \cdot l}{E_s \cdot A_s + E_b \cdot A_b}$$

$$\therefore \qquad E = \left[\frac{E_s A_s + E_b A_b}{A_s + A_b}\right]$$

EXAMPLE 2 *A load of 2 MN is applied on a short column of concrete 500 mm × 500 mm. The column is reinforced with four steel bars of 10 mm diameter, one in each corner. Find the stress in the concrete and steel bars. Take E for steel as* 2.1×10^5 *N/mm*^{2} *and for concrete* 1.4×10^5 *N/mm*^{2} **(UPTU-2004)**

SOLUTION Given P = load applied = 2 MN

$= 2 \times 10^6$ Newton

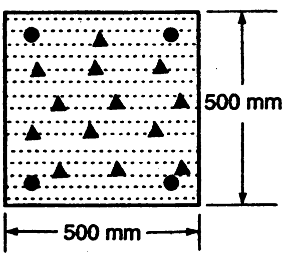


Fig.1.69

Area of column = (500×500) mm^2

$= 250000$ mm^2

Area of 4 steel bars (As) $= \cancel{4} \times \frac{\pi}{\cancel{4}}(10)^2$

$= 314.159$ mm^2

Area of concrete (Ac) = (Area of column – Area of steel bars)

$= 250000 - 314.159$

$= 249685.841$ mm^2

Now, strain in steel = strain in concrete

$$\varepsilon_{steel} = \varepsilon_{concrete}$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\therefore \qquad \sigma_s = \sigma_c \cdot \frac{E_s}{E_c}$$

$$= \sigma_c \times \frac{2.1 \times 10^5}{1.4 \times 10^4}$$

$$= 15\,\sigma_c \qquad \ldots(1)$$

Now, by figure we have,

Load on steel + load on concrete = Total load

$$\sigma_s A_s + \sigma_c A_c = P$$

$$15\,\sigma_c \times 314.159 + \sigma_c \times 249685.84 = 2000000$$

$$\boxed{\sigma_c = 7.86\ \text{N/mm}^2} \qquad \textbf{Ans.}$$

Putting the value of σ_c in the above equation (1).

$$\sigma_s = 15 \times 7.86\ \text{N/mm}^2$$

$$= 117.92\ \text{N/mm}^2 \qquad \textbf{Ans.}$$

EXAMPLE 3 *A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel respectively.*

$$E_{\text{concrete}} = 14\ \text{GPa}$$

$$E_{\text{steel}} = 200\ \text{GPa}$$

SOLUTION Given

$$\sigma_{co} = 6\,\text{MPa}$$

and $\sigma_{st} = 120$ MPa

$$E_{co} = 14\ \text{GPa}$$

and $E_{st} = 200$ GPa

Strain in column = Strain in steel

i.e. $\varepsilon_{co} = \varepsilon_{st}$

$$\frac{\sigma_{co}}{E_{con}} = \frac{\sigma_{st}}{E_{st}} \qquad [\because \sigma = \varepsilon \cdot E]$$

$$\frac{\sigma_{co}}{14 \times 10^3} = \frac{\sigma_{st}}{200 \times 10^3}$$

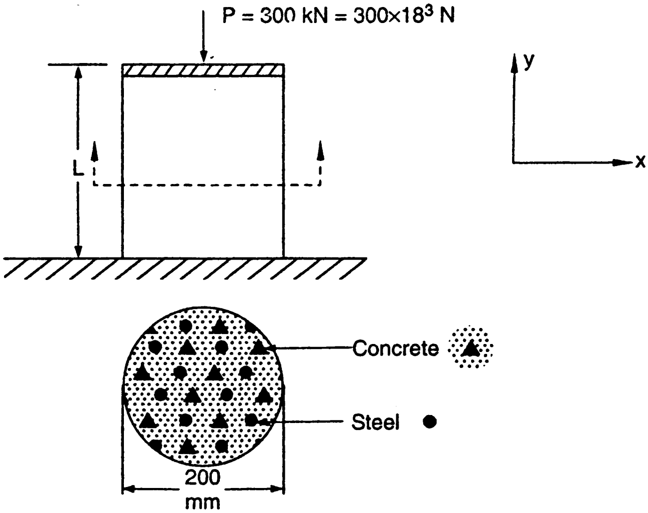


Fig. 1.70 Sectional Plan

$\therefore \quad 200\sigma_{co} = 14\sigma_{st}$

or, $\quad 100\sigma_{co} = 7\sigma_{st}$

when $\quad \sigma_{st} = 120 \text{ MPa}$

then $\quad 100\sigma_{co} = 7 \times 120 \text{ MPa}$

$$\therefore \quad \sigma_{co} = \frac{7 \times 120}{100} = 8.4 \text{ MPa} > 6 \text{ MPa}$$

when $\quad \sigma_{co} = 6 \text{ MPa}$

then $\quad 100 \times 6 = 7 \times \sigma_{st}$

$$\therefore \quad \sigma_{st} = \frac{600}{7} = 85.71 < 120 \text{ MPa}$$

Using $\sigma_{co} = 6$ MPa and $\sigma_{st} = 85.71$ MPa

Now, $\Sigma F_y = 0$ gives $P_{\text{steel}} + P_{\text{concrete}} = 300$

$$\sigma_{st} \cdot A_{st} + \sigma_{co} \cdot A_{co} = 300 \times 10^3$$

$$85.71 \times A_{st} + 6 \times \left[\frac{\pi}{4}(200)^2 - A_{st}\right] = 3 \times 10^5$$

$$79.71 A_{st} + 6000\pi = 3 \times 10^5$$

$$A_{st} = \frac{3 \times 10^5 - 6000\pi}{79.71}$$

$$= 1397.92$$

$$= 1398 \text{ mm}^2$$ **Ans.**

EXAMPLE 4 *Two brass rods and one steel rod together support a load as shown in figure 1.71. If the stresses in brass and steel are not to exceed 60 N/mm² and 120 N/mm², find the safe load that can be supported.*

Given, $E_{steel} = 2 \times 10^5 \, N/mm^2$

$E_{brass} = 1 \times 10^5 \, N/mm^2$

Cross-sectional area of steel rod = 1500 mm²

Cross-section area of each brass rod = 1000 mm²

SOLUTION We know that decrease in the length of steel rod, should be equal to the decrease in length of brass rod *i.e.* $\delta_{\text{steel}} = \delta_{\text{brass}}$.

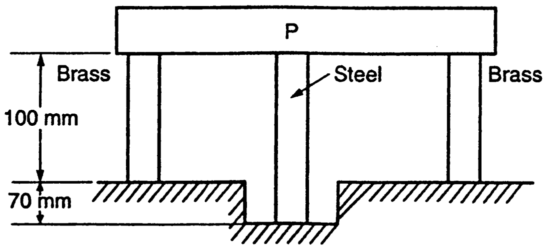


Fig. 1.71

Now, δ_{steel} = Strain in steel Rod × Length of steel rod

$= \varepsilon_s \times L_s$

Similarly, $\delta_{\text{brass}} = \varepsilon_b \times L_b$

where ε_s and ε_b are the strain in steel and brass rod respectively.

As $\delta_{\text{steel}} = \delta_{\text{brass}}$

$\therefore \quad \varepsilon_{\text{steel}} \cdot L_{\text{steel}} = \varepsilon_{\text{brass}} \cdot L_{\text{brass}}$

$$\frac{\varepsilon_{\text{steel}}}{\varepsilon_{\text{brass}}} = \frac{L_{\text{brass}}}{L_{\text{steel}}} = \frac{100}{170}$$

Now, $\sigma_{\text{steel}} = \varepsilon_{\text{steel}} \times E_{\text{steel}}$

and, $\sigma_{\text{brass}} = \varepsilon_{\text{brass}} \times E_{\text{brass}}$

$$\therefore \quad \frac{\sigma_{\text{steel}}}{\sigma_{\text{brass}}} = \frac{\varepsilon_{\text{steel}} \times E_{\text{steel}}}{\varepsilon_{\text{brass}} \times E_{\text{brass}}}$$

$$= \frac{100}{170} \times \frac{2 \times 10^5}{1 \times 10^5}$$

$$= 1.176$$

Total load (P) = load on steel rod + load on brass rods

$$\frac{\sigma_{\text{steel}}}{\sigma_{\text{brass}}} = 1.176$$

$\therefore \quad \sigma_{steel} = 1.176 \times \sigma_{brass}$

$= 1.176 \times 60$ (As $\sigma_{brass} \not> 60$ N/mm^2 given)

$= 70.56$ N/mm^2 < 120 N/mm^2 of steel given hence accepted

Therefore, safe load (P) = $\sigma_s . As + \sigma_b \cdot A_b$

$= 70.56 \times 1500 + 60 \times (2 \times 1000) = 105840 + 120000$

$= 225840$ Newton $= 225.84$ kN **Ans.**

EXAMPLE 5 *A rigid block of mass M is supported by three symmetrically spaced roads as shown in Fig. 1.72. Each copper rod has an area of 900 mm²; E = 120 GPa and the allowable stress is 70 MPa. The steel rod has an area of 1200mm², E = 200 GPa and the allowable stress is 140 MPa.*

Determine the largest mass M which can be supported.

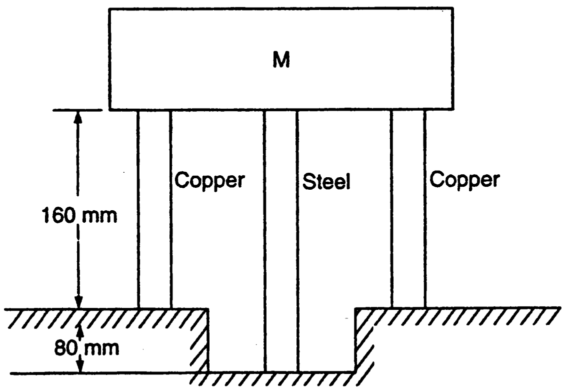


Fig. 1.72

SOLUTION It is evident that mass M causes the rods to deform equally

i.e. $\quad \delta_{co} = \delta_{st}$

$$\left(\frac{\sigma \cdot L}{E}\right)_{co} = \left(\frac{\sigma \cdot L}{E}\right)_{st}$$

$$\frac{\sigma_{co} \times 160}{120 \times 10^3} = \frac{\sigma_{st} \times 240}{200 \times 10^3}$$

$$\sigma_{co} = \frac{240 \times 120 \times 10^3}{160 \times 200 \times 10^3}\sigma_{st} = 0.96_{st}$$

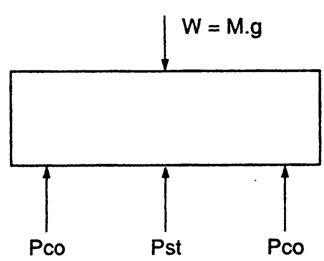


Fig. 1.73

when $\sigma_{st} = 140$ MPa, then $\sigma_{co} = 0.9 \times 140$ MPa

$= 126 \text{ MPa} > 70 \text{ MPa}$ Not OK

when $\sigma_{co} = 70$ MPa

then $\sigma_{st} = \frac{10}{9} \times \sigma_{co}$

$= \frac{10}{9} \times 70$ MPa

$= 77.78 \text{ MPa} < 140 \text{ MPa}$ Hence OK

Therefore using $\sigma_{co} = 70$ MPa

and $\sigma_{st} = 77.78$ MPa

Now, applying $\Sigma F_y = 0$ gives

$$2P_{co} + P_{st} = W$$

$$2(\sigma_{co} \times A_{co}) + \sigma_{st} \times A_{st} = M.g$$

$$2(70 \times 900) + 77.78 \times 1200 = M \times 9.81$$

$$M = \frac{2 \times 70 \times 900 + 77.78 \times 1200}{9.81}$$

$$= \frac{126000 + 93336}{9.81} = 22358.4 \text{ kg}$$ **Ans.**

EXAMPLE 6 *A rigid plateform shown in figure has negligible mass and rests on two steel bars each 250 mm long. There centre bar is aluminium and 249.90 mm long. Compute the stress in the aluminium bar after the centre load P =400 kN has been applied. For each steel bar the cross sectional area is 1200 mm² and G = 200 GPa and for the aluminium brass the area is 2400 mm² and E = 70 GPa.*

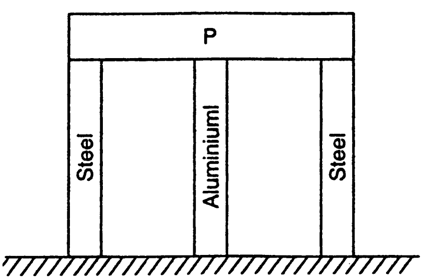


Fig. 1.74

SOLUTION Given $L_{\text{steel}} = 250$ mm

$L_{\text{aluminium}} = 249.90$ mm

Since applied load will deform rods equally so we have,

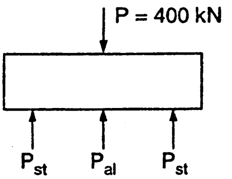


Fig. 1.75

$$\sigma_{st} = \delta_{al} + 0.10$$

$$\left(\frac{\sigma \cdot L}{E}\right)_{co} = \left(\frac{\sigma \cdot L}{E}\right)_{al} + 0.10$$

$$\frac{\sigma_{st} \cdot L_{st}}{E_{st}} = \frac{\sigma_{al} \cdot L_{al}}{E_{al}} + 0.10$$

$$\frac{\sigma_{st} \times 250}{2 \times 10^5} = \frac{\sigma_{al} \times 249.90}{70 \times 10^3} \times 0.10$$

$$1.25 \times 10^{-3} \sigma_{st} = 3.57 \times 10^{-3} \sigma_{al} + 0.10$$

$$\sigma_{st} = 2.856\, \sigma_{al} + 80 \qquad \text{...(I)}$$

$\Sigma F_y = 0$ gives

$$P_{al} + 2P_{st} = 4 \times 10^5$$

$$\sigma_{al} \times A_{al} + 2\sigma_{st} \times A_{st} = 4 \times 10^5$$

$$\sigma_{al} \times 2400 + 2\,(2.856\, \sigma_{al} + 80) \times 1200 = 4 \times 10^5$$

$$2400\sigma_{al} + 6854.4\, \sigma_{al} + 192000 = 4 \times 10^5$$

$$9254.54\, \sigma_{al} + 192000 = 4 \times 10^5$$

$$9254.46\, \sigma_{al} = 208000$$

$$\therefore \qquad \sigma_{al} = \left(\frac{208000}{9254.46}\right)$$

$$= 22.475 \text{ MPa} \qquad \textbf{Ans.}$$

1.23 ELONGATION DUE TO ROTATION

A uniform slender rod of length 'l' and cross sectional area A is rotating in a horizontal plane about a vertical axis through one end. If the unit mass of the rod is ρ and it is rotating at a constant angular velocity of ω rad/sec, show that the total elongation of the rod is $\left(\frac{\rho\omega^2 l^3}{3E}\right)$.

Proof: Let us consider a small element of thickness dx at a distance from origin O. Due to rotation, dP = Centrifugal force of differential mass

$$= dM \cdot \omega^2 x$$

$$= (\rho A dx)\, \omega^2 x$$

$$= \rho A \omega^2 x dx$$

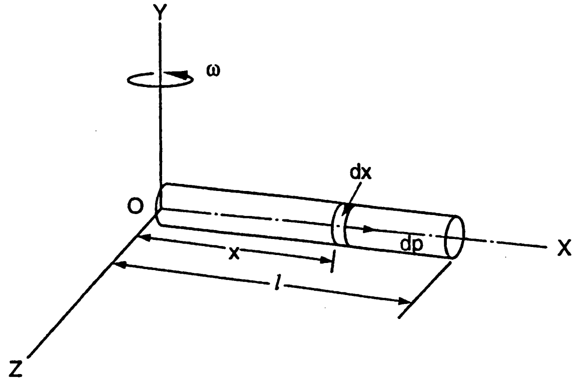


Fig. 1.76

Also, elemental deformation

$$d\delta = \frac{(\rho A\omega^2 x dx) \cdot x}{A \cdot E}$$

$$\int d\delta = \frac{\rho\omega^2}{E}\int_0^1 x^2 dx = \frac{\rho\omega^2}{E}\left[\frac{x^3}{3}\right]_0^l$$

$$\delta = \frac{\rho\omega^2}{E} \times \frac{l^3}{3}$$

$$= \frac{\rho\omega^2 l^3}{3E}$$

HIGHLIGHTS

1. Stress $(\sigma) = \frac{P}{A}$ assuming uniform distribution over the cross-section. Stress is a tensor. As long as the stress (σ) is less than the yield strength (σ_y) the material behaves elastically and obey's Hooke's law $\sigma = E\varepsilon$. When σ reaches the value σ_y, the material starts yielding and keeps deforming plastically under a constant load. If load is removed, unloading takes place along line *MN* parallel to the initial portion *OY* of the loading curve. The segment *ON* of the horizontal axis represents the strain corresponding to the permanent set or plastic deformation as a result from the loading and unloading of the

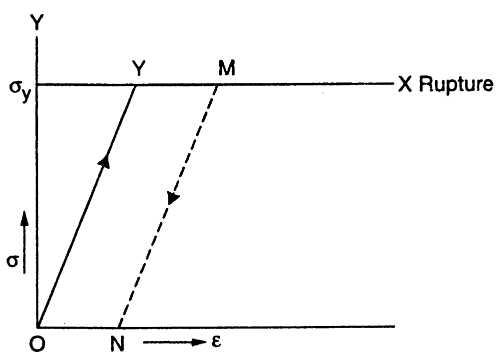


Fig. 1.77

specimen. No material behaves exactly as shown in the Fig. 1.77, this stress-strain diagram is useful in discussing plastic deformation of ductile material such as mild steel.

2. Strain $(\varepsilon) = \dfrac{\text{Change in dimension}}{\text{Original dimension}}$

   Strain may be—
   (a) Tensile strain (ε_t)
   (b) Compressive strain (ε_c)
   (c) Volumetric strain (ε_v)
   (d) Shear strain ϕ or γ
3. Stress may be—
   (a) Tensile stress (σ_t)
   (b) Compressive stress (σ_c)
   (c) Shear stress (τ)

4. Modulus of elasticity $(E) = \dfrac{\sigma}{\varepsilon}$ with the limit of Hook's law

$$= \frac{P/A}{\delta/l} = \frac{P}{A} \times \frac{l}{\delta}$$

   $\therefore$ Change in dimension (linear) of uniform section *i.e.* prismatic bar

$$\delta = \left[\frac{P \cdot l}{A \cdot E}\right]$$

5. Factor of safety (f.o.s) $= \dfrac{\text{Ultimate stress}}{\text{Working stress}}$

6. *Principle of superposition*
   Total change in length of a bar of different length and different diameter when subjected to an axial load P is given by

$$dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} + \cdots\right] \text{ when } E \text{ is same}$$

   In general $dL = \Sigma \dfrac{P_n L_n}{A_n E_n}$ where n = number of bar

$$= P\left[\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} + \frac{L_3}{E_3 A_3} + \cdots\right] \text{ where } E \text{ is different.}$$

7. The total deformation of a uniformly tapering circular rod of diameter D_1 and D_2 when the rod is subjected to an axial load P is given by

$$dL = \frac{4 \cdot P \cdot L}{\pi E D_1 D_2}$$

8. Total elongation (deformation) of a uniformly tapering rectangular bar when subjected to an axial load P is given by

$$dL = \frac{PL}{Et\,(a-b)} \cdot \log_e\left(\frac{a}{b}\right)$$

where L = total length of bar
t = thickness of bar
b = width at smaller end
a = width at bigger end
E = Young's modulus.

9. Elongation of a bar due to its own weight is given by

$$dL = \frac{WL}{2AE}$$

where W = weight of bar
L = length of bar
A = Area of cross-section of bar
E = modulus of elasticity of bar material.

10. In composite bar of equal length

(a) strain in each bar is equal

$$\varepsilon_1 = \varepsilon_2$$

i.e.

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

(b) Total load on the composite bar is equal to the sum of loads carried by each different material

i.e. $P = P_1 + P_2$

11. The stress induceed in the body due to change in temperature when the body is not allowed to expand or contract freely are known as thermal stress.

Thermal strain $(\varepsilon_{\text{temp}}) = \alpha \cdot T$

Thermal stress $(\sigma_{\text{th}}) = E \cdot \alpha \cdot \Delta T$

12. When a body is loaded axially it deforms longitudinally as well as transversely at right angle to the longitudinal direction.

The strain occurring in the longitudinal direction is known as longitudinal strain and that occurring in the transverse direction is known as lateral strain.

13. Within elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's Ratio and is denoted by $\frac{1}{m}$ or μ or ν.

14. Longitudinal strain $(\varepsilon_l) = \frac{\delta l}{l}$

Lateral strain $(\varepsilon_b \text{ or } \varepsilon_d) = \frac{\delta_b}{b} \text{ or } \frac{\delta_d}{d}$

where δl = change in length
δb = change in width
δd = change in depth or diameter.

15. Volumetric strain $(\varepsilon_v) = \dfrac{\delta_V}{V}$

Volumetric strain (ε_v) for a rectangular bar subjected to an axial load P is given by

$$\varepsilon_v = \frac{\delta l}{l}(1 - 2v)$$

16. Volumetric strain for a rectangular bar subjected to three mutually perpendicular stresses is given by

$$\varepsilon_v = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\nu)$$

where σ_x, σ_y and σ_z are the stresses in x, y and z direction respectively.

17. Principle of complementary shear stress states that a set of shear stresses across a plane is always accompanied by a set of balancing shear stresses of the same magnitude across the plane and normal to it.
i.e. $\tau = \tau'$

18. When an element is subjected to simple shear stresses then :
   (i) The planes of maximum normal stresses are perpendicular to each other.
   (ii) The planes of maximum normal stresses are inclined at an angle of 45° to the plane of pure shear.
   (iii) One of the maximum normal stress is tensile while the other maximum normal stress is compressive.
   (iv) The maximum normal stress are of the same magnitude are equal to the shear stress on the plane of pure shear.

19. Volumetric strain of a cylindrical rod, subjected to an axial tensile load is given by

$$\varepsilon_v = \text{longitudinal strain} - 2 \times \text{strain of diameter}$$
$$= \frac{\delta l}{l} - 2 \cdot \frac{\delta d}{d}$$

20. The ratio of normal stress to the corresponding volumetric strain is called the Bulk Modulus and denoted by K.

21. The relation between Young's modulus and bulk modulus is given by
$$E = 3K(1 - 2\nu)$$

22. The relation between modulus of elasticity and modulus of rigidity is given by
$$E = 2G(1 + \nu)$$
modulus of rigidity is also denoted by G.

23. Relation between Young's modulus, Bulk modules and modulus of rigidity is following

$$E = \frac{9GK}{3K + G}$$

PROBLEMS FOR PRACTICE
(THEORETICAL)

1. Define the term — stress; strain; elasticity; elastic limit; Young's modulus and Modulus of rigidity.
2. State Hook's law.
3. Four section of a bar having different lengths and different diameter are subjected to axial pull P.
   Determine the total change in length of the bar. Assume Young's modulus of different sections same.
4. Define modular ratio; thermal stresses; thermal strain and Poisson's ratio.
5. What do you mean by a bar of uniform strength?
6. Find an expression for the rotal elongation of a bar due to its own weight when the bar is fixed at its upper end hanging freely at the lower end.

(NUMERICAL)

1. Find the maximum value of P in drawing if $P \not> 140$ MPa in steel and in Aluminium 90 MPa or in Bronze of 100 MPa.

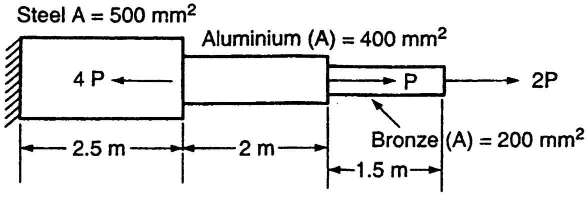


Fig. 1.78

2. Two blocks of wood width 'W' and thickness t are glued together along the joint inclined at an angle α as shown in Fig. 1.79. Using free body diagram show that the shearing stress on the glued joint is

$$\tau = \frac{P\alpha \sin^2 \alpha}{2A}$$

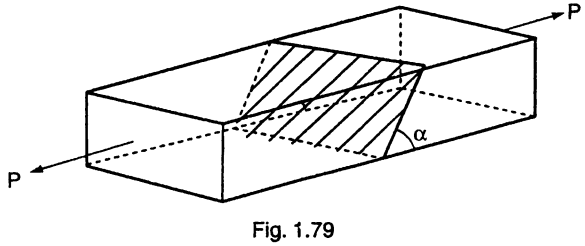


Fig. 1.79

3. A machine member is formed by connecting a steel bar to an aluminium bar as shown in Fig. 1.80. Assuming that the bars are prevented from buckling sideways, calculate the magnitude of force P, that will cause the total length of the member to decrease 0.30 mm. The values of elastic modulus for steel and aluminimum are $2 \times 10^5\ \text{N/mm}^2$ and $6.5 \times 10^4\ \text{N/mm}^2$ respectively.

[Ans. 406.22 kN]

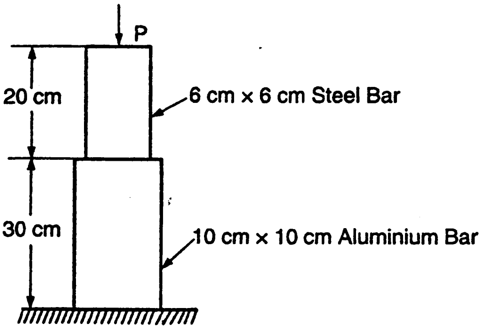
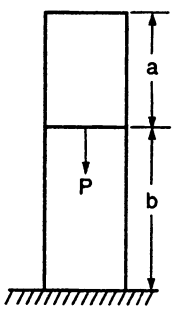


Fig. 1.80

4. Determine by taking into account the weight of bar, the displacement of the free end of bar shown in Fig.1.81. If its cross-sectional area is A the modulus of elasticity E and specific weight of material γ.



$$\left[\textbf{Ans. } \frac{Pb}{AE} + \frac{\gamma\,(a+b)^2}{2E}\right]$$

Fig. 1.81

5. Determine the displacement of section xx of the bar as shown in figure if its cross-section is A, modulus of elasticity E, and the specific weight of the material γ.

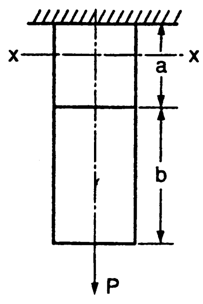


Fig. 1.82

$$\left[\textbf{Ans. } \frac{(P + \gamma\, Ab)\, a}{A \cdot E} + \frac{\gamma\, a^2}{2E}\right]$$

OBJECTIVE QUESTIONS

1. The factor of safety is the ratio of
   (a) working load or stress to ultimate load or stress
   (b) ultimate load or stress to yield load or stress
   (c) ultimate load or stress to working load or stress
   (d) yield load or stress to ultimate load or stress
2. Gauge length
   (a) is the total length of the specimen rod
   (b) is that part of the length over which measurement is made
   (c) there is no such length
   (d) none of the above
3. A prismatic bar is a bar of
   (a) maximum ultimate strength (b) maximum yield strength
   (c) uniform cross-section (d) varying cross-section
4. The gauge length is usually taken to be
   (a) 100 mm (b) 75 mm
   (c) 50 mm (d) 25 mm
5. The specimen rod under tension test has the following parameters
   (a) gauge length = 50 mm; diameter = 10 mm
   (b) gauge length = 50 mm; diameter = 12.5 mm
   (c) guage length = 25 mm; diameter = 25 mm
   (d) none of the above
6. The failure criteria for ductile materials is based on the following factor
   (a) ultimate strength (b) shear strength
   (c) yield strength (d) limit of proportionality
7. E and G are related by the equation
   (a) $E = 2G(1 - \nu)$ (b) $E = 2G(1 + \nu)$
   (c) $E = 3G(1 - \nu)$ (d) $E = 3G(1 + \nu)$
8. E and K are related by the equation
   (a) $E = 3K(1 + 2\nu)$ (b) $E = 3K(2 - \nu)$
   (c) $E = 3K(1 - 2\nu)$ (d) $E = 3E(1 - 2\nu)$
9. The material becomes harder due to strain hardening. Strain hardening in case of structural steel occurs
   (a) between yield strength and ultimate strength
   (b) between limit of proportionality and yield strength
   (c) between ultimate strength and fracture point
   (d) none of the above
10. Structural steel forms neck before it breaks. Neck formation starts
    (a) before limit of proportionality (b) after yield strength
    (c) before ultimate strength (d) at ultiamte strength
11. Limiting values of Poission's ratio are
    (a) 0 to (+) 0.5 (b) 0 to (–) 0.5
    (c) 1 to (+) 0.5 (d) – 1 to (+) 0.5

12. For mild steel, the ratio of modulus of elasticity in tension and compression (E_t/E_c) is equal to
(a) 0.5 (b) 1 (c) 1.2 (d) 1.3
13. Ductile fracture generally takes place along planes on which the shear stress is
(a) maximum (b) minimum (c) positive (d) negative
14. The property of a material to undergo large uniform elongation before fracture (in tension) is known as
(a) super elasticity (b) super plasticity
(c) visco elasticity (d) visco plasticity
15. The percentage reduction in area during the tension test on a cast iron test pecimen is
(a) 5 – 10% (b) 10 – 15% (c) 0 – 3% (d) 0 – 5%
16. The phenomenon under which the strain in a material varies under constant stress is called
(a) strain hardening (b) Bauschinger's effect
(c) creep (d) fatigue
17. The length, coefficient of thermal expansion and Young's Modulus of bar '*A*' are twice that of bar '*B*'. If the temperature of both bars is increased by the same amount while preventing any expansion, then the ratio of stress developed in bar *A* to that in bar *B* will be
(a) 2 (b) 4 (c) 8 (d) 16
18. In the figure shear stress is 'τ' and shear strain $(\phi) = \left(\frac{\tau}{G}\right)$, then the diagonal strain will be
(a) $\frac{\phi}{2}$ (b) $\frac{\phi}{4}$ (c) $\frac{\phi}{6}$ (d) $\frac{\phi}{8}$
19. If all the dimensions of a prismatic bar of cross-section suspended freely from the celing of a roof are doubled then the total elongation produced by its own weight will increase
(a) eight times (b) four times (c) three times (d) two times
20. A solid bar of uniform diameter D and length l is hung vertically from a ceiling. If the density of the material of the bar is ρ and the modulus of elasticity is E then the total elongation of the bar due to its own weight is
(a) $\frac{\rho l}{2E}$ (b) $\frac{\rho l^2}{2E}$ (c) $\frac{\rho E}{2l}$ (d) $\frac{\rho E}{2l^2}$
21. In terms of bulk modulus (K) and modulus of rigidity (G), the Poisson's ratio can be expressed as
(a) $\frac{3K - 2G}{6K + 4G}$ (b) $\frac{3K + 4G}{6K - 4G}$ (c) $\frac{3K - 2G}{6K + 2G}$ (d) $\frac{3K + 2G}{6K - 2G}$
22. A square section tapered bar of length l with sides l_1 and l_2 at its bigger and smaller end respectively is subjected to an axial pull. Taking E as the modulus of elasticity of the bar material, the elongation of the bar will be

(a) $\frac{2Pl}{El_1 l_2}$ (b) $\frac{P \cdot l}{2El_1 l_2}$ (c) $\frac{P \cdot l}{2E(l_1 + l_2)}$ (d) $\frac{P \cdot l}{E \cdot l_1 \cdot l_2}$

23. The elongation of conical bar of base diameter D hanging vertically, due to its own weight (when ρ = density of the material of the bar; l = length and E = (modulus of elasticity of the bar material) will be

(a) $\frac{\rho l^2}{2E}$ (b) $\frac{\rho l^2}{3E}$ (c) $\frac{\rho l^2}{4E}$ (d) $\frac{\rho \cdot l^2}{6E}$

24. A prismatic bar fixed at both ends is loaded by an axial load plating at a distance '*a*' from one of the supports. The reaction at the supports will be

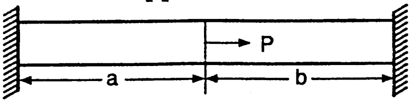


Fig. 1.83

(a) $\frac{Pb}{a+b}; \frac{Pa}{a+b}$ (b) $\frac{P}{a(a+b)}; \frac{P}{b(a+b)}$

(c) $\frac{2P}{a(a+b)}; \frac{2P}{b(a+b)}$ (d) none of the above

25. For an isotropic elastic material, the number of independent elastic constant is

(a) 1 (b) 2 (c) 3 (d) 4

26. For most metals, Poisson's ratio is approximately

(a) 0.3 (b) 3 (c) 35

(d) none of the above

27. A material is called perfectly elastic if

(a) while loading or unloading, the deformation and recovery are instantaneous

(b) the recovery is complete and immediate

(c) the load-deformation curve has the same shape while loading or unloading

(d) All the above

ANSWERS

1. (c)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (a)
7. (b)	**8.** (c)	**9.** (a)	**10.** (d)	**11.** (d)	**12.** (b)
13. (a)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (a)
19. (b)	**20.** (a)	**20.** (a)	**21.** (c)	**22.** (d)	**23.** (d)
24. (a)	**25.** (b)	**26.** (a)	**27.** (d)		

— ❖❖ —