Chapter

1

Analysis of Stress

1.1 DEFINITION OF STRESS

Consider an elastic body subjected to point forces, body forces, surface tractions, and thermal expansion. These external loads transmit their effects throughout the body and cause the body to deform. Now we will analyse quantitatively, the deformation and the mode of transmission of the external loads throughout the body.

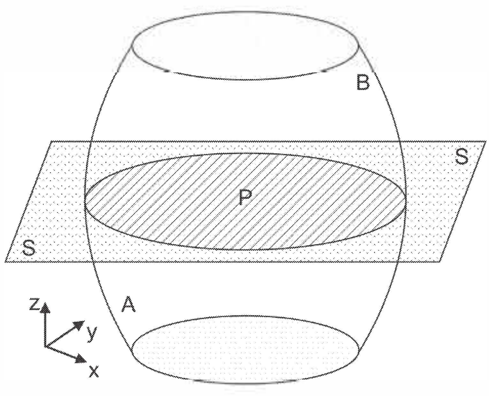

Figure 1.1 Body with a mathematical cut

Suppose that we pass a hypothetical plane S through an elastic body as shown in Fig. 1.1. If we consider the free body diagram of part A shown in Fig. 1.2, we have to determine the force distribution that is transmitted from portion A of the body to the portion B, through the interface S.

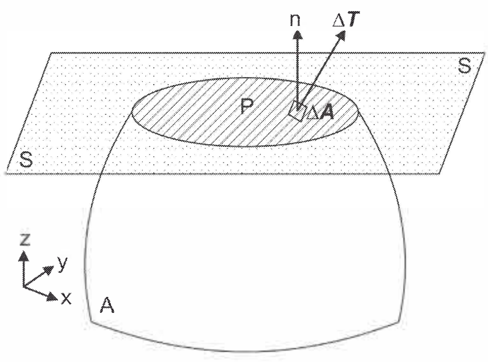

Figure 1.2 Force vector $\Delta \boldsymbol{T}$ on area ΔA at point P

Let P be the point on interface S with coordinates (x, y, z) as shown in Fig. 1.2. Let ΔA be a small area surrounding point P with outward drawn normal n. The action of part B of the body on ΔA at P can be represented by the force vector $\Delta \boldsymbol{T}$. We assume that as ΔA tends to zero the ratio $\Delta T/\Delta A$ tends to a definite limit; and further the moment of the forces acting on area ΔA about any point within the area vanishes in the limit. Expressed analytically

$$\lim_{\Delta A \to 0} \frac{\Delta T}{\Delta A} = \frac{dT}{dA} = \boldsymbol{T} \tag{1.1}$$

where $\boldsymbol{T}$ is called stress vector and represents force per unit area acting at point P on a plane with outward drawn normal n.

In general, the stress vector $\boldsymbol{T}$ at a point depends not only on the location of the point (identified by coordinates x, y, z) but also on the plane passing through the point (identified by direction cosines n_x, n_y, n_z) of the outward normal n.

The stress vector $\boldsymbol{T}$ can be resolved into two components, one along the normal n called the normal stress denoted by σ and the other perpendicular to n called the tangential stress or shear stress denoted by τ. We have the relation

$$|T| = \sigma^2 + \tau^2 \tag{1.2}$$

where $|T|$ is the magnitude of the stress vector.

The stress vector $\boldsymbol{T}$ can also be resolved into three components parallel to the x-, y-, z-axes denoted by T_x, T_y, T_z. We then have

$$|T|^2 = T_x^2 + T_y^2 + T_z^2 \tag{1.3}$$

It should be noticed that the definition of the stress is not restricted to solids only. They can apply to any continuous medium exhibiting viscosity or rigidity.

1.2 STRESS NOTATION

We shall concentrate on means of specifying stresses on an orthogonal set of interfaces at a point. Accordingly, consider interfaces at a point O parallel to the reference planes of an xyz reference, as shown in Fig. 1.3. Stresses on a plane with outward normal parallel to the x are denoted by $\sigma_{xx}, \tau_{xy}, \tau_{xz}$; on a plane with outward normal parallel to y are denoted by $\sigma_{yy}, \tau_{yx}, \tau_{yz}$; and on a plane with outward normal parallel to the z are denoted by $\sigma_{zz}, \tau_{zx}, \tau_{zy}$. The stresses for the orthogonal interfaces are shown on the enlarged diagram in Fig. 1.4.

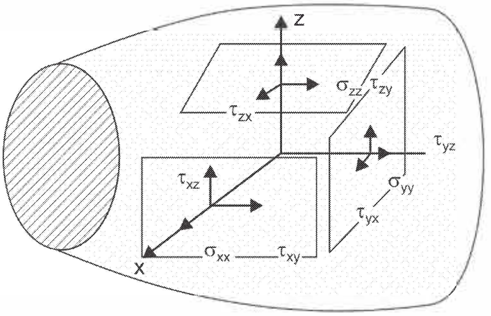

Figure 1.3 Orthogonal interfaces extracted from point O in a body

The nine stresses on three orthogonal interfaces at a point enable us to determine the stresses for any interface at the point.

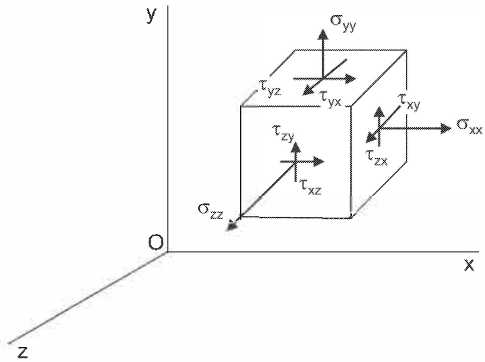

Figure 1.4 Rectangular stress components

We may consider the set of nine stresses parallel to the coordinate axes as rectangular components of a quantity which is called a *second-order tensor;* the *stress tensor*.

$$\tau_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \tag{1.4}$$

1.3 THE STATE OF STRESS AT A POINT

Since an infinite number of planes can be drawn through a point, we get an infinite number of stress vectors acting at a given point, each stress vector associated with the plane on which it is acting. The totality of all stress vectors acting on every possible plane passing through the point is defined to be the state of stress at the point.

1.4 STRESS ON AN ARBITRARY PLANE

Figure 1.5 shows a tetrahedron at point P generating three orthogonal interfaces at O and face ABC with arbitrary orientation.

Let the orthogonal planes be the x, y and z planes and the arbitrary plane be identified by its outward drawn normal n whose direction cosines are n_x, n_y and n_z. Consider a small tetrahedron at point P as shown in Fig. 1.5. Let h be the perpendicular distance from P to the inclined face ABC. Let T be the resultant stress vector on face ABC. This can be resolved into components T_x, T_y, T_z, parallel to the three axes x, y and z. Stress components σ_{xx}, τ_{xy}, τ_{xz} act on face PBC; Stress components σ_{yy}, τ_{yx}, τ_{yz} act on face PBC; stress components σ_{zz}, τ_{zx}, τ_{zy} act on face PBC. The area of faces BPC, CPA, APB are An_x, An_y, An_z where A is the area of inclined face ABC. For equilibrium of the tetrahedron PABC, the sum of forces in x, y and z directions must individually vanish. The result is Cauchy's stress formula.

$$T_x^{\,n} = n_x\sigma_{xx} + n_y\tau_{yx} + n_z\tau_{zx} \tag{1.5}$$

$$T_y^{\,n} = n_x\tau_{xy} + n_y\sigma_{yy} + n_z\tau_{zy} \tag{1.6}$$

$$T_z^{\,n} = n_z\tau_{xz} + n_y\tau_{yz} + n_z\sigma_{zz} \tag{1.7}$$

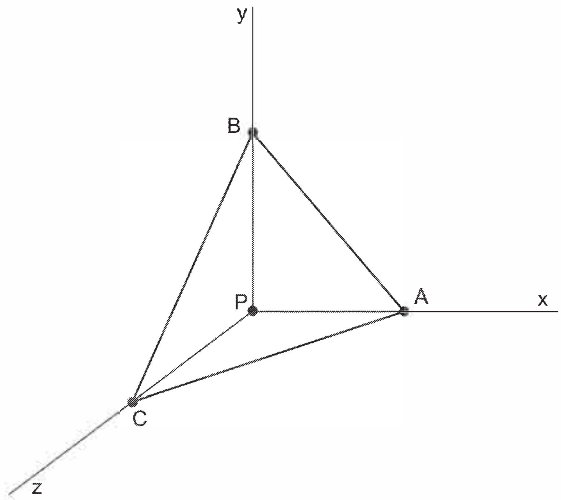

Figure 1.5 Tetrahedron at P generating three orthogonal interfaces at P and face ABC with arbitrary orientation

Equations (1.5)–(1.7) prove that the nine stress components at P will enable one to determine the stresses on any arbitrary plane through the point P.

If $\boldsymbol{T}^n$ is the stress vector on plane ABC,

$$|\boldsymbol{T}^n|^2 = (T_x^{\,n})^2 + (T_y^{\,n})^2 + (T_z^{\,n})^2 \tag{1.8}$$

If σ_n and τ_n are the normal and shear stress components, we have

$$|\boldsymbol{T}^n|^2 = \sigma_n^{\,2} + \tau_n^{\,2} \tag{1.9}$$

where

$$\sigma_n = n_x T_x^{\,n} + n_y T_y^{\,n} + n_z T_z^{\,n} \tag{1.10}$$

1.5 TRANSFORMATION EQUATIONS FOR STRESSES

Consider a point O somewhere inside a solid body. A reference xyz is shown at this point. Imagine an infinitesimal interface at O with a normal in the x direction. Consider stresses σ_{xx}, τ_{xy}, and τ_{xz} on this interface. Now move the interface away from point O along x-axis for convince viewing, as shown in Fig. 1.6. Do the same for interfaces in y and z directions as have been shown in the diagram. We can prove that if we know nine stresses at a point, we can determine normal and shear stresses on an interface at point O having any orientation relative to the axes xyz.

Let us consider an inclined interface at point O having any orientation n relative to the axes xyz. We can determine normal and shear stresses on this interface using the following equations:

$$\sigma_{nn} = \sigma_{xx}a_{nx}^2 + \tau_{xy}a_{nx}a_{ny} + \tau_{xz}a_{nx}a_{nz} + \tau_{yx}a_{ny}a_{nx} + \sigma_{yy}a_{ny}^2 + \tau_{yz}a_{ny}a_{nz} +$$
$$\tau_{zx}a_{nz}a_{nx} + \tau_{zy}a_{nz}a_{ny} + \sigma_{zz}a_{nx}^2 \tag{1.11}$$

where a_{nx}, a_{ny}, a_{nz} denote direction cosines of n.

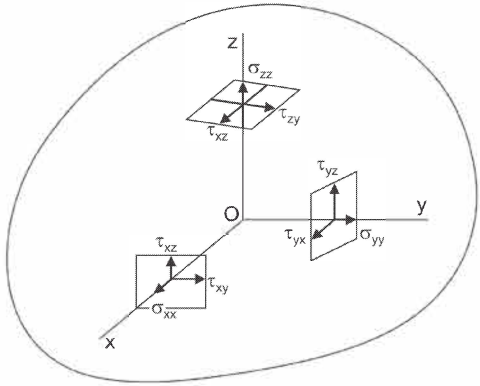

Figure 1.6 Stresses on orthogonal interfaces at the point O

We now proceed to calculate shear stress in some chosen direction *s* on the inclined interface. The required equation is

$$\tau_{ns} = \sigma_{xx}a_{nx}a_{sx} + \tau_{xy}a_{nx}a_{sy} + \tau_{xz}a_{nx}a_{sz} + \tau_{yx}a_{ny}a_{sx} + \sigma_{yy}a_{ny}a_{sy} + \tau_{yz}a_{ny}a_{sz} + \tau_{zx}a_{nz}a_{sx} + \tau_{zy}a_{nz}a_{sy} + \sigma_{zz}a_{nz}a_{sz} \tag{1.12}$$

where a_{sx}, a_{sy}, a_{sz} denote direction cosines of *s*.

Equations (1.11) and (1.12) thus permit us to calculate stresses on any given interface at a point provided that we know the nine stresses on an orthogonal setoff interfaces at that point.

We can now proceed to compute nine stress components for a reference $x'y'z'$ rotated relative to xyz at a point O (Fig. 1.7). Instead of using n to denote the direction of an inclined interface, we shall use x'. Then Eq. (1.11) can be used to give the stress component $\sigma_{x'x'}$ by replacing n by x' as follows

$$\sigma_{x'x'} = \sigma_{xx}a^2_{x'x} + \tau_{xy}a_{x'x}a_{x'y} + \tau_{xy}a_{x'x}a_{x'y} + \tau_{xz}a_{x'x}a_{x'z} + \tau_{yz}a_{x'y}a_{x'a} + \sigma_{yy}a^2_{x'y} + \tau_{yz}a_{x'y}a_{x'z} + \tau_{zx}a_{x'z}a_{x'x} + \tau_{zy}a_{x'z}a_{x'y} + \sigma_{zz}a^2_{x'z} \tag{1.13}$$

where $a_{x'x}$ is the direction on cosine between the x′-axis and x-axis, and so on. By repeating this procedure, we can determine other normal stresses $\sigma_{y'y'}$ and $\sigma_{z'z'}$. Equation (1.13) may be called the *seminal equation* for generating normal stress for orthogonal interfaces corresponding to a reference $x'y'z'$ rotated relative to xyz at a point.

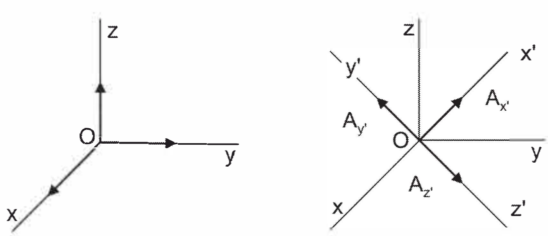

Figure 1.7 Reference coordinate axes

In a similar manner, the shear stress $\tau_{x'y'}$ can be found from Eq. (1.12) by replacing n by x' and s by y'. Thus we get

$$\tau_{x'y'} = \sigma_{xx}a_{x'x}a_{y'x} + \tau_{xy}a_{x'x}a_{x'y} + \tau_{xz}a_{x'x}a_{x'z} + \tau_{yx}a_{x'y}a_{y'x} + \sigma_{yy}a_{x'y}a_{y'y} +$$

$$\tau_{yz}a_{x'y}a_{y'z} + \tau_{zx}a_{x'z}a_{y'x} + \tau_{zy}a_{x'z}a_{y'y} + \sigma_{zz}a_{x'z}a_{y'z} \tag{1.14}$$

We can calculate other shear stresses $\tau_{x'z'}$, $\tau_{y'x'}$, $\tau_{y'z'}$, $\tau_{z'x'}$, $\tau_{z'y'}$. For orthogonal interfaces associated with axes $x'y'z'$.

Thus Eq. (1.14) is the parent equation for determining shear stresses for the reference $x'y'z'$ rotated relative to xyz at a point.

1.6 PRINCIPAL STRESSES

There exist three orthogonal planes through a point on which only normal stresses exist and shear stresses are zero called principal planes. The direction cosines of outward normal on these planes define the principal axes. The magnitude of normal stresses on the principal planes are called *principal stresses*. Given the nine stress components at a point, our objective is to determine the principal stresses, and the direction cosines of principal axes. This is discussed in this section.

Let us assume that there is a plane n with direction cosines n_x, n_y and n_z on which the stress vector $\boldsymbol{T}^n$ is wholly normal. Let σ be the magnitude of this stress vector. Then we have

$$\boldsymbol{T}^n = \sigma n \tag{1.15}$$

The components of along the x-, y- and z-axes are

$$T_x^n = \sigma n_x ; T_y^n = \sigma n_y ; T_z^n = \sigma n_z \tag{1.16}$$

Also, from Cauchy's formula

$$T_x^n = \sigma_{xx}n_x + \tau_{xy}n_y + \tau_{xz}n_z$$

$$T_y^n = \tau_{xy}n_x + \sigma_{yy}n_y + \tau_{yz}n_z$$

$$T_z^n = \tau_{xz}n_x + \tau_{yz}n_y + \sigma_{zz}n_z \tag{1.17}$$

Subtracting Eq. (1.17) from Eq. (1.16), we get

$$(\sigma_{xx} - \sigma)n_x + \tau_{xy}n_y + \tau_{xy}n_z = 0$$

$$\tau_{xy}n_x + (\sigma_{yy} - \sigma)n_y + \tau_{yz}n_z = 0$$

$$\tau_{xz}n_x + \tau_{yz}n_y + (\sigma_{zz} - \sigma)n_z = 0 \tag{1.18}$$

These are three simultaneous equations involving the unknowns n_x, n_y, and n_z. For the existence of a nontrivial solution.

$$\begin{vmatrix} (\sigma_{xx} - \sigma) & \tau_{xy} & \tau_{zz} \\ \tau_{xy} & (\sigma_{yy} - \sigma) & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & (\sigma_{zz} - \sigma) \end{vmatrix} = 0 \tag{1.19}$$

On expanding, the above determinant leads to a cubic equation in σ as

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0 \tag{1.20}$$

where,
$$I_1 = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \tag{1.21}$$
$$I_2 = (\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2) \tag{1.22}$$
$$I_3 = (\sigma_{xx}\,\sigma_{yy}\,\sigma_{zz} + 2\,\tau_{xy}\,\tau_{yz}\,\sigma_{xz} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{zx}^2 - \sigma_{zz}\tau_{xy}^2) \tag{1.23}$$

The three real roots of the cubic equation designated as σ_1, σ_2, and σ_3 are the principal stresses.

Substituting any one of these (σ_1 or σ_2 or σ_3) in Eq. (1.18), we can solve for the corresponding entities n_x, n_y and n_z. In the sequel to avoid the trivial solution, the condition
$$n_x^2 + n_y^2 + n_z^2 = 1 \tag{1.24}$$
is used along with any two equations from the set of equations [Eq. (1.18)].

The planes on each of which the stress vector is wholly normal are called the principal planes, and the corresponding stresses are the principal stresses. The normal to a principal plane is called the principal stress axis.

The quantities I_1, I_2 and I_3 are known as the first, second and third invariants of stress respectively. An invariant is one whose value does not change when the frame of reference is changed.

In terms of the principal stresses, the invariants are
$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{1.25}$$
$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \tag{1.26}$$
$$I_3 = \sigma_1\sigma_2\sigma_3 \tag{1.27}$$

1.7 OCTAHEDRAL STRESSES

A plane that is equally inclined to the three axes x, y, z is called an *octahedral plane*. Such a plane will have $n_x = n_y = n_z$. Since $n_x^2 + n_y^2 + n_z^2 = 1$, an octahedral plane will be defined by $n_x = n_y = n_z = \pm \dfrac{1}{\sqrt{3}}$. There are eight such planes.

The normal and shear stresses on these planes are called the *octahedral normal stress* σ_{oct} and *octahedral shear stress* τ_{oct}. They are calculated by the following formulae.
$$\sigma_{oct} = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \tag{1.28}$$
$$\tau_{oct}^2 = \frac{1}{9}\left[\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + \left(\sigma_{zz} - \sigma_{xx}\right)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right] \tag{1.29}$$

They can also be expressed in terms of the principal stress and stress invariants as
$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3} I_1 \tag{1.30}$$
$$\tau_{oct}^2 = \frac{1}{9}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right] \tag{1.31}$$
or
$$\tau_{oct} = \frac{\sqrt{2}}{3}(I_1^2 - 3I_2)^{1/2} \tag{1.32}$$

The octahedral normal stress σ_{oct} can be interpreted as the mean normal stress at a given point in a body. If in state of stress, the first invariant $I_1 = (\sigma_1 + \sigma_2 + \sigma_3)$ is zero, then the normal stresses on the octahedral planes will be zero and only the shear stresses will act.

1.8 HYDROSTATIC AND PURE SHEAR STATES

An arbitrary state of stress can be resolved into a hydrostatic state and a state of pure shear. Let the given state referred to a Cartesian system *xyz* be

$$[\tau_{ij}] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} \tag{1.33}$$

Let $$p = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) = \frac{1}{3} I_1 \tag{1.34}$$

The given stress state can be resolved into two different states

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} + \begin{bmatrix} (\sigma_{xx} - p) & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & (\sigma_{yy} - p) & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & (\sigma_{zz} - p) \end{bmatrix} \tag{1.35}$$

The pure shear state of stress is also known as the *deviatoric state of stress*.

1.9 EQUATIONS OF EQUILIBRIUM

A rectangular parallelepiped under the action of stress and body forces is shown in Fig. 1.8. We wish to establish with this element the requirement for the manner in which stresses must vary with position to guarantee equilibrium in a domain (i.e. we wish to derive the differential form of the equations of equilibrium). Summing forces in the x-direction, we get

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + B_x = 0 \tag{1.36}$$

Using the complementary property of shear stresses ($\tau_{xy} = \tau_{yz}$, $\tau_{yz} = \tau_{zy}$, $\tau_{zx} = \tau_{zx}$) making similar computations in the y and z directions, we may give the equations of equilibrium at a point in the body as follows:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + B_x = 0 \tag{1.37}$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + B_y = 0 \tag{1.38}$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + B_z = 0 \tag{1.39}$$

where B_x, B_y, B_z are the body forces components per unit volume.

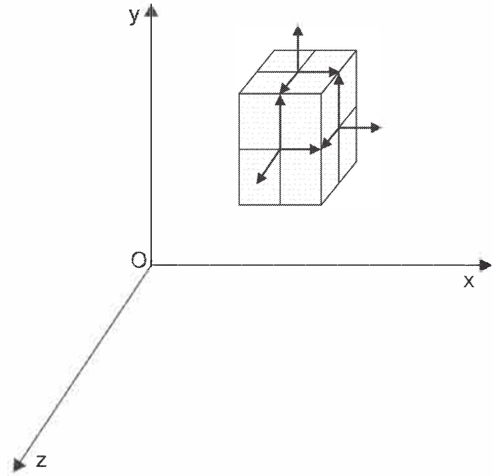

Figure 1.8 A rectangular parallelepiped under the action of stresses

Notice that the equations of equilibrium now involve stresses and body forces are functions of position. These are so called field variables. Also; we have here a set of partial differential equations for these field variables.

If in a given state of stress, there exists a coordinate system xyz such that

$$\sigma_{zz} = 0,\ \tau_{xz} = 0,\ \tau_{yz} = 0, \tag{1.40}$$

then that it is said to be a plane stress state parallel to the xy plane. This stress state is also known as a *two-dimensional state of stress*. The differential equations of equilibrium for plane stress state (in the xy plane) become

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + B_x = 0 \tag{1.41}$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + B_y = 0 \tag{1.42}$$

1.10 EQUATIONS OF EQUILIBRIUM IN CYLINDRICAL COORDINATES

Till now, we have used a Cartesian frame of reference for analysis of stress. Numerous problems exist where it is more convenient to use polar or cylindrical coordinates.

Consider an elastic body of revolution as shown in Fig. 1.9, the axis of revolution is taken as the z-axis. The two other coordinates are r and θ, where θ is measured counterclockwise.

The stress components at a point P (r, θ, z) are σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, $\tau_{r\theta}$, $\tau_{\theta z}$, τ_{zr}. These are shown acting on the faces of a radial element at point P in Fig. 1.9. σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} are called radial, circumferential and axial stress respectively. If the stresses vary from point to point, one can derive the differential equation of equilibrium.

Let B_r, B_θ and B_z denote the body force components per unit volume. For equilibrium of the element shown in Fig. 1.9, the sum of forces in r, θ and z directions must vanish individually. We can derive the differential equations of equilibrium expressed in polar coordinates as

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{(\sigma_{rr} - \sigma_{\theta\theta})}{r} + B_r = 0 \tag{1.43}$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\tau_{zz}}{r} + B_z = 0 \tag{1.44}$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2\tau_{r\theta}}{r} + B_\theta = 0 \tag{1.45}$$

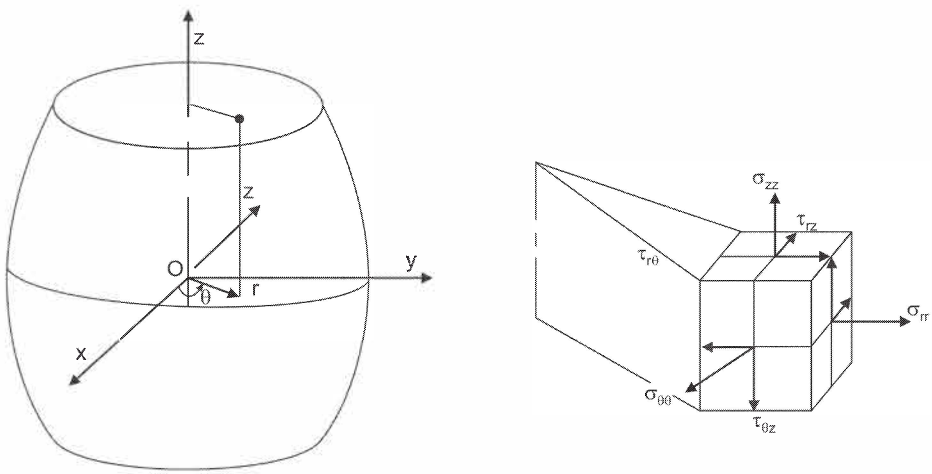

Figure 1.9 Cylindrical coordinates ($r\theta z$) and stresses on an element

Axisymmetric Case

If a body of revolution is loaded symmetrically, the stress components do not depend on θ. Since the deformation are axisymmetric, $\tau_{r\theta}$ at $\tau_{\theta z}$ do not exist. Consequently, the above equations in the absence of body forces are reduced to

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{(\sigma_{rr} - \sigma_{\theta\theta})}{r} + \frac{\partial \tau_{rz}}{\partial z} = 0 \tag{1.46}$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\tau_{rz}}{r} = 0 \tag{1.47}$$

However, the state of stress is now characterized by the components $\sigma_{rr}(r, z)$, $\sigma_{\theta\theta}(r, z)$, $\sigma_{zz}(r, z)$, and $\tau_{rz}(r, z)$.

Plane Stress Case

If the state of stress is a two-dimensional plane stress, then only σ_{rr}, $\sigma_{\theta\theta}$, $\tau_{r\theta}$ exist. The other components vanish.

The equation of equilibrium reduced to

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{(\sigma_{rr} - \sigma_{\theta\theta})}{r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} = 0 \tag{1.48}$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \tag{1.49}$$

1.11 SUMMARY

In this chapter, we have presented the concept of stress as a means of describing how an external force (point load, body force, surface traction, temperature change) distributes through an elastic body. In particular, we introduced notation for nine stress components. These form the rectangular components of a second order tensor. Not all nine components of the stresses are arbitrary. Using Newton's law, it was shown that shear stresses with reverse indices at a point are equal, i.e. the complementing property of shear, so only six of the nine stress terms are independent at a point. Finally, for equilibrium, the six stress components had to vary with position in a certain manner established by the equations of equilibrium, a set of partial differential equations. We shall consider next how to measure the primary effects of the applied forces, namely the deformation of a body.

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