1

Design of Curved Beams

1.1 INTRODUCTION

A beam is a structural member whose length is large (longer than the width and the thickness) compared to its cross-sectional area which is loaded and supported in the direction transverse to its axis.

Curved beams in the form of C-clamps, press frames, chain links and brackets are used as machine elements. As the name indicates the beam is initially curved before the bending moment is applied. When such members are subjected to bending moment, the stress distribution is not linear since the stress increases more rapidly on the inner side.

1.2 DIFFERENCE BETWEEN A STRAIGHT BEAM AND A CURVED BEAM

Straight Beam (Fig. 1.1):

- Here the beam is initially straight.
- In a straight beam, there are infinite number of layers of equal length and parallel to each other.
- The neutral axis (NA) coincides with the centroidal axis.
- The stress distribution is linear.
- The expression for straight beam

$$\sigma_b = \frac{M}{I} c \qquad \text{... 1.1(b)/Pg 2, DHB}$$

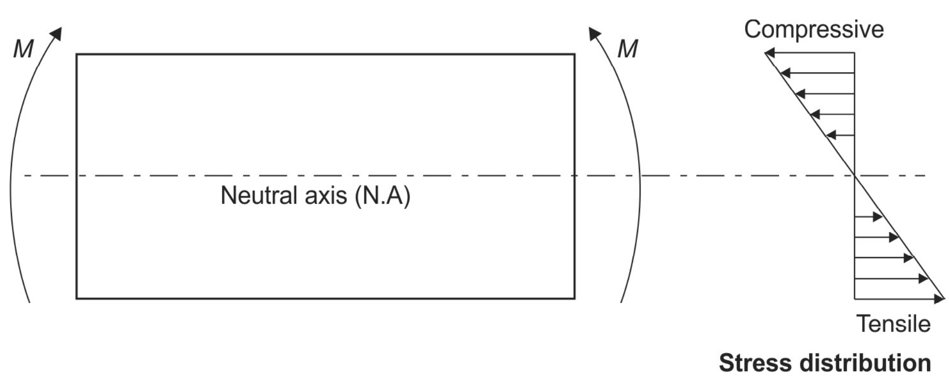

Fig. 1.1: A straight beam

where,

σ_b = bending stress
M = applied bending moment
I = moment of inertia
c = distance of layer from neutral axis

Curved Beam (Fig. 1.2):

- Here the beam is initially curved.
- The neutral axis does not coincide with the centroidal axis but is shifted towards the center of curvature of the beam.
- Stress distribution is not linear but is hyperbolic since neutral axis is initially curved.
- Fibres on one side of the neutral axis are in tension while on the other side the layers are in compression.
- The expression for bending stress

$$\sigma_i = \frac{Mc_i}{AeR_i} \text{ or } \sigma_o = \frac{-Mc_o}{AeR_o} \qquad \text{... 10.1(b) and (c)/Pg 159, DHB}$$

where,

$\sigma_{i,o}$ = bending stress at inner at outer fibers respectively
M = applied bending moment
c_i = distance from centroidal axis to the inner fiber
c_o = distance from centroidal axis to the outer fiber
e = distance from centroidal axis to neutral axis
R_i = radius of curvature of inner fiber
R_o = radius of curvature of outer fiber

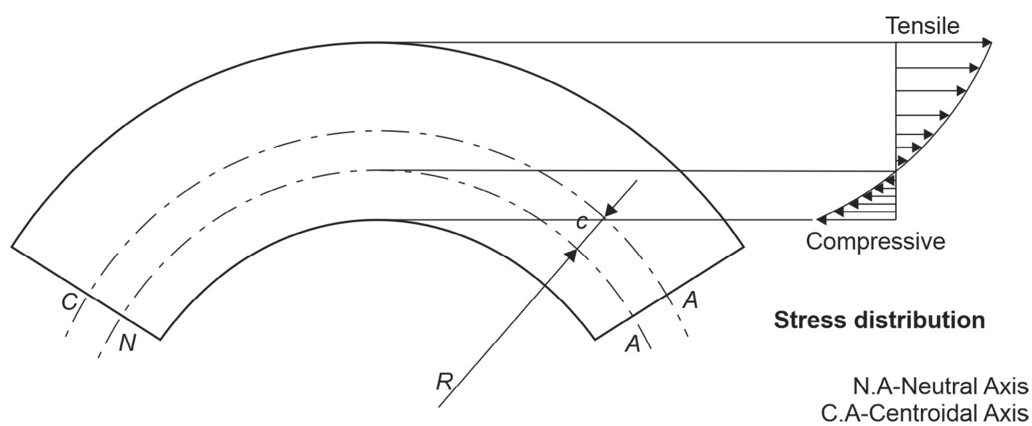

Fig. 1.2: A curved beam

1.3 STRESSES IN CURVED BEAMS (WINKLER-BACH EQUATION)

Assumptions:

- The material of the beam is perfectly homogeneous and isotropic.
- The material of the beam obeys Hooke's law.
- Young's modulus is same in tension and compression.

- Each layer of the beam is free to expand or contract independent of the layer above or below it.
- The transverse sections of the beam which are plane before bending remain plane even after bending.
- Stresses induced are within the elastic limit.

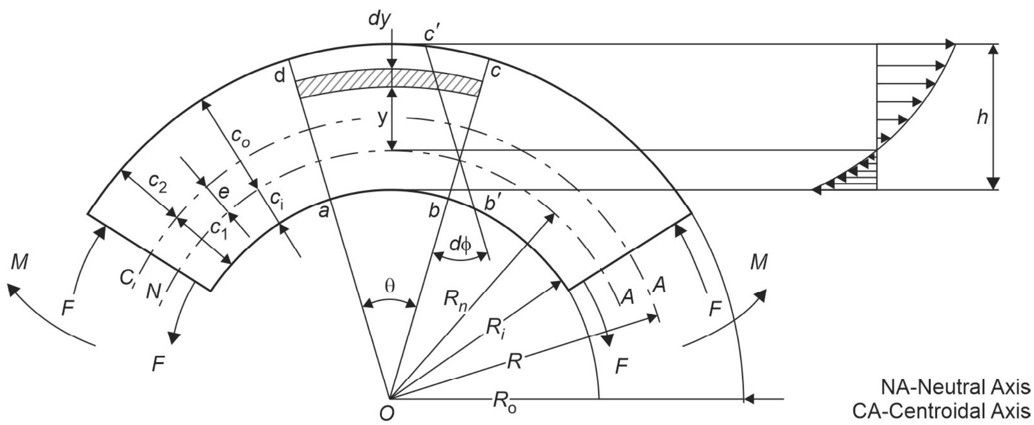

Fig. 1.3: Stress analysis in curved beam **(Fig. 10.1/Pg 160, DHB)**

Let,

F = applied load
M = applied bending moment
A = cross-sectional area
e = distance from centroidal axis to neutral axis
R = radius of curvature of centroidal axis = $(R_i + c_i)$
R_n = radius of curvature of neutral axis = $(R_i + c_i)$ or $(R_o - c_o)$
R_i = radius of curvature of inside fiber
R_o = radius of curvature of outside fiber = $(R + c_2)$
c_i = distance from neutral axis to inner fiber = $(c_1 - e)$ ... **10.1(d)/Pg 159, DHB**
c_o = distance from neutral axis to outer fiber = $(c_2 + e)$ or $(h - c_i)$ ... **10.1(d)/Pg 159, DHB**
c_1 = distance from centroidal axis to inner fiber
c_2 = distance from centroidal axis to outer fiber
h = depth of cross-section = $(c_1 + c_2)$ or $(c_i + c_o)$
y = distance from neutral axis to fiber under consideration

Consider a segment *abcd* subtending an angle θ at the center of curvature. When the beam is subjected to a bending moment as shown in **Fig. 1.3**, the side *bc* undergoes rotation through an angle $d\phi$ about neutral axis and takes a new position $c'b'$. Due to rotation, the inner fibers are stretched while the outer fibers are compressed.

Consider a strip of thickness dy at a distance y from neutral axis and having an area dA.

The original length of strip = $(R_n + y)\, d\theta$ ... (Eq. 1.1)

and the elongation experienced by the strip = $y \cdot d\phi$ ... (Eq. 1.2)

$\therefore$ the strain experienced by the strip, $\varepsilon = \dfrac{y \cdot d\phi}{(R_n + y)\, d\theta}$ ... (Eq. 1.3)

According to Hooke's law $\sigma = \varepsilon E$

$$\sigma = E\frac{y \cdot d\phi}{(R_n + y)d\theta} = \left[E\frac{d\phi}{d\theta}\right]\left(\frac{y}{R_n + y}\right) \qquad \text{... (Eq. 1.4)}$$

Now force responsible for strain in the strip $dF = \sigma \cdot dA$

$$dF = \left[E\frac{d\phi}{d\theta}\right]\left(\frac{y}{R_n + y}\right)dA \qquad \text{... (Eq. 1.5)}$$

For the beam to be in equilibrium $\Sigma F = 0$

i.e. $$\int dF = 0$$

$$\int\left[E\frac{d\phi}{d\theta}\right]\left(\frac{y \cdot dA}{R_n + y}\right) = 0$$

$$E\left[\frac{d\phi}{d\theta}\right]\int\left(\frac{y \cdot dA}{R_n + y}\right) = 0$$

Since $$E\left[\frac{d\phi}{d\theta}\right] \neq 0$$

we have $$\int\frac{y \cdot dA}{(R_n + y)} = 0 \qquad \text{... (Eq. 1.6)}$$

Taking moments about neutral axis (NA) for the strip

$$dM = dF \cdot y$$

$$= \left[E\frac{d\phi}{d\theta}\right]\left(\frac{y^2 \cdot dA}{R_n + y}\right) \qquad \text{... (Eq. 1.7)}$$

Thus the total bending moment is $M = \int dM$

$$= \int\left[E\frac{d\phi}{d\theta}\right]\left(\frac{y^2 \cdot dA}{R_n + y}\right)$$

$$= \left[E\frac{d\phi}{d\theta}\right]\int\left(\frac{y^2 \cdot dA}{R_n + y}\right)$$

$$= \left[E\frac{d\phi}{d\theta}\right]\int\left[y - \left(\frac{y \cdot R_n}{R_n + y}\right)\right]dA$$

$$= \left[E\frac{d\phi}{d\theta}\right]\left\{\int y \cdot dA - R_n\int\left(\frac{y}{R_n + y}\right)dA\right\}$$

$$M = \left[E\frac{d\phi}{d\theta}\right]\left\{\int y \cdot dA - 0\right\} \qquad \text{... using (Eq. 1.6)}$$

But $\int y \cdot dA$ is the moment of inertia, which may be replaced with Ae, [i.e. the product of total area and distance e from centroidal axis to neutral axis].

$$\therefore \qquad M = \left[E\frac{d\phi}{d\theta}\right]Ae$$

$$\left[E\frac{d\phi}{d\theta}\right] = \frac{M}{Ae} \qquad \text{... (Eq. 1.8)}$$

Substituting Eq. (1.8) in Eq. (1.4)

$$\boldsymbol{\sigma = \frac{M}{Ae}\left(\frac{y}{R_n + y}\right)} \qquad \text{... (Eq. 1.9)} \ \textbf{10.1(a)/Pg 159, DHB}$$

(Eq. 1.9) gives the stress induced in any fibre at a distance y from the neutral axis.
For the existing beam (before applying the moment)

y is negative, when measured towards center of curvature ($-c_i$)

y is positive, when measured away from center of curvature (c_o)

At inner fiber, $y = -c_i$

(Eq. 1.9) yields...
$$\sigma_i = \frac{M}{Ae}\left(\frac{-c_i}{R_n - c_i}\right)$$

$$\therefore \qquad \sigma_i = \frac{-Mc_i}{AeR_i} \qquad (\because R_n - c_i = R_i) \qquad \text{... (Eq. 1.10)}$$

At outer fiber, $y = +c_o$

(Eq. 1.9) yields...
$$\sigma_o = \frac{M}{Ae}\left(\frac{c_o}{R_n + c_o}\right)$$

$$\sigma_o = \frac{Mc_o}{AeR_o} \qquad (\because R_n + c_o = R_o) \qquad \text{... (Eq. 1.11)}$$

Based on the applied bending moment, outer fibers are subjected to compression (negative) and inner fibers are subjected to tension (positive).

(Eq. 1.10) yields...
$$\boldsymbol{\sigma_i = \frac{Mc_i}{AeR_i}} \qquad \text{... (Eq. 1.12)} \ \textbf{10.1(b)/Pg 159, DHB}$$

(Eq. 1.11) yields...
$$\boldsymbol{\sigma_o = \frac{-Mc_o}{AeR_o}} \qquad \text{... (Eq. 1.13)} \ \textbf{10.1(c)/Pg 159, DHB}$$

Note:

- *The bending stress in a curved beam is zero at a point other than at centroidal axis.*
- *If the section is symmetrical such as circular, rectangular, I-beam with equal flanges, then maximum bending moment will always occur at the inside fiber.*
- *If the section has an axial load in addition to bending, then it should be added to the bending stress to obtain the resultant stress on the section.*
- *On the other hand, if the line of action of force does not pass through the centre of gravity (C_g) of the section, it is referred to as* ***eccentric load****. To analyze such problems, we replace the eccentric load by an equal and parallel force through C_g of the cross-section together with a couple (C) in opposite direction as shown in* ***Fig. 1.4.***

  Direct stress (tensile/compressive) $\sigma_D = \pm\dfrac{F}{A}$ ... **1.1(a)/Pg 2, DHB**

  This couple $C = Fx$ produces bending stress in the cross-section and hence is a bending moment with respect to centroidal axis.

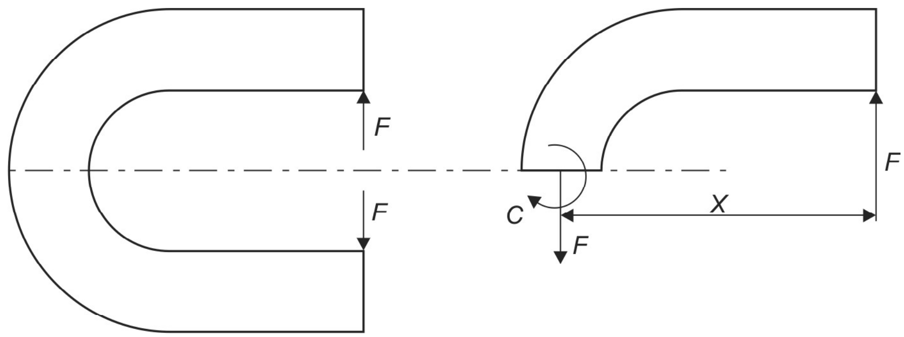

Fig. 1.4: Eccentric load

1.4 STEPS TO SOLVE CURVED BEAM PROBLEMS

1. Locate the position of C_g of cross-section with respect to the innermost fiber.

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} \quad \text{or} \quad \bar{y} = \frac{\Sigma ay}{\Sigma a}$$

2. Replace the eccentric load by an equal and parallel force through C_g of the cross-section together with a couple in opposite direction.
3. Evaluate the direct stresses (σ_D) produced.
4. Evaluate the extreme fiber bending stress due to couple.
5. Evaluate the resultant stresses in the extreme fibers as

   Resultant stress in the inner most fiber, $\sigma_i)_r = \sigma_D + \sigma_i$

   Resultant stress in the outer most fiber, $\sigma_o)_r = \sigma_D + \sigma_o$

Note: a. Step 2 may be skipped, if not required.

b. While solving problems, *ln* (natural log) is used in place of log$_e$.

Problems of Type I: To Find Stress

1. Determine the maximum stress induced in a punch press as shown in Fig. 1.5(a).

Solution: $F = 120 \times 10^3$ N, $\sigma_i)_r$, $\sigma_o)_r$ = ?

a. To find $\bar{x}$:

$a_1 = 30 \times 10 = 300$ mm^2, $\quad x_1 = 10/2 = 5$ mm,

$a_2 = 8 \times 20 = 160$ mm^2, $\quad x_2 = (20/2) + 10 = 20$ mm,

$a_3 = 16 \times 4 = 64$ mm^2, $\quad x_3 = (4/2) + 30 = 32$ mm,

$A = \Sigma a = a_1 + a_2 + a_3 = 524$ mm^2

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{(300 \times 5) + (160 \times 20) + (64 \times 32)}{(300 + 160 + 64)} = 12.87 \text{ mm}$$

b. Replace the given eccentric force by an equal and parallel force through C_g along with a couple as shown in **Fig. 1.5(b).**

c. Direct stress $\sigma_D = \frac{F}{A} = \frac{120 \times 10^3}{524} = 229$ N/mm^2 or MPa ... **1.1(a)/Pg 2, DHB**

d. Bending stresses (σ_i, σ_o):

- Comparing the given cross-section with **Fig. 10 – Tb 10.1/Pg 164, DHB,** we have $B = 30$ mm, $b_1 = 16$ mm, $d = 10$ mm, $d_1 = 4$ mm, $b_2 = 8$ mm, $H = 10 + 20 + 4 = 34$ mm

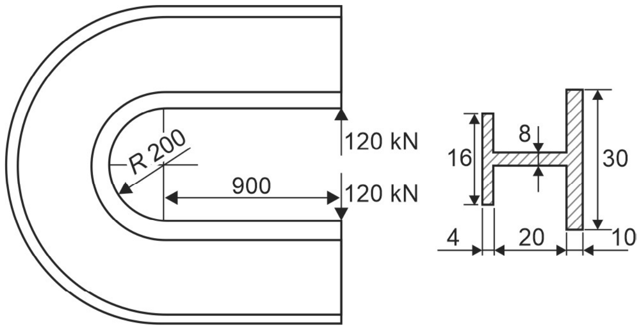

Fig. 1.5(a): Problem 1

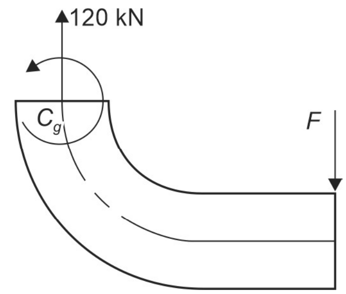

Fig. 1.5(b): Problem 1

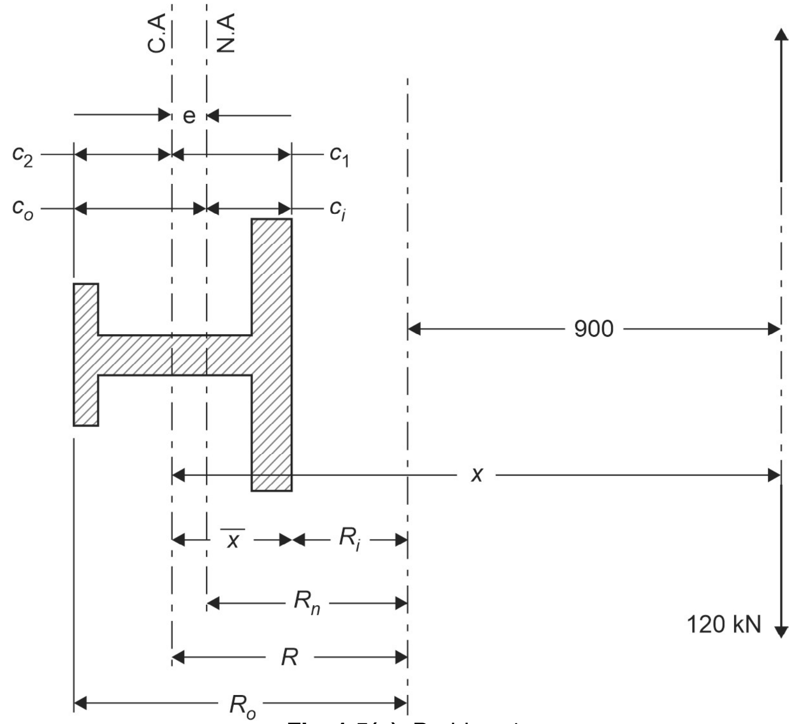

Fig. 1.5(c): Problem 1

- From **Fig. 1.5(c)**, we have

  $c_1 = \bar{x} = 12.87$ mm, $\quad R_i = 200$ mm

  $c_2 = H - c_1 = 34 - 12.87 = 21.13$ mm

  $R = c_1 + R_i = 12.87 + 200 = 212.87$ mm

  $R_o = R_i + H = 200 + 34 = 234$ mm

- Moment, $M = F \cdot x = F(900 + R_i + \bar{x})$

  $= 120 \times 10^3 (900 + 200 + 12.87)$

  $M = 133.5 \times 10^6$ N-mm

- $e = R - R_n$ ... **Fig. 10 – Tb 10.1/Pg 164, DHB**

$$e = R - \frac{A}{B\ln\left(\frac{R+d-c_1}{R-c_1}\right) + b_2\ln\left(\frac{R+c_2-d_1}{R+d-c_1}\right) + b_1\ln\left(\frac{R+c_2}{R+c_2-d_1}\right)}$$

$$= 212.87 - \frac{524}{\left[30\ln\left(\frac{212.87+10-12.87}{212.87-12.87}\right)\right] + \left[8\ln\left(\frac{212.87+21.13-4}{212.87+10-12.87}\right)\right] + \left[16\ln\left(\frac{212.87+21.13}{212.87+21.13-4}\right)\right]}$$

$e = 0.496$ mm

Now $c_i = c_1 - e = 12.87 - 0.496 = 12.37$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 21.13 + 0.496 = 21.63$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(133.5\times10^6)\times12.37}{524\times0.496\times200} = 31.77 \text{ kN/mm}^2$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(133.5\times10^6)\times21.63}{524\times0.496\times234} = -47.48 \text{ kN/mm}^2$$ ... **10.1(c)/Pg 159, DHB**

e. Resultant stresses

- Resultant stress in the inner most fiber

$$\sigma_i)_r = \sigma_D + \sigma_i = 229 + (31.17 \times 10^3) = 31.40 \text{ kN/mm}^2$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = 229 - (47.48 \times 10^3) = -47.25 \text{ kN/mm}^2$$

The resultant stresses are plotted in **Fig. 1.5(d)**.

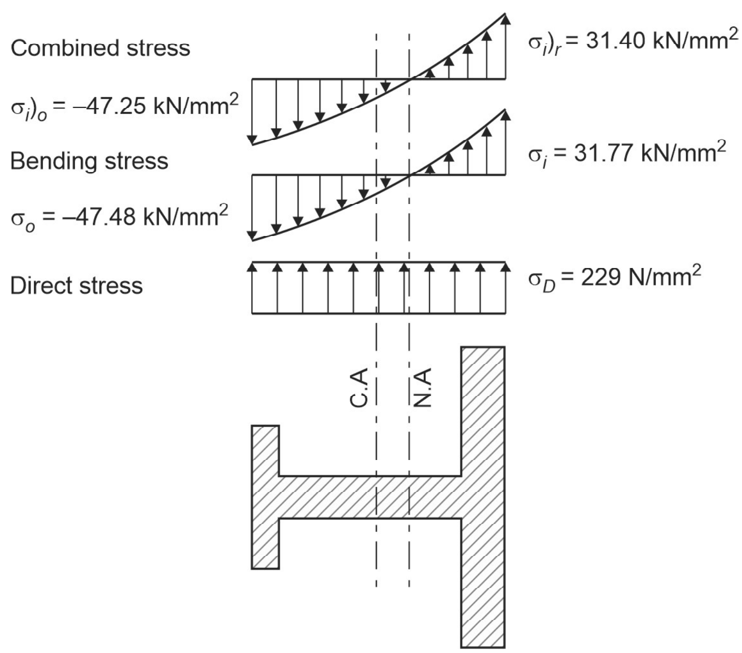

Fig. 1.5(d): Problem 1

2. Fig. 1.6(a) shows a frame of punching machine and its various dimensions. Determine the combined stress at the inner and outer fibers. Also find the maximum shear stress.

VTU – June/July 2013 – 15 Marks [Similar: July 2006 – 16 Marks]

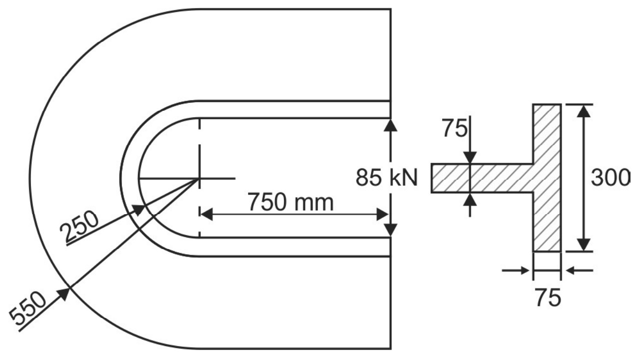

Fig. 1.6(a): Problem 2

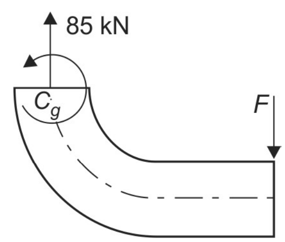

Fig. 1.6(b): Problem 2

Solution: $F = 85 \times 10^3$ N, $R_i = 250$ mm, $R_o = 550$ mm, $\sigma_i)_r$, $\sigma_o)_r = ?$

a. To find $\bar{x}$:

$a_1 = 300 \times 75 = 22500 \text{ mm}^2$ $\quad a_2 = 75 \times [550 - (250 + 75)] = 75 \times 225 = 16875 \text{ mm}^2$

$x_1 = 75/2 = 37.5$ mm $\quad x_2 = (225/2) + 75 = 187.5$ mm

$A = \Sigma a = a_1 + a_2 = 39375 \text{ mm}^2$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{(22500 \times 37.5) + (16875 \times 187.5)}{39375} = 101.79 \text{ mm}$$

b. Replace the given eccentric force by an equal and parallel force through C_g along with a couple as shown in **Fig. 1.6(b)**.

c. Direct stress: $\sigma_D = \dfrac{F}{A} = \dfrac{85 \times 10^3}{39375} = 2.16$ MPa ... **1.1(a)/Pg 2, DHB**

d. Bending stresses (σ_i, σ_o):

- Comparing the given cross-section with **Fig. 8 – Tb. 10.1/Pg 164, DHB**, we have

  $B = 300$ mm, $a = 75$ mm, $d = 75$ mm

- From **Fig. 1.6(c)**, we have

  $R_i = 250$ mm, $R_o = 550$ mm, $c_1 = \bar{x} = 101.79$ mm, $H = R_o - R_i = 550 - 250 = 300$ mm

  $R = c_1 + R_i = 101.79 + 250 = 351.79$ mm, $\quad c_2 = H - c_1 = 300 - 101.79 = 198.21$ mm

- Moment $\quad M = Fx = F(750 + R_i + \bar{x})$

  $= 85 \times 10^3 \times (750 + 250 + 101.79)$

  $M = 93.65 \times 10^6$ N-mm

- $e = R - R_n$ ... **Fig. 8 – Tb. 10.1/Pg 164, DHB**

$$e = R - \frac{A}{B \ln\left(\dfrac{R + d - c_1}{R - c_1}\right) + a \ln\left(\dfrac{R + c_2}{R + d - c_1}\right)}$$

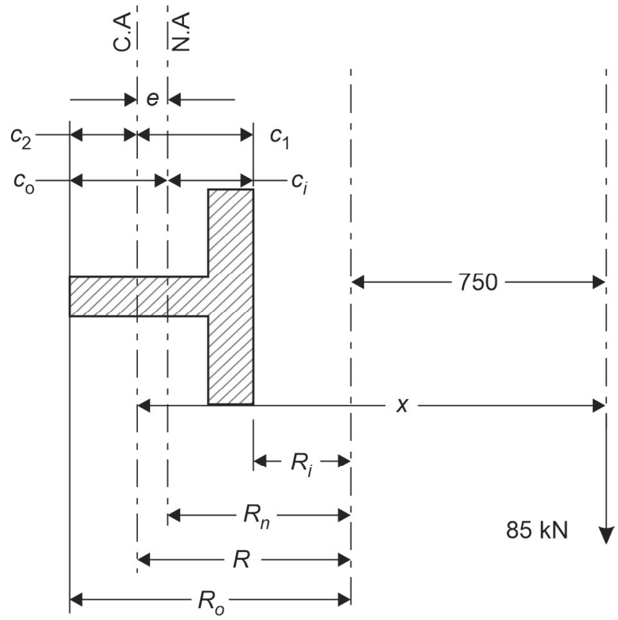

Fig. 1.6(c): Problem 2

$$= 351.79 - \frac{39375}{\left[300 \ln\left(\frac{351.79 + 75 - 101.79}{351.79 - 101.79}\right)\right] + \left[75 \ln\left(\frac{351.79 + 198.21}{351.79 + 75 - 101.79}\right)\right]}$$

$e = 18.57$ mm

Now $c_i = c_1 - e = 101.79 - 18.57 = 83.22$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 198.21 + 18.57 = 216.78$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(93.65 \times 10^6) \times 83.22}{39375 \times 18.57 \times 250} = 42.63 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(93.65 \times 10^6) \times 216.78}{39375 \times 18.57 \times 550} = -50.48 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

e. Resultant stresses:

- Resultant stress in the inner most fiber

$$\sigma_i)_r = \sigma_D + \sigma_i = 2.16 + 42.63 = 44.79 \text{ MPa}$$

- Resultant stress in the outer most fiber

$$\sigma_o)_r = \sigma_D + \sigma_o = 2.16 - 50.48 = -48.32 \text{ MPa}$$

- Maximum shear stress

$$\tau_{max} = \frac{\sigma_{max}}{2} = \frac{44.79}{2} = 22.40 \text{ MPa}$$

The resultant stresses are plotted in **Fig. 1.6(d)**.

3. A crane hook has a trapezoidal cross-section as:

Inside width = 87.5 mm, outside width = 25 mm, depth = 112.5 mm

The line of action of load passes through the center of curvature. The radius of curvature of inner side = 62.5 mm. Calculate the maximum stresses developed under a load of 90 kN. Also draw the stress distribution.

VTU– June /July 2008 – 16 Marks: [Similar: Jan/Feb. 2003 – 15 Marks]

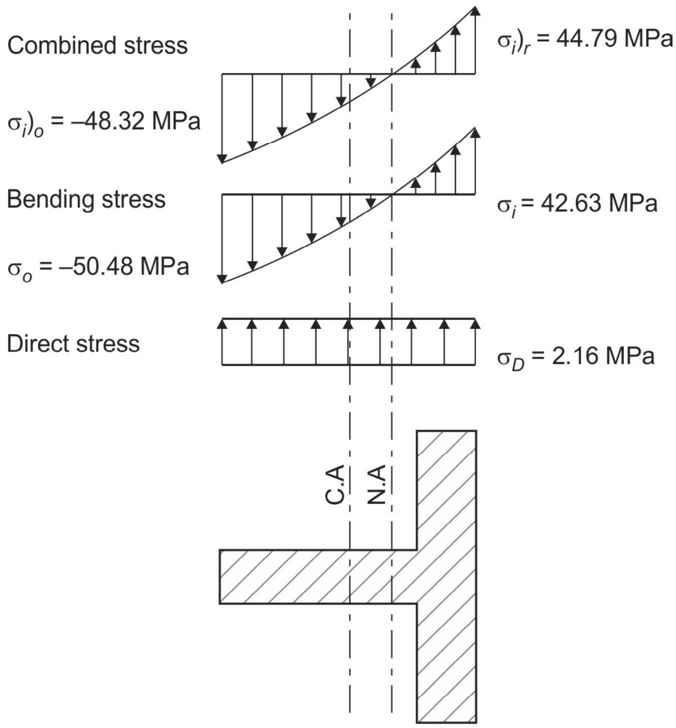

Fig. 1.6(d): Problem 2

Solution: $F = 90 \times 10^3$ N$\cdot\sigma_i)_r$, $\sigma_o)_r$ = ?

Based on given data, the crane hook is shown in **Fig. 1.7(a).**

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. f–Tb. 1.3(a)/Pg 13, DHB**, we have

$$b = 25 \text{ mm}, b_1 = 87.5 \text{ mm}, h = 112.5 \text{ mm}$$

$$\therefore \qquad b_o = b_1 - b = 87.5 - 25 = 62.5 \text{ mm}$$

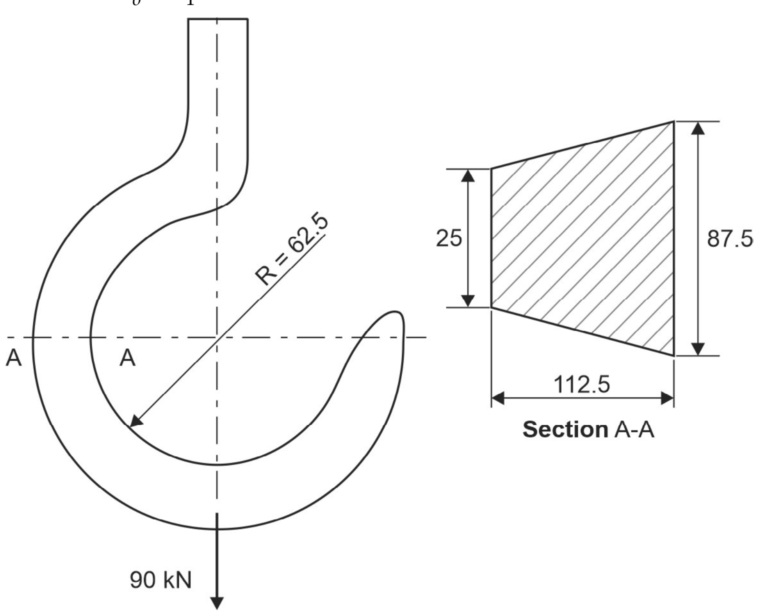

Fig. 1.7(a): Problem 3

$$c = \frac{(3b+2b_o)h}{3(2b+b_o)}$$

$$= \frac{[(3\times 25)+(2\times 62.5)]\times 112.5}{3[(2\times 25+62.5]} = 66.67 \text{ mm (from outer fiber)}$$

... Fig f – Tb 1.3(a)/Pg 13, DHB

$$\bar{x} = h - c = 112.5 - 66.67 = 45.83 \text{ mm (from inner fiber)}$$

b. Replace the given eccentric force by an equal and parallel force through C_g along with a couple (similar to **Fig. 1.6(b)**).

c. Direct stress, $\sigma_D = \dfrac{F}{A} = \dfrac{90\times 10^3}{6328.13} = 14.22 \text{ MPa}$ **... 1.1(a)/Pg 2, DHB**

where $A = \dfrac{h(b_1+b)}{2} = \dfrac{112.5\times(87.5+25)}{2} = 6328.13 \text{ mm}^2$

d. Bending stresses (σ_i, σ_o)

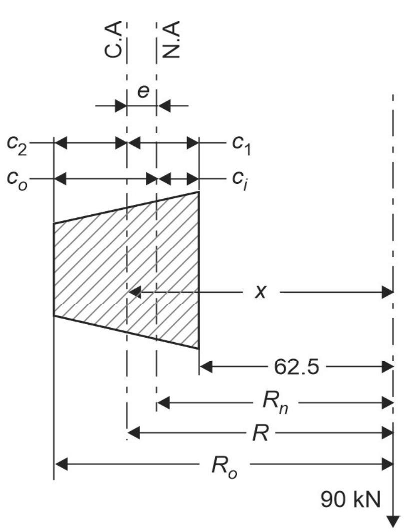

Fig. 1.7(b): Problem 3

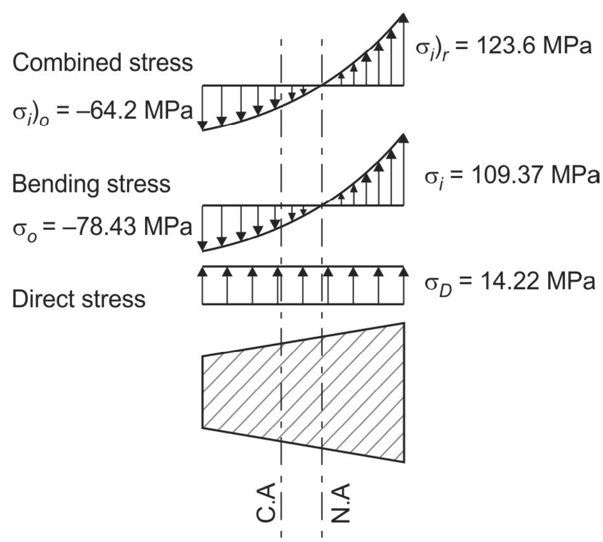

Fig. 1.7(c): Problem 3

- From **Fig. 1.7(b)**, we have

$$c_1 = \bar{x} = 45.83 \text{ mm}$$

$$c_2 = h - c_1 = 112.5 - 45.83 = 66.67 \text{ mm} = c$$

$$R_i = 62.5 \text{ mm}$$

$$R = R_i + c_1 = 62.5 + 45.83 = 108.33 \text{ mm}$$

$$R_o = R_i + h = 62.5 + 112.5 = 175 \text{ mm}$$

- Moment, $M = F\cdot x = F(R_i + \bar{x})$

$$= 90\times 10^3 \times [62.5 + 45.83]$$

$$M = 9.75\times 10^6 \text{ N-mm}$$

- $e = R - R_n$ ... **Fig. 7 – Tb 10.1/Pg 163, DHB**

$$= R - \frac{A}{\left[\left(\frac{b_1(R+c_2) - b(R-c_1)}{h}\right)\ln\left(\frac{R+c_2}{R-c_1}\right)\right] - (b_1 - b)}$$

$$= 108.33 - \frac{6328.13}{\left[\left(\frac{87.5(108.33+66.67) - 25(108.33-45.83)}{112.5}\right)\ln\left(\frac{108.33+66.67}{108.33-45.83}\right)\right] - (87.5-25)}$$

$e = 8.43$ mm

Now,

$c_i = c_1 - e = 45.83 - 8.43 = 37.4$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 66.67 + 8.43 = 75.1$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(9.75\times10^6)\times37.4}{6328.13\times8.43\times62.5} = 109.37 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(9.75\times10^6)\times75.1}{6328.13\times8.43\times175} = -78.43 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

e. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_{i})_r = \sigma_D + \sigma_i = 14.22 + 109.37 = 123.6 \text{ MPa}$$

- Resultant stress in the outer most fiber,

$$\sigma_{o})_r = \sigma_D + \sigma_o = 14.22 - 78.43 = -64.21 \text{ MPa}$$

Thus the inner most fiber is subjected to a maximum bending stress of 123.6 MPa
The resultant stresses are plotted as shown in **Fig. 1.7(c).**

4. If the cross-section in problem 3 is a rectangle as shown in Fig. 1.8(a), with the load being 20 kN, find the resultant stresses. The radius of curvature on inner side is 50 mm and that on the outer side is 150 mm (Fig. 1.8).

VTU – [Similar: June/July 2018 – 10 Marks]

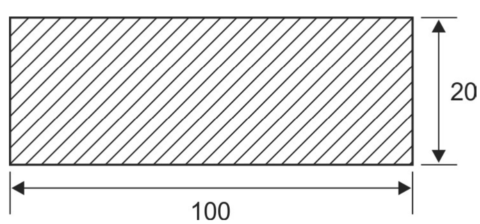

Fig. 1.8(a): Problem 4

Solution: $F = 20 \times 10^3$ N, $R_i = 50$ mm, $R_o = 150$ mm. $\sigma_{i})_r$, $\sigma_{o})_r$ = ?

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. 1 – Tb 10.1/Pg 162, DHB**, we have

$b = 20$ mm, $h = 100$ mm

$$\bar{x} = \frac{h}{2} = \frac{100}{2} = 50 \text{ mm}$$

b. Direct stress: $\sigma_D = \frac{F}{A} = \frac{F}{bh} = \frac{20\times10^3}{20\times100} = 10$ MPa ... **1.1(a)/Pg 2, DHB**

c. Bending stresses (σ_i, σ_o):

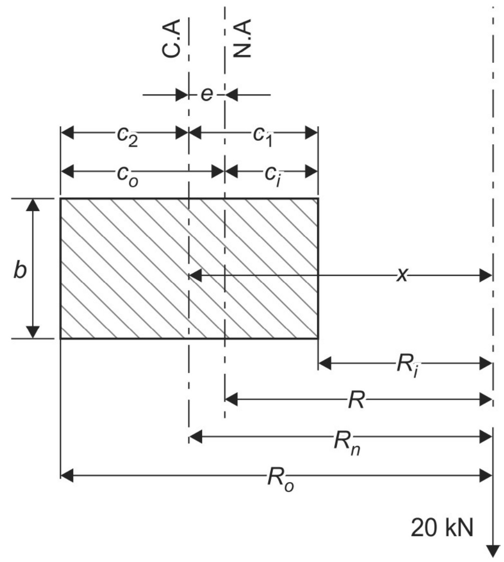

Fig. 1.8(b): Problem 4

- From **Fig. 1.8(b)**, we have

$$c_1 = \bar{x} = 50 \text{ mm}$$
$$c_2 = h - c_1 = 100 - 50 = 50 \text{ mm} \Rightarrow c_1 = c_2 = c = 50 \text{ mm}$$
$$R_i = 50 \text{ mm}$$
$$R = R_i + c_1 = 50 + 50 = 100 \text{ mm}$$
$$R_o = R_i + h = 50 + 100 = 150 \text{ mm}$$

- Moment, $M = F \cdot x = F(R_i + \bar{x}) = 20 \times 10^3 \times [50 + 50]$

$$M = 2 \times 10^6 \text{ N-mm}$$

- $e = R - R_n$ ... **Fig. 1 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{h}{\ln\left(\dfrac{R+c}{R-c}\right)} = 100 - \frac{100}{\ln\left(\dfrac{100+50}{100-50}\right)}$$

$e = 8.98$ mm

Now $c_o = c_2 + e = 50 + 8.98 = 58.98$ mm ... **10.1(d)/Pg 159, DHB**

$c_i = c_1 - e = 50 - 8.98 = 41.02$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(2 \times 10^6) \times 41.02}{(20 \times 100) \times 8.98 \times 50} = 91.36 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(2 \times 10^6) \times 58.98}{(20 \times 100) \times 8.98 \times 150} = -43.78 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = 10 + 91.36 = 101.36 \text{ MPa}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = 10 - 43.78 = -33.78 \text{ MPa}$$

The resultant stresses are plotted as shown in **Fig. 1.8(c).**

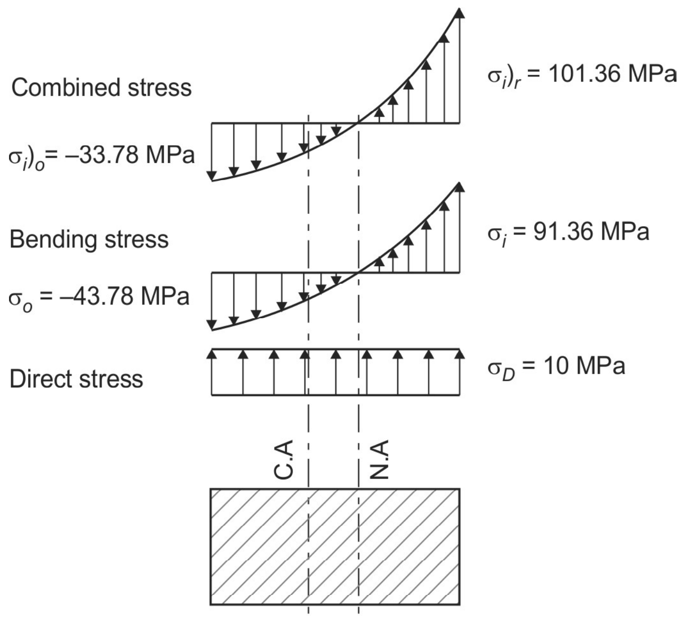

Fig. 1.8(c): Problem 4

5. **A curved beam of rectangular cross-section of width 20 mm and depth 40 mm is subjected to a pure bending moment of 600 N-m. The mean radius of curvature is 50 mm. Determine the location of neutral axis, maximum and minimum stress, ratio of maximum to minimum stress.**

VTU – June/July 2015 – 10 Marks

Solution: $M = 600 \times 10^3$ N-mm, $R = 50$ mm. $\sigma_i)_r$, $\sigma_o)_r = ?$, $\sigma_{max}/\sigma_{min} = ?$

Based on given data, the cross-section is as shown in **Fig. 1.9**.

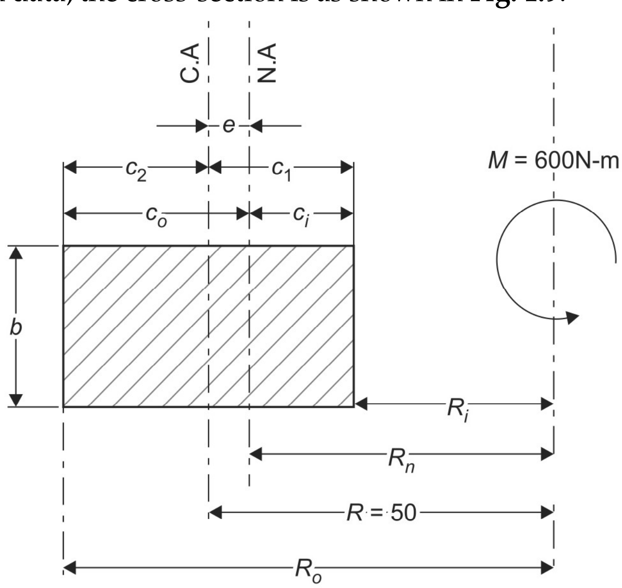

Fig. 1.9: Problem 5

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. 1 – Tb 10.1/Pg 162, DHB**, we have

$$b = 20 \text{ mm}, h = 40 \text{ mm}$$

$$\bar{x} = \frac{h}{2} = \frac{40}{2} = 20 \text{ mm}$$

b. Direct stress: In this case $\sigma_D = 0$.

c. Bending stresses (σ_i, σ_o):

- From **Fig. 1.9**, we have

$$c_1 = \bar{x} = 20 \text{ mm}$$

$$c_2 = h - c_1 = 40 - 20 = 20 \text{ mm}$$

$$R = R_i + c_1 \Rightarrow R_i = R - c_1 = 50 - 20 = 30 \text{ mm}$$

$$R_o = R_i + h = 30 + 40 = 70 \text{ mm}$$

- Moment, $M = 600 \times 10^3$ N-mm ... (data)
- $e = R - R_n$ ... **Fig. 1 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{h}{\ln\left(\frac{R+c}{R-c}\right)} = 50 - \frac{40}{\ln\left(\frac{50+20}{50-20}\right)}$$

$$e = 2.79 \text{ mm}$$

Now $c_i = c_1 - e = 20 - 2.79 = 17.21$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 20 + 2.79 = 22.79$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(600 \times 10^3) \times 17.21}{(20 \times 40) \times 2.79 \times 30} = 154.21 \text{ MPa}$$

... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(600 \times 10^3) \times 22.79}{(20 \times 40) \times 2.79 \times 70} = -87.52 \text{ MPa}$$

... **10.1(c)/Pg 159, DHB**

- Ratio $\frac{\sigma_{max}}{\sigma_{min}} = \left|\frac{154.21}{87.52}\right| = 1.76$

6. Compute the combined stresses at the inner and outer fibres in the critical section of a crane hook which is required to lift loads up to 25 kN. The hook has trapezoidal cross-section with parallel sides 60 mm and 30 mm, the distance between them being 90 mm. The inner radius of the hook is 100 mm. The load line is nearer to the inner surface of the hook by 25 mm than the center of curvature at the critical section. What will be the stresses at the inner and outer fibre, if the beam is treated as straight beam for the given load?

VTU – June/July 2017 – 16 Marks, Dec. 2012 – 16 Marks, Dec. 06/Jan. 07 – 12 Marks, [Similar: Dec. 2010 – 10 Marks]

Solution: $F = 25 \times 10^3$ N, $\sigma_i)_r$, $\sigma_o)_r$ = ? for case 1: curved beam; case 2: straight beam. Based on given data, the crane hook is shown in **Fig. 1.10(a).**

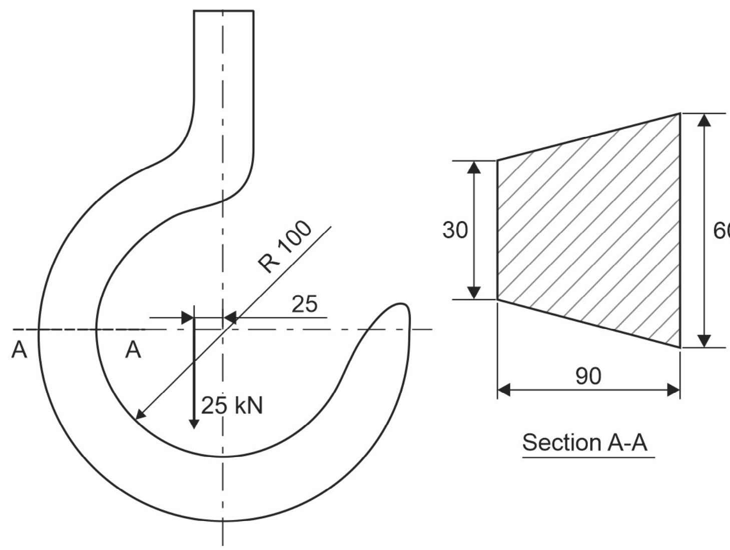

Fig. 1.10(a): Problem 6

Case 1: ***Curved beam:***

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. f – Tb 1.3(a)/Pg 13, DHB,** we have

$$b = 30 \text{ mm}, b_1 = 60 \text{ mm}, h = 90 \text{ mm}$$

$$\therefore \quad b_o = b_1 - b = 60 - 30 = 30 \text{ mm}$$

$$c = \frac{(3b + 2b_o)h}{3(2b + b_o)}$$

$$= \frac{[(3 \times 30) + (2 \times 30)] \times 90}{3 \times [(2 \times 30) + 30]} = 50 \text{ mm (from outer fiber)}$$

... **Fig. f – Tb 1.3(a)/Pg 13, DHB**

$$\bar{x} = h - c = 90 - 50 = 40 \text{ mm (from inner fiber)}$$

b. Direct stress, $\sigma_D = \dfrac{F}{A} = \dfrac{25 \times 10^3}{4050} = 6.17 \text{ MPa}$ ... **1.1(a)/Pg 2, DHB**

where $A = \dfrac{h}{2}(b_1 + b) = \dfrac{90 \times (60 + 30)}{2} = 4050 \text{ mm}^2$

c. Bending stresses (σ_i, σ_o)

- From **Fig. 1.10(b)**, we have

$$c_1 = \bar{x} = 40 \text{ mm}$$

$$R_i = 100 \text{ mm}$$

$$c_2 = h - c_1 = 90 - 40 = 50 \text{ mm} = c$$

$$R = R_i + c_1 = 100 + 40 = 140 \text{ mm}$$

$$R_o = R_i + h = 100 + 90 = 190 \text{ mm}$$

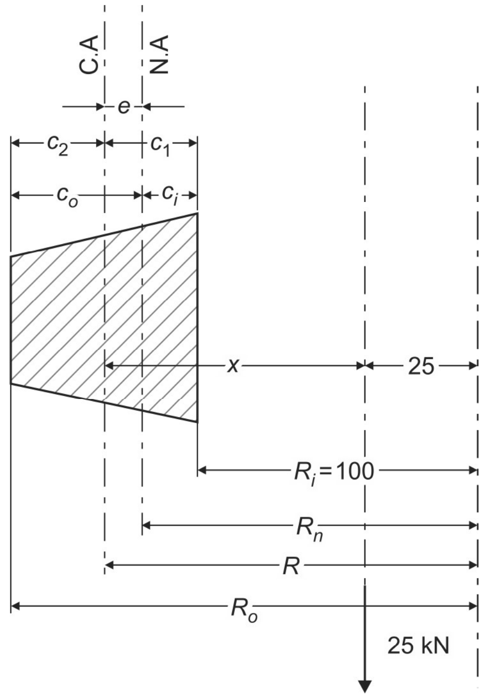

Fig. 1.10(b): Problem 6

- Moment, $M = F{\cdot}x = F[(R_i - \text{offset}) + \bar{x}]$

$$= 25 \times 10^3 \times [(100 - 25) + 40]$$

$$M = 2.88 \times 10^6 \text{ N-mm}$$

- $e = R - R_n$ ... **Fig. 7 – Tb 10.1/Pg 163, DHB**

$$= R - \frac{A}{\left[\left(\frac{b_1(R+c_2) - b(R-c_1)}{h}\right)\ln\left(\frac{R+c_2}{R-c_1}\right)\right] - (b_1 - b)}$$

$$= 140 - \frac{4050}{\left[\left(\frac{60(140+50) - 30(140-40)}{90}\right)\times \ln\left(\frac{140+50}{140-40}\right)\right] - (60-30)}$$

$e = 4.58$ mm

Now, $c_i = c_1 - e = 40 - 4.58 = 35.42$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 50 + 4.58 = 54.58$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(2.88\times 10^6)\times 35.42}{4050\times 4.58\times 100} = 55 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(2.88\times 10^6)\times 54.58}{4050\times 4.58\times 190} = -44.6 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = 6.17 + 55 = 61.17 \text{ MPa}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = 6.17 - 44.6 = -38.43 \text{ MPa}$$

The resultant stresses are plotted as shown in **Fig. 1.10(c)**.

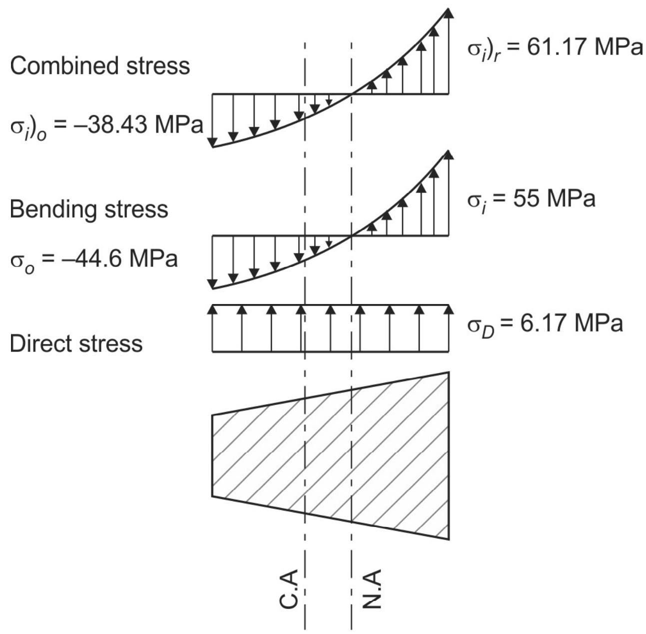

Fig. 1.10(c): Problem 6: For a curved beam

Case 2: ***Straight beam:***

- We know that,

$$\frac{M}{I} = \frac{\sigma}{c} = \frac{E}{R}$$

$$\Rightarrow \qquad \sigma = \frac{Mc}{I} \qquad \text{... 1.1(b)/Pg 2, DHB}$$

But

$$I = \frac{\left(6b^2 + 6bb_o + b_o^2\right)h^3}{36\left(2b + b_o\right)} \qquad \text{... Fig. f – Tb 1.3(a)/Pg 13, DHB}$$

$$= \frac{\left(6 \times 30^2 + 6 \times 30 \times 30 + 30^2\right)90^3}{36\left(2 \times 30 + 30\right)}$$

$$I = 2632500 \text{ mm}^4$$

Also, $c = \bar{x} = 40$ mm, $c_2 = h - c_1 = 90 - 40 = 50$ mm ... (*from case 1*)

- Bending stress at inner fiber $\sigma_i = \dfrac{Mc_1}{I} = \dfrac{\left(2.88 \times 10^6\right) \times 40}{2632500} = 43.76$ MPa
- Bending stress at outer fiber $\sigma_o = \dfrac{-Mc_2}{I} = \dfrac{-\left(2.88 \times 10^6\right) \times 50}{2632500} = -54.7$ MPa

a. Resultant stresses

- Resultant stress in the inner most fiber

$$\sigma_i)_r = \sigma_D + \sigma_i = 6.17 + 43.76 = 49.93 \text{ MPa}$$

- Resultant stress in the outer most fiber

$$\sigma_o)_r = \sigma_D + \sigma_o = 6.17 - 54.7 = -48.53 \text{ MPa}$$

The resultant stresses are plotted as shown in **Fig. 1.10(d).**

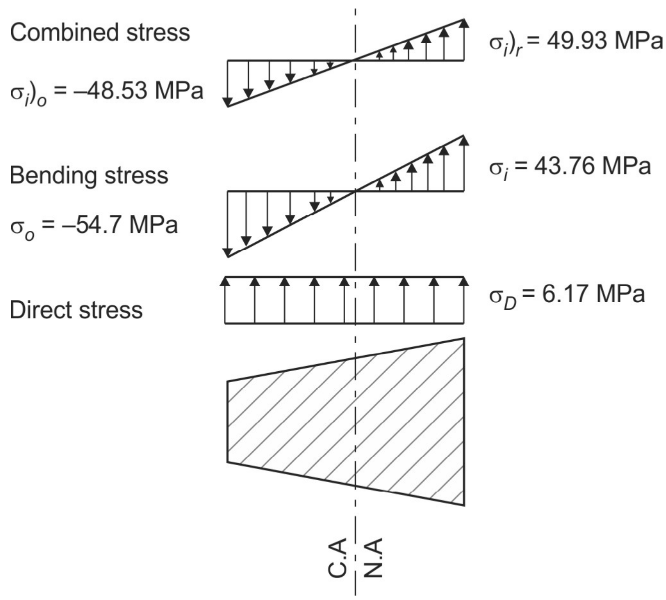

Fig. 1.10(d): Problem 6: For a straight beam

7. Determined the combined stresses in the inner and outer fibers at the critical section in a crane hook which is required to lift loads up to 50 kN. The hook has trapezoidal section with inner and outer sides of 90 mm and 40 mm respectively, depth is 120 mm. The center of curvature of the section is at a distance of 100 mm from the inner side of the section and the load line passes through the center of curvature. Also determine the factor of safety according to maximum shear stress theory, if τ_{all} = 80 MPa.

VTU – Dec. 2014/Jan. 2015 – 12 Marks, May/June 2010 – 12 Marks,
[Similar: June 2012 – 15 Marks; Dec. 2008/Jan. 2009 – 12 Marks;
Dec. 2007/Jan. 2008 – 12 Marks

Solution: $F = 50 \times 10^3$ N, $\sigma_i)_r$, $\sigma_o)_r$ = ?

Based on given data, the crane hook is shown in **Fig. 1.11(a).**

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. f – Tb 1.3(a)/Pg 13, DHB,** we have

$$b = 40 \text{ mm}, b_1 = 90 \text{ mm}, h = 120 \text{ mm}$$

$$\therefore \quad b_o = b_1 - b = 90 - 40 = 50 \text{ mm}$$

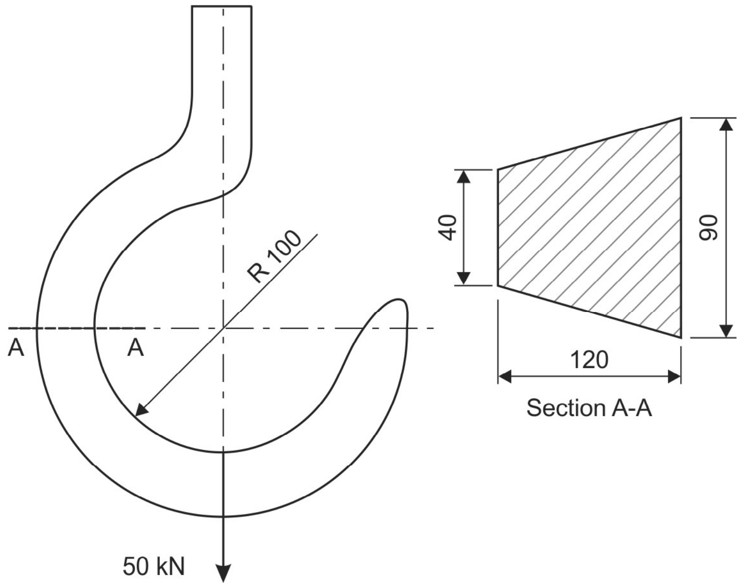

Fig. 1.11(a): Problem 7

$$c = \frac{(3b + 2b_o)h}{3(2b + b_o)}$$

$$= \frac{[(3 \times 40) + (2 \times 50)] \times 120}{3 \times [(2 \times 40) + 50]} = 67.60 \text{ mm}$$ ... **Fig. f – Tb 1.3(a)/Pg 13, DHB**

$$\bar{x} = h - c = 120 - 67.70 = 52.30 \text{ mm}$$

b. Direct stress, $\sigma_D = \frac{F}{A} = \frac{50 \times 10^3}{7800} = 6.41 \text{ MPa}$ ... **1.1(a)/Pg 2, DHB**

where $A = \frac{h}{2}(b_1 + b) = \frac{120 \times (90 + 40)}{2} = 7800 \text{ mm}^2$

c. Bending stresses (σ_i, σ_o)

- From **Fig. 1.11(b)**, we have

  $c_1 = \bar{x} = 52.30$ mm $R_i = 100$ mm $c_2 = h - c_1 = 120 - 52.30 = 67.70$ mm $= c$

  $R = R_i + c_1 = 100 + 52.30 = 152.30$ mm $R_o = R_i + h = 100 + 120 = 220$ mm

- Moment, $M = F \cdot x = F(R_i + \bar{x})$

  $= 50 \times 10^3 \times (100 + 52.30)$

  $= 7.62 \times 10^6$ N-mm

- $e = R - R_n$ ... **Fig. 7 – Tb 10.1/Pg 163, DHB**

$$= R - \frac{A}{\left[\left(\frac{b_1(R + c_2) - b(R - c_1)}{h}\right)\ln\left(\frac{R + c_2}{R - c_1}\right)\right] - (b_1 - b)}$$

$$= 152.30 - \frac{7800}{\left[\left(\frac{90 \times (152.30 + 67.70) - 40 \times (152.30 - 52.30)}{120}\right) \times \ln\left(\frac{152.30 + 67.70}{152.30 - 52.30}\right)\right] - (90 - 40)}$$

$e = 7.36$ mm

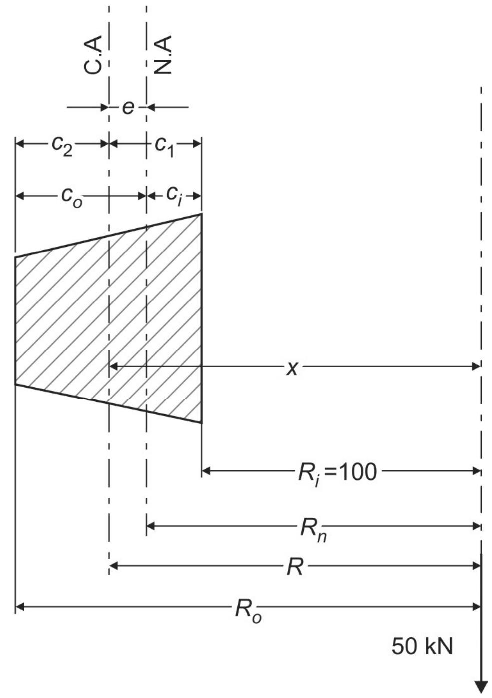

Fig. 1.11(b): Problem 7

Now, $c_i = c_1 - e = 52.30 - 7.36 = 44.94$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 67.70 + 7.36 = 75.06$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(7.62\times 10^6)\times 44.94}{7800\times 7.36\times 100} = 59.65 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber

$$s_o = \frac{-Mc_o}{AeR_o} = \frac{-(7.62\times 10^6)\times 75.06}{7800\times 7.36\times 220} = -45.29 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber

$$\sigma_i)_r = \sigma_D + \sigma_i = 6.41 + 59.65 = 66.06 \text{ MPa}$$

- Resultant stress in the outer most fiber

$$\sigma_o)_r = \sigma_D + \sigma_o = 6.41 - 45.29 = -38.88 \text{ MPa}$$

The resultant stresses are plotted as shown in **Fig. 1.11(b)**.

e. Factor of safety

$$\frac{\sigma_e}{n} = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

8. The frame of a punch press is shown in Fig 1.12. Find the stresses at the inner and outer fibers at section X-X of the frame, if *W* = 5000 N.

VTU – May/June 2010 – 10 Marks

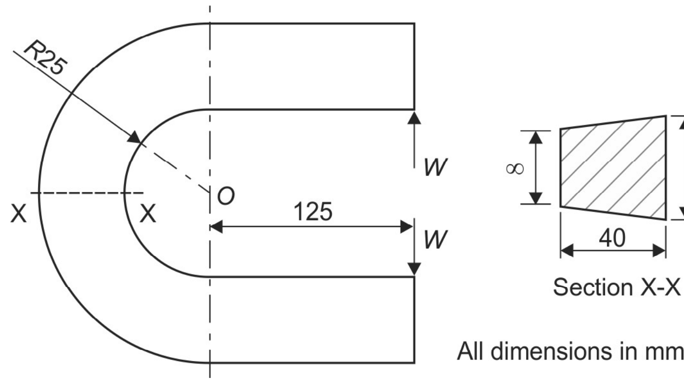

Fig. 1.12: Problem 8

Solution: $F = 5000$ N, $\sigma_i)_r$, $\sigma_o)_r$ = ?

Proceeding on similar line to problem 7, we have

- $c = 22.86$ mm
- $\bar{x} = h - c = 17.14$
- $\sigma_D = 8.93$ MPa
- $M = 8.36 \times 10^5$ N-mm
- $e = 2.92$ mm
- $c_i = 14.23$ mm, $c_o = 25.77$ mm
- Bending stress at inner fiber, $\sigma_i = 291.03$ MPa
- Bending stress at outer fiber, $\sigma_o = -202.82$ MPa
- Resultant stress in the inner most fiber, $\sigma_i)_r = 299.96$ MPa
- Resultant stress in the outer most fiber, $\sigma_o)_r = -193.89$ MPa

9. **The horizontal cross-section of a crane hook is an isosceles triangle of 120 mm deep, the inner width being 90 mm. The hook carries a load of 50 kN. Inner radius of curvature is 100 mm. The line of action of load passes through the center line of curvature. Determine the stress at the extreme fibres.**

VTU – June/July 09 – 12 Marks

Solution: $F = 50 \times 10^3$ N, $R_i = 100$ mm, $\sigma_i)_r$, $\sigma_o)_r$ = ?

a. To find $\bar{x}$: $\bar{x} = \frac{h}{3} = \frac{120}{3} = 40$ mm

b. Direct stress $\sigma_D = \frac{F}{A} = \frac{50 \times 10^3}{5400} = 9.26$ MPa ... **1.1(a)/Pg 2, DHB**

where $A = \frac{90 \times 120}{2} = 5400$ mm^2

c. Bending stresses (σ_i, σ_o)

- Comparing the given cross-section with **Fig. 6 – Tb 10.1/Pg 163, DHB**, we have

  $b = 90$ mm, $h = 120$ mm
- From **Fig. 1.13(a)**, we have

$$c_1 = \bar{x} = 40 \text{ mm}$$
$$R_i = 100 \text{ mm}$$
$$c_2 = h - c_1 = 120 - 40 = 80 \text{ mm}$$
$$R = R_i + c_1 = 100 + 40 = 140 \text{ mm}$$
$$R_o = R_i + h = 100 + 120 = 220 \text{ mm}$$

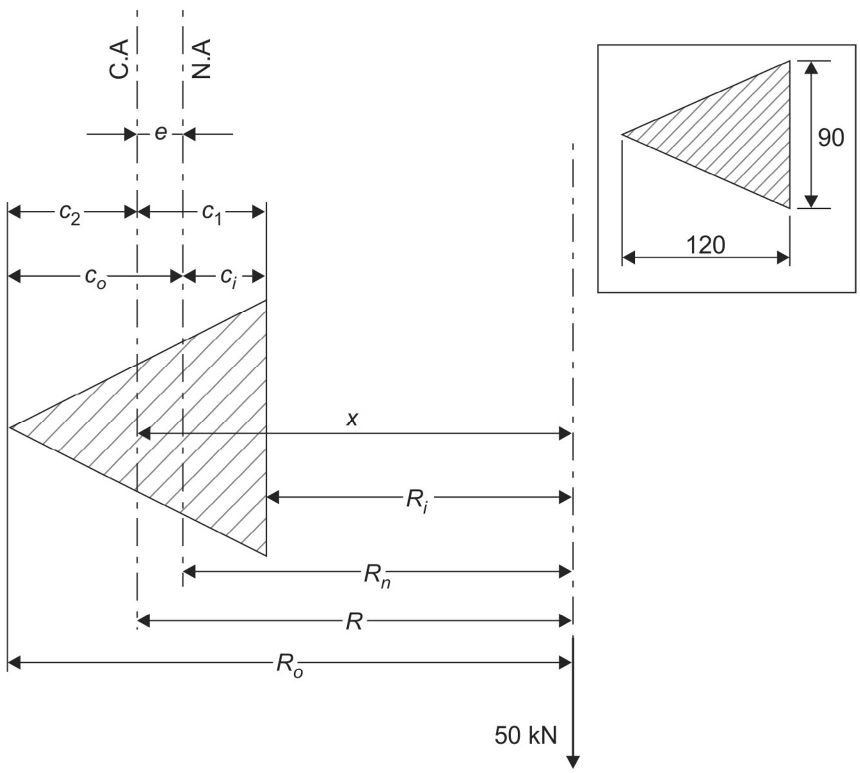

Fig. 1.13(a): Problem 9

- Moment, $M = F \cdot x = F(R_i + \bar{x})$

$$= 50 \times 10^3 \times (100 + 40)$$

$$M = 7 \times 10^6 \text{ N-mm}$$

- $e = R - R_n$ ... **Fig. 6 – Tb 10.1/Pg 163, DHB**

$$= R - \frac{h^2/2}{\left[(R+c_2)\ln\left(\frac{R+c_2}{R-c_1}\right)\right] - h} = 140 - \frac{\left(120^2/2\right)}{\left[(140+80)\times\ln\left(\frac{140+80}{140-40}\right)\right] - 120}$$

$e = 5.32$ mm

Now, $c_i = c_1 - e = 40 - 5.32 = 34.68$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 80 + 5.32 = 85.32$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{\left(7\times10^6\right)\times 34.68}{5400\times5.32\times100} = 84.5 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-\left(7\times10^6\right)\times 85.32}{5400\times5.32\times220} = -94.50 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = 9.26 + 84.5 = 93.76 \text{ MPa}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = 9.26 - 94.50 = -85.24 \text{ MPa}$$

- Maximum shear stress,

$$\tau_{max} = \frac{\sigma_{max}}{2} = \frac{93.76}{2} = 46.88 \text{ MPa}$$

The resultant stresses are plotted as shown in **Fig. 1.13(b)**

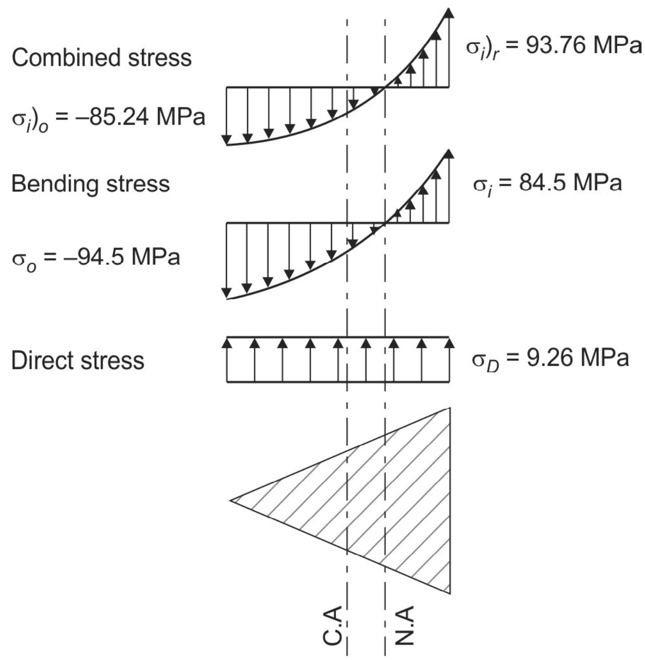

Fig. 1.13(b): Problem 9

10. Determine the maximum tensile stress and maximum shear stress of the component shown in Fig. 1.14(a) and indicate the location.

VTU – June/July 2014 – 10 Marks; Dec. 2013/Jan. 2014 – 10 Marks

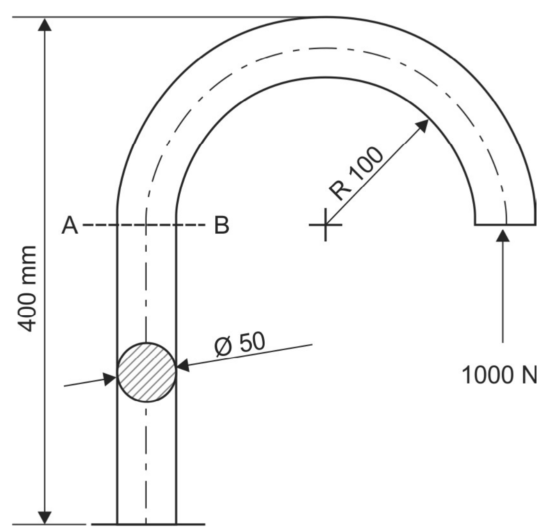

Fig. 1.14(a): Problem 10

Solution: $F = 1000$ N, $D = 50$ mm, $R_i = 100$ mm, $\sigma_i)_r$, $\sigma_o)_r = ?$, $\tau_{max} = ?$

a. To find $\bar{x}$: $\bar{x} = \frac{D}{2} = \frac{50}{2} = 25$ mm

b. Direct stress $\sigma_D = \frac{F}{A} = \frac{1000}{1963.50} = 0.51$ MPa ... **1.1(a)/Pg 2, DHB**

where $A = \frac{\pi D^2}{4} = \frac{\pi \times 50^2}{4} = 1963.50 \text{ mm}^2$

c. Bending stresses (σ_i, σ_o)

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB,** we have

$$D = 90 \text{ mm}, c_1 = c_2 = 25 \text{ mm} = \bar{x}$$

- From **Fig. 1.14(b)**, we have

$c_1 = c_2 = 25 \text{ mm} = \bar{x}$ $\quad R_i = 100$ mm

$R = R_i + c_1 = 100 + 25 = 125$ mm $\quad R_o = R_i + D = 100 + 50 = 150$ mm

- Moment, $M = F \cdot x = F[(D/2) + 2R_i + \bar{x}]$ or $M = F(D + 2R_i)$

$$= 1000 \times [(25 + (2 \times 100) + 25)]$$

$$M = 250 \times 10^3 \text{ N-mm}$$

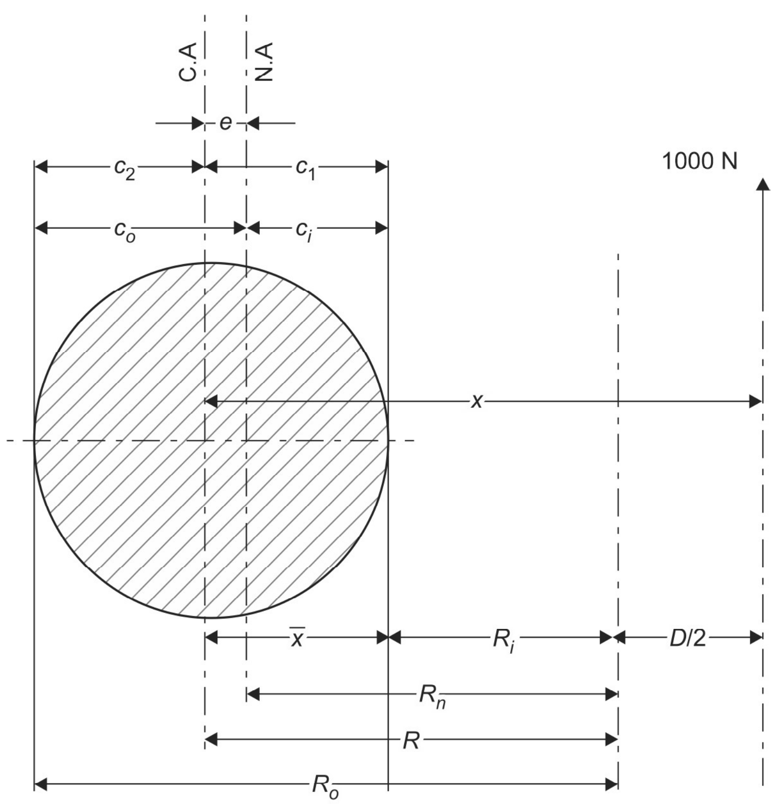

Fig. 1.14(a): Problem 10

- $$e = R - R_n$$ ... **Fig. 2 – Tb. 10.1/Pg 162, DHB**

$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 125 - \frac{25^2/2}{125 - \sqrt{125^2 - 25^2}}$$

$$e = 1.26 \text{ mm}$$

Now, $c_i = c_1 - e = 25 - 1.26 = 23.74$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 25 + 1.26 = 26.26$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(250 \times 10^3) \times 23.74}{1963.50 \times 1.26 \times 100} = 23.98 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(250 \times 10^3) \times 26.26}{1963.50 \times 1.26 \times 150} = -17.69 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber

$$\sigma_i)_r = \sigma_D + \sigma_i = 0.51 + 23.98 = 24.49 \text{ MPa}$$

- Resultant stress in the outer most fiber

$$\sigma_o)_r = \sigma_D + \sigma_o = 0.51 - 17.69 = -17.18 \text{ MPa}$$

- Maximum shear stress,

$$\tau_{max} = \frac{\sigma_{max}}{2} = \frac{24.49}{2} = 12.25 \text{ MPa}$$

The resultant stresses are plotted as shown in **Fig. 1.14(c).**

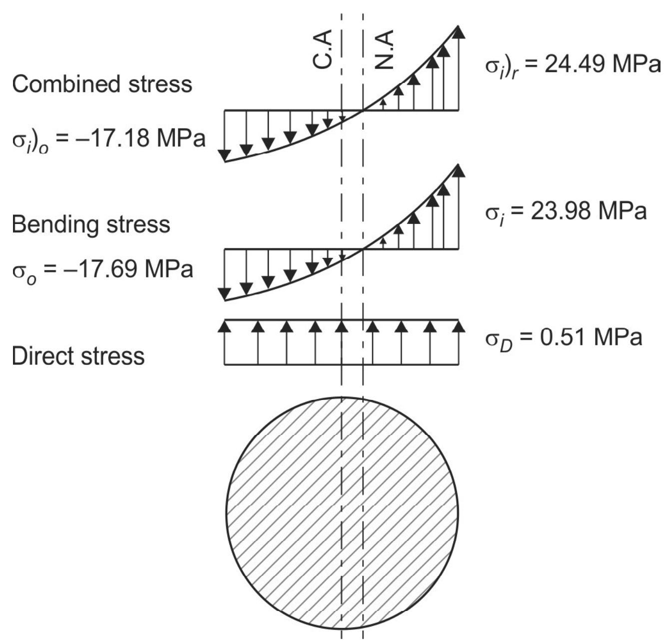

Fig. 1.14(c): Problem 10

11. Calculate the stresses at the points A and B for a circular beam as shown in Fig. 1.15(a). The circular beam is subjected to a compressive load of 6 kN.

[VTU – July 07 – 10 Marks]

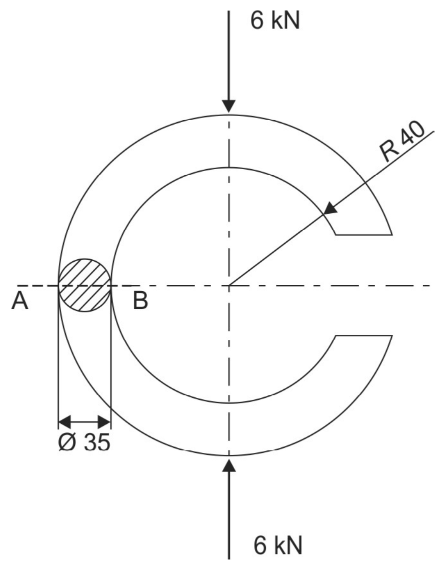

Fig. 1.15(a): Problem 10

Solution: $F = -6000$ N (compressive), $R_i = 40$ mm, $d = 35$ mm, $\sigma_i)_r$, $\sigma_o)_r = ?$

a. To find $\bar{x}$: $\bar{x} = \frac{D}{2} = \frac{35}{2} = 17.5$ mm

b. Direct stress, $\sigma_D = -\frac{F}{A} = \frac{-6\times 10^3}{962.11} = -6.24$ MPa (Compressive) ... **1.1(a)/Pg2, DHB**

where $A = \frac{\pi D^2}{4} = \frac{\pi \times 35^2}{4} = 962.11 \text{ mm}^2$

c. Bending stresses (σ_i, σ_o)

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB,** we have

$$D = 35 \text{ mm}, c_1 = c_2 = 17.5 \text{ mm} = \bar{x}$$

- From **Fig. 1.15(b)**, we have

$$c_1 = c_2 = 17.5 \text{ mm} = \bar{x} \qquad R_i = 40 \text{ mm}$$

$$R = R_i + c_1 = 40 + 17.5 = 57.5 \text{ mm}$$

$$R_o = R_i + D = 40 + 35 = 75 \text{ mm}$$

- Moment, $M = F \cdot x = F(R_i + \bar{x})$

$$= 6000 \times (40 + 17.5)$$

$$M = 345 \times 10^3 \text{ N-mm}$$

- $e = R - R_n$ ... **Fig. 2 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 57.5 - \frac{17.5^2/2}{57.5 - \sqrt{57.5^2 - 17.5^2}}$$

$$e = 1.36 \text{ mm}$$

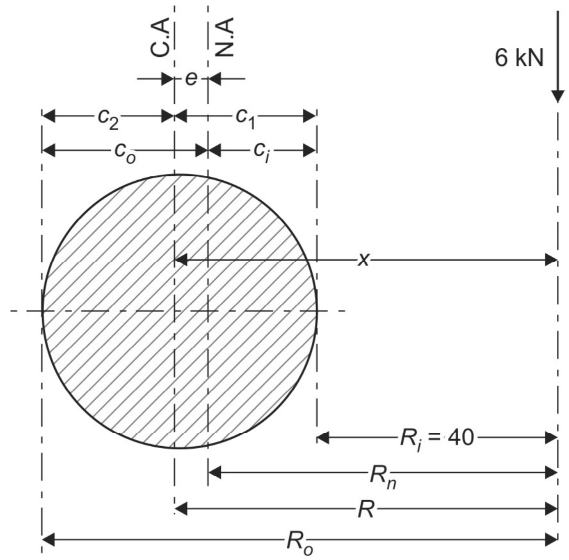

Fig. 1.15(b): Problem 11

Now, $c_i = c_1 - e = 17.5 - 1.36 = 16.14$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 17.5 + 1.36 = 18.86$ mm ... **10.1(d)/Pg 159, DHB**

Here inner fibers are in compression, while outer fibers are in tension.

- Bending stress at inner fiber,

$$\sigma_i = \frac{-Mc_i}{AeR_i} = \frac{-\left(345\times10^3\right)\times16.14}{962.11\times1.36\times40} = -106.38 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{Mc_o}{AeR_o} = \frac{\left(345\times10^3\right)\times18.86}{962.11\times1.36\times75} = 66.30 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = -6.24 - 106.38 = -112.62 \text{ MPa}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = -6.24 + 66.30 = 60.06 \text{ MPa}$$

12. An open S-link made from a rod of 25 mm is as shown in Fig. 1.16(a). Calculate the stresses at sections A – A and B – B.

Solution: $F = 900$ N, $D = 25$ mm, $\sigma_i)_r$, $\sigma_o)_r = ?$

Case 1: ***Section A – A:***

a. To find $\bar{x}$: $$\bar{x} = \frac{D}{2} = \frac{25}{2} = 12.5 \text{ mm}$$

b. Direct stress: $$\sigma_D = \frac{F}{A} = \frac{900}{490.87} = 1.83 \text{ MPa}$$ ... **1.1(a)/Pg 2, DHB**

where $$A = \frac{\pi D^2}{4} = \frac{\pi\times25^2}{4} = 490.87 \text{ mm}^2$$

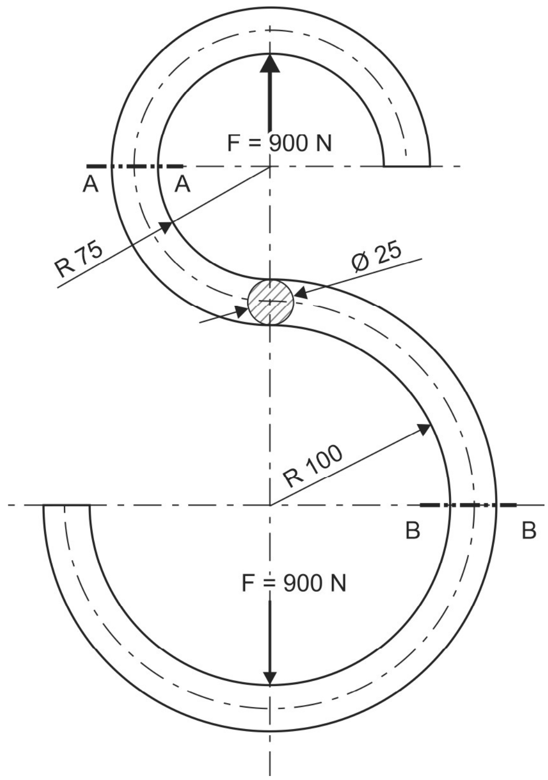

Fig. 1.16(a): Problem 12

c. Bending stresses (σ_i, σ_o):

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB,** we have

$$D = 25 \text{ mm}, c_1 = c_2 = 12.5 \text{ mm} = \bar{x}$$

- From **Fig. 1.16(b)**, we have

$$c_1 = c_2 = 12.5 \text{ mm} = \bar{x} \qquad R_i = 75 \text{ mm}$$

$$R = R_i + c_1 = 75 + 12.5 = 87.5 \text{ mm} \quad R_o = R_i + D = 75 + 25 = 100 \text{ mm}$$

- Moment, $M = F \cdot x = F(R_i + \bar{x})$

$$= 900 \times (75 + 12.5)$$

$$M = 78750 \text{ N-mm}$$

- $e = R - R_n$ ... **Fig 2 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 87.5 - \frac{12.5^2/2}{87.5 - \sqrt{87.5^2 - 12.5^2}}$$

$$e = 0.45 \text{ mm}$$

Now, $c_i = c_1 - e = 12.5 - 0.45 = 12.05$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 12.5 + 0.45 = 12.95$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(78750) \times 12.05}{490.87 \times 0.45 \times 75} = 57.28 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

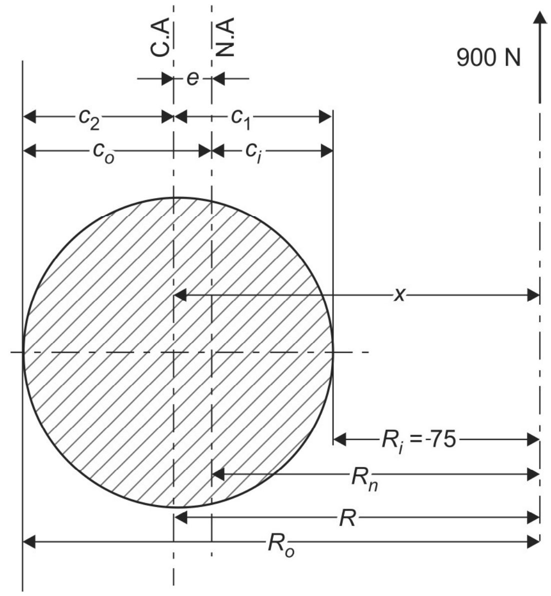

Fig. 1.16(b): Problem 12: Section A – A

- Bending stress at outer fiber

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(78750)\times 12.05}{490.87\times 0.45\times 100} = -46.16 \text{ MPa} \qquad \text{... \textbf{10.1(c)/Pg 159, DHB}}$$

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = 1.83 + 57.28 = 59.11 \text{ MPa}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = 1.83 - 46.16 = -44.33 \text{ MPa}$$

- Maximum shear stress,

$$\tau_{max} = \frac{\sigma_{max}}{2} = \frac{59.11}{2} = 29.56 \text{ MPa}$$

Case 2: ***Section B – B:***

e. Here $\bar{x} = 12.5$ mm and $\sigma_D = 1.83$ MPa (from Section A-A)

f. Bending stresses (σ_i, σ_o)

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB,** we have

$$D = 25 \text{ mm}, c_1 = c_2 = 12.5 \text{ mm} = \bar{x}$$

- From **Fig. 1.16(c)**, we have

$$c_1 = c_2 = 12.5 \text{ mm} = \bar{x} \qquad R_i = 100 \text{ mm}$$

$$R = R_i + c_1 = 100 + 12.5 = 112.5 \text{ mm}$$

$$R_o = R_i + D = 100 + 25 = 125 \text{ mm}$$

- Moment, $M = F\cdot x = F(R_1 + \bar{x})$

$$= 900 \times (100 + 12.5)$$

$$M = 101250 \text{ N-mm}$$

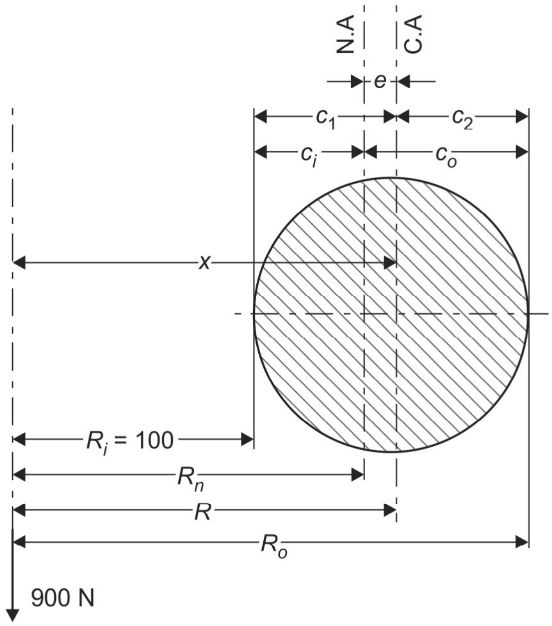

Fig. 1.16(c): Problem 12: Section B – B

- $e = R - R_n$ ... **Fig 2 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 112.5 - \frac{12.5^2/2}{112.5 - \sqrt{112.5^2 - 12.5^2}}$$

$$= 0.348 \text{ mm}$$

Now, $c_i = c_1 - e = 12.5 - 0.348 = 12.152$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 12.5 + 0.348 = 12.848$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(101250) \times 12.152}{490.87 \times 0.348 \times 100} = 72.03 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(101250) \times 12.848}{490.87 \times 0.348 \times 125} = -60.92 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

g. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = 1.83 + 72.03 = 73.86 \text{ MPa}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = 1.83 - 60.92 = -59.09 \text{ MPa}$$

- Maximum shear stress,

$$\tau_{max} = \frac{\sigma_{max}}{2} = \frac{73.86}{2} = 36.93 \text{ MPa}$$

13. A curved link mechanism made from a round steel bar is shown in Fig. 1.17(a). The material for the link is plain carbon steel 30C8 with an allowable yield strength of 400 MPa. Determine the factor of safety.

VTU – Dec. 2013/Jan. 2014 – 10 Marks

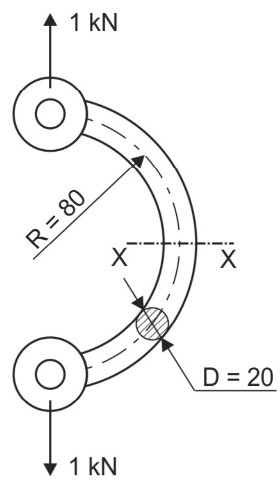

Fig. 1.17(a): Problem 13

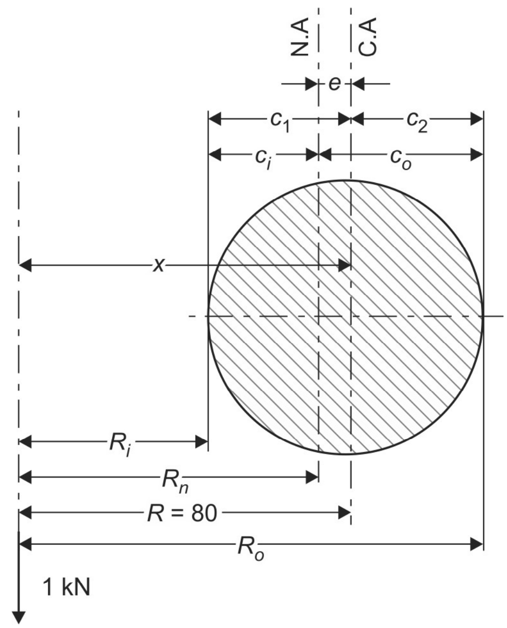

Fig. 1.17(b): Problem 13

Solution: $F = 1000$ N, $D = 20$ mm, $n = ?$

a. To find $\bar{x}$: $\quad \bar{x} = \frac{D}{2} = \frac{20}{2} = 10$ mm

b. Direct stress. $\quad \sigma_D = \frac{F}{A} = \frac{1000}{314.16} = 3.18$ MPa ... **1.1(a)/Pg 2, DHB**

where $\quad A = \frac{\pi D^2}{4} = \frac{\pi \times 20^2}{4} = 314.16 \text{ mm}^2$

c. Bending stresses (σ_i, σ_o):

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB,** we have

$$D = 20 \text{ mm}, c_1 = c_2 = 10 \text{ mm} = \bar{x}$$

- From **Fig. 1.17(b)**, we have

$$c_1 = c_2 = 10 \text{ mm} = \bar{x} \qquad R = 80 \text{ mm}$$

$$R = R_i + c_1 \Rightarrow R_i = R - c_1 = 80 - 10 = 70 \text{ mm}$$

$$R_o = R_i + D = 70 + 20 = 90 \text{ mm}$$

- Moment, $\quad M = F \cdot x = FR = 1000 \times 80 = 80000$ N-mm
- $e = R - R_n$ ... **Fig 2 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 80 - \frac{10^2/2}{80 - \sqrt{80^2 - 10^2}}$$

$$= 0.314 \text{ mm}$$

Now, $c_i = c_1 - e = 10 - 0.314 = 9.686$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 10 + 0.314 = 10.314$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(80000)\times 9.686}{314.16\times 0.314\times 70} = 112.22 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(80000)\times 10.314}{314.16\times 0.314\times 90} = -92.94 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = 3.18 + 112.22 = 115.40 \text{ MPa}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = 3.18 - 92.94 = -89.76 \text{ MPa}$$

e. Factor of safety

$$n = \frac{\sigma_{yt}}{\sigma_{max}} = \frac{400}{115.40} = 3.466 \approx 3.5$$

14. A trough 25 mm thick and 200 mm long is as shown in Fig. 1.18(a). Determine the magnitude and location of maximum tension, maximum compression and maximum shear stresses.

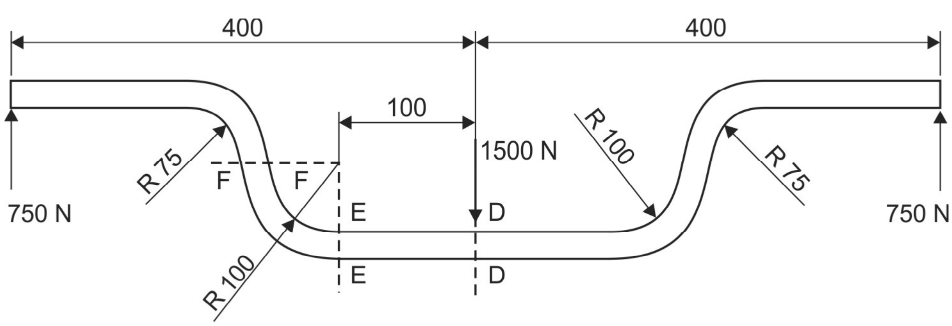

Fig. 1.18(a): Problem 14

Solution: $F = 750$ N, distance $x = 400$ mm, $x_1 = 100$ mm from section D–D, $R_i = 100$ mm at section E–E, $R_i = 75$ mm at section F–F, $\sigma_i)_r$, $\sigma_o)_r = ?$, $\tau_{max} = ?$

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. 1 – Tb 10.1/Pg 162, DHB**, we have

$b = 200$ mm, $h = 25$ mm

$$\bar{x} = \frac{h}{2} = \frac{25}{2} = 12.5 \text{ mm}$$

b. Bending stresses (σ_i, σ_o)

Section D – D:

Here the beam is straight, hence, $\sigma_b = \dfrac{Mc}{I}$ ... **1.1(b)/Pg 2, DHB**

But, $M = F \cdot x = 750 \times 400 = 3 \times 10^5$ N-mm

$c = h/2 = 25/2 = 12.5$ mm

$$I = \frac{bh^3}{12} = \frac{200 \times 25^3}{12} = 260.42 \times 10^3 \text{ mm}^4 \quad \text{... } \textbf{Tb 1.3(a)/Pg 12, DHB}$$

$$\sigma_b = \frac{\left(3 \times 10^5\right) \times 12.5}{260.42 \times 10^3} = 14.4 \text{ MPa}$$

Here the transverse shear stress is zero at the inner and outer fiber, as it is a straight beam.

Section E – E (R_i = 100 mm):

- From **Fig. 1.18(b)**, we have

$c_1 = c_2 = c = \bar{x} = 12.5$ mm, $R_i = 100$ mm

$R = R_i + c_1 = 100 + 12.5 = 112.5$ mm

$R_o = R_i + h = 100 + 25 = 125$ mm

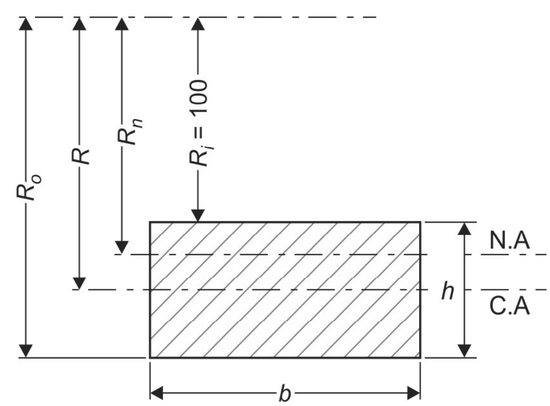

Fig. 1.18(b): Problem 14

- Moment, $M = F \cdot (x - x_1) = 750 \times (400 - 100)$

$= 225 \times 10^3$ N-mm

- $e = R - R_n$ ... **Fig. 1 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{h}{\ln\left(\dfrac{R+c}{R-c}\right)} = 112.5 - \frac{25}{\ln\left(\dfrac{112.5+12.5}{112.5-12.5}\right)}$$

$= 0.46$ mm

Now, $c_i = c_1 - e = 12.5 - 0.46 = 12.04$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 12.5 + 0.46 = 12.96$ mm ... **10.1(d)/Pg 159, DHB**

Here inner fibers are in compression, while outer fibers are in tension.

- Bending stress at inner fiber,

$$\sigma_i = \frac{-Mc_i}{AeR_i} = \frac{(225 \times 10^3) \times 12.04}{(25 \times 200) \times 0.46 \times 100} = -11.78 \text{ MPa} \quad \text{... } \textbf{10.1(b)/Pg 159, DHB}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{Mc_o}{AeR_o} = \frac{(225 \times 10^3) \times 12.96}{(25 \times 200) \times 0.46 \times 125} = 10.14 \text{ MPa} \qquad \text{... } \mathbf{10.1(c)/Pg\ 159,\ DHB}$$

Section F – F:

- From **Fig. 1.18(c)**, we have

$$c_1 = c_2 = c = \bar{x} = 12.5 \text{ mm} \qquad R_i = 75 \text{ mm}$$
$$R = R_i + c_1 = 75 + 12.5 = 87.5 \text{ mm}$$
$$R_o = R_i + h = 75 + 25 = 100 \text{ mm}$$

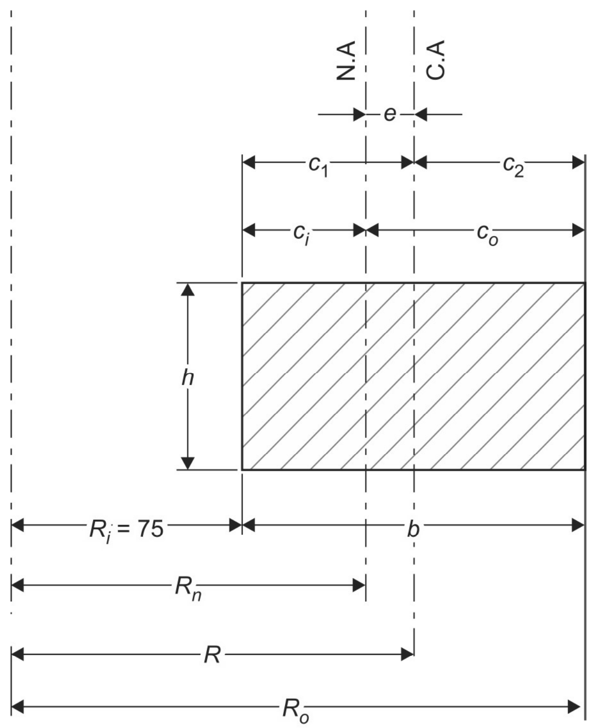

Fig. 1.18(c): Problem 14

- Moment, $M = F[(x - x_1 - R_i)_{E-E} - \bar{x}]$

$$= 750 \times [400 - (100 - 100 - 12.5)]$$
$$M = 140.63 \times 10^3 \text{ N-mm}$$

- $e = R - R_n$ ... **Fig. 1 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{h}{\ln\left(\frac{R+c}{R-c}\right)} = 87.5 - \frac{25}{\ln\left(\frac{87.5+12.5}{87.5-12.5}\right)}$$
$$= 0.6 \text{ mm}$$

Now, $c_i = c_1 - e = 12.5 - 0.6 = 11.90$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 12.5 + 0.6 = 13.10$ mm ... **10.1(d)/Pg 159, DHB**

Here inner fibers are in tension, while outer fibers are in compression.

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(140.63\times 10^3)\times 11.90}{(25\times 200)\times 0.6\times 75} = 7.46 \text{ MPa} \quad \ldots\mathbf{10.1(b)/Pg\ 159,\ DHB}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{(140.63\times 10^3)\times 13.10}{(25\times 200)\times 0.6\times 100} = -6.14 \text{ MPa} \quad \ldots\mathbf{10.1(c)/Pg\ 159,\ DHB}$$

c. Maximum stresses
- Maximum tensile stress occurs at $D - D$, $\sigma_{max} = 14.4$ MPa
- Maximum compressive stress occurs at $E - E$, $\sigma_{max} = -11.78$ MPa
- Maximum shear stress, $\tau_{max} = \frac{\sigma_{max}}{2} = \frac{14.4}{2} = 7.2$ MPa

15. An offset bar has forces applied as shown in Fig. 1.19(a). The bar is (25 × 50) mm. The offset of two applied forces is a pure couple that causes the same bending moment at every section of the beam. Determine the magnitude and location of maximum tension, maximum compression and maximum shear stresses.

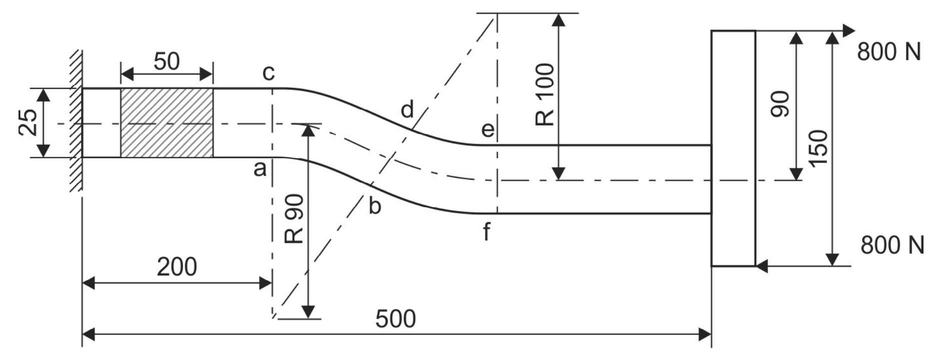

Fig. 1.19(a): Problem 15

Solution: F = 800 N, distance x = 150 mm, R = 90 mm at section a–c, R = 100 mm at section e–f, $\sigma_i)_r$, $\sigma_o)_r$ = ?, τ_{max} = ?

a. To find $\bar{x}$:
- Comparing the given cross-section with **Fig. 1 – Tb 10.1/Pg 162, DHB**, we have

$$b = 50 \text{ mm}, h = 25 \text{ mm}$$

$$\bar{x} = \frac{h}{2} = \frac{25}{2} = 12.5 \text{ mm}$$

b. Bending stresses (σ_i, σ_o)

Section a – c:

- From **Fig. 1.19(b),** we have

$$c_1 = c_2 = c = \bar{x} = 12.5 \text{ mm}, \quad R = 90 \text{ mm}$$

$$R_i = R - c_1 = 90 - 12.5 = 77.5 \text{ mm}$$

$$R_o = R_i + h = 77.5 + 25 = 102.5 \text{ mm}$$

- Moment, $M = F\cdot x = 800 \times 150 = 1.20 \times 10^5$ N-mm
- $e = R - R_n$ **Fig. 1 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{h}{\ln\left(\frac{R+c}{R-c}\right)} = 90 - \frac{25}{\ln\left(\frac{90+12.5}{90-12.5}\right)}$$

$$= 0.58 \text{ mm}$$

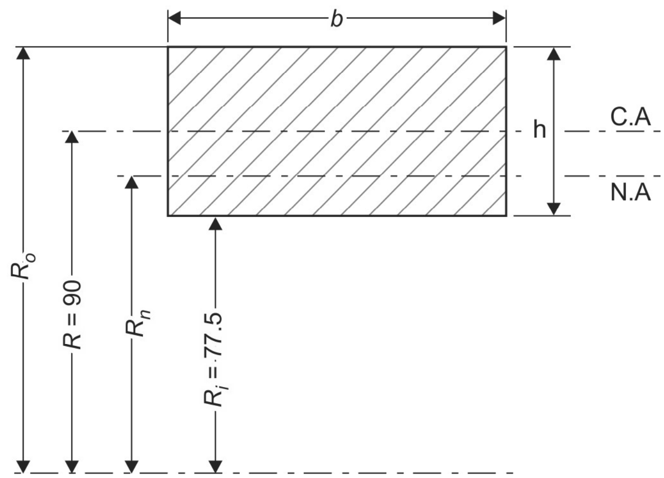

Fig. 1.19(b): Problem 15

Now, $c_i = c_1 - e = 12.5 - 0.58 = 11.92$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 12.5 + 0.58 = 13.08$ mm ... **10.1(d)/Pg 159, DHB**

Here inner fibers are in compression, while outer fibers are in tension.

- Bending stress at inner fiber,

$$\sigma_i = \frac{-Mc_i}{AeR_i} = \frac{(1.2\times10^5)\times11.92}{(25\times50)\times0.58\times77.5} = -25.45 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{Mc_o}{AeR_o} = \frac{(1.2\times10^5)\times13.08}{(25\times50)\times0.58\times102.5} = 21.12 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

Section e – f:

- From **Fig. 1.19(c),** we have

$$c_1 = c_2 = c = \bar{x} = 12.5 \text{ mm}, \quad R = 100 \text{ mm}$$
$$R_i = R - c_1 = 100 - 12.5 = 87.5 \text{ mm}$$
$$R_o = R_i + h = 87.5 + 25 = 112.5 \text{ mm}$$

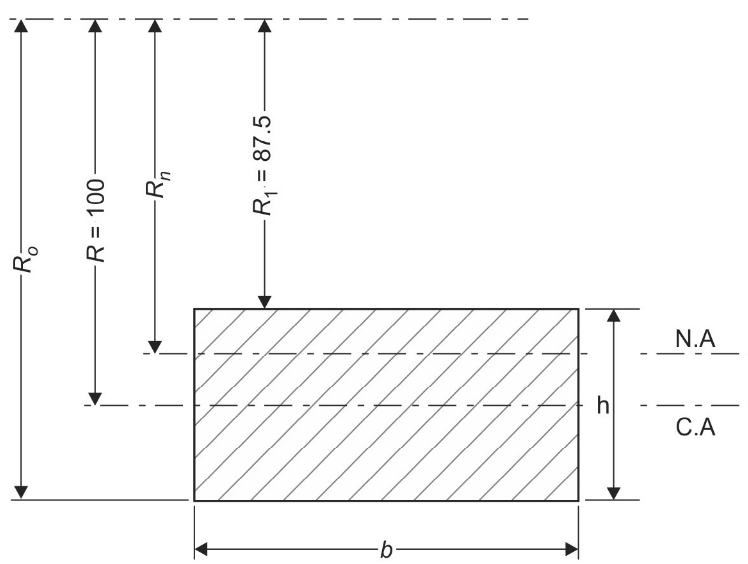

Fig. 1.19(c): Problem 15

- Moment, $M = 1.20 \times 10^5$ N-mm (same for both sections)
- $e = R - R_n$ ... **Fig. 1 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{h}{\ln\left(\frac{R+c}{R-c}\right)} = 100 - \frac{25}{\ln\left(\frac{100+12.5}{100-12.5}\right)}$$

$e = 0.52$ mm

Now, $c_i = c_1 - e = 12.5 - 0.52 = 11.98$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 12.5 + 0.52 = 13.02$ mm ... **10.1(d)/Pg 159, DHB**

Here inner fibers are in tension, while outer fibers are in compression.

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(1.2 \times 10^5) \times 11.92}{(25 \times 50) \times 0.52 \times 87.5} = 25.14 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(1.2 \times 10^5) \times 13.02}{(25 \times 50) \times 0.52 \times 112.5} = -21.37 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

c. Maximum stresses
- Maximum tensile stress occurs at $a - c$, $\sigma_{max} = 25.14$ MPa
- Maximum compressive stress occurs at $e - f$, $\sigma_{max} = -21.37$ MPa
- Maximum shear stress, $\tau_{max} = \frac{\sigma_{max}}{2} = \frac{25.14}{2} = 12.57$ MPa

1.5 STRESSES IN CLOSED RINGS

Consider a closed ring as shown in **Fig. 1.20(a)**, subjected to a tensile load F. Due to symmetry, we consider only one quadrant as shown in **Fig. 1.20(b)**. The applied load tends to stretch the vertical dimension and reduce the horizontal dimension.

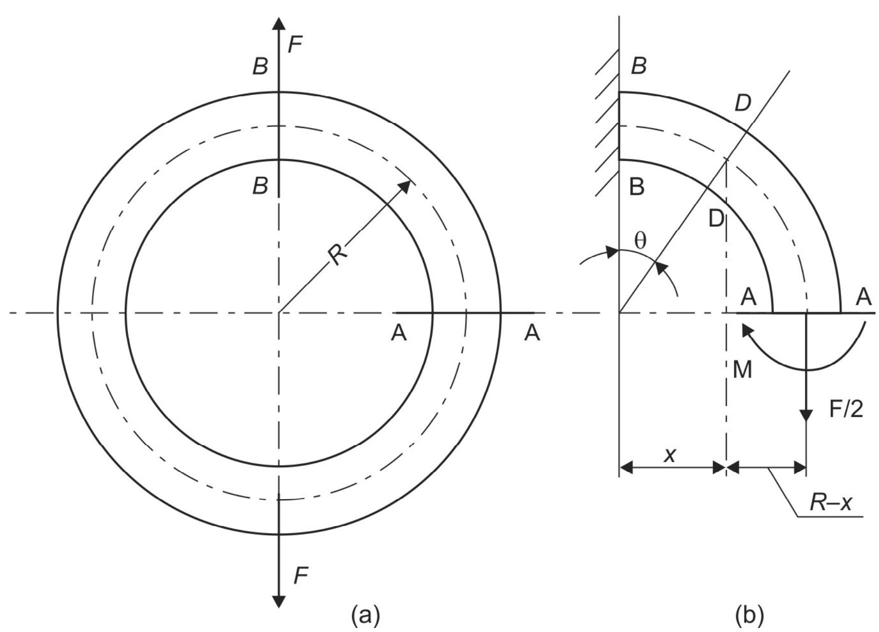

Fig. 1.20: Closed ring in tension **(Fig. 10.4/Pg 161, DHB)**

- Bending moment at any cross-section of the ring is,

$$M = F \cdot R\left[\frac{1}{\pi} - \frac{\sin\theta}{2}\right]$$

$$= \frac{FR}{2}\left[\frac{2}{\pi} - \sin\theta\right] \quad \text{... (Eq. 1.14)}$$

where R = mean radius of the ring
θ = angle with respect to load line (vertical)
F = load

- At section B – B, $\theta = 0$ (with respect to vertical)
(Eq. 1.14) yields..., $\boldsymbol{M_B = 0.318\,FR}$ ... (Eq. 1.15) **10.5/Pg 160, DHB**
- At section A – A, $\theta = 90°$ (with respect to vertical)
(Eq. 1.14) yields..., $\boldsymbol{M_A = -0.182\,FR}$ ... (Eq. 1.16) **10.6/Pg 161, DHB**
- Stress at D – D, $\sigma_D = \dfrac{F\sin\theta}{2A}$

The stress at any point in the cross-section is

$$\sigma = \sigma_D + \sigma_b \quad \text{... (Eq. 1.17) } \textbf{10.8/Pg 161, DHB}$$

where, $$\sigma_b = \frac{Mc_i}{AeR_i} \text{ or } \frac{Mc_o}{AeR_o} \quad \text{... } \textbf{10.1(b), 10.1(c)/Pg 159, DHB}$$

Section B – B:
At load line (θ = 0), with respect to vertical:

The inner fibers are subjected to compression while the outer fibers are subjected to tension (in σ_b), hence

- Resultant stress in the inner fiber, $\sigma_i)_r = \sigma_D + \dfrac{-M_B c_i}{AeR_i}$ ... **10.8/Pg 161, DHB**
- Resultant stress in the outer fiber, $\sigma_o)_r = \sigma_D + \dfrac{M_B c_o}{AeR_o}$ ... **10.8/Pg 161, DHB**

Since the load line is acting at section B-B itself, σ_D = 0 [i.e. there is no direct stress]

Section A – A:
Away from load line (θ = 90°, with respect to vertical):

The inner fibers are subjected to compression while the outer fibers are subjected to tension (in σ_b), hence

- Resultant stress in the inner fiber, $\sigma_i)_r = \sigma_D + \dfrac{-M_A c_i}{AeR_i}$ ... **10.8/Pg 161, DHB**
- Resultant stress in the outer fiber, $\sigma_o)_r = \sigma_D + \dfrac{M_A c_o}{AeR_o}$ ... **10.8/Pg 161, DHB**

16. Determine the stresses induced in a closed ring as shown in Fig. 1.21(a).

Solution: $F = 25 \times 10^3$ N, $D = 40$ mm, $R_i = 40$ mm, $\sigma_i)_r$, $\sigma_o)_r$ = ?

a. To find $\bar{x}$: $$\bar{x} = \frac{D}{2} = \frac{40}{2} = 20 \text{ mm}$$

where $$A = \frac{\pi D^2}{4} = \frac{\pi \times 40^2}{4} = 1256.64 \text{ mm}^2$$

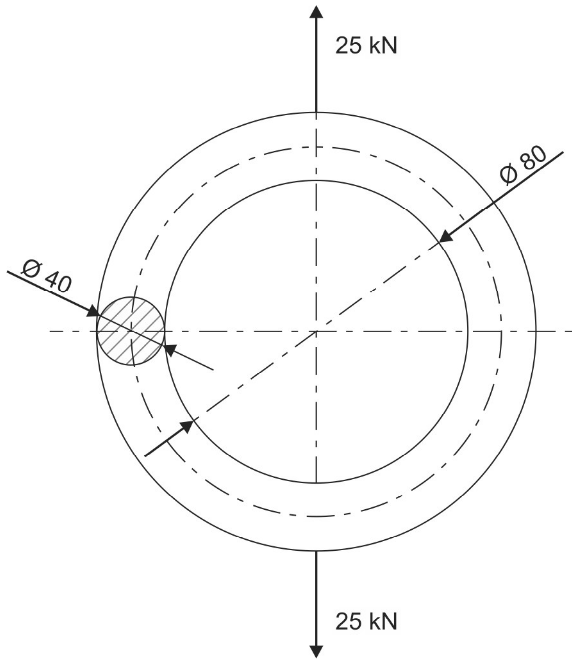

Fig. 1.21(a): Problem 16

b. Resultant stresses:

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB,** we have

$$D = 40 \text{ mm}, c_1 = c_2 = 20 \text{ mm} = \bar{x}$$

- From **Fig. 1.21(b)**, we have $c_1 = c_2 = 20 \text{ mm} = \bar{x}, \quad R_i = 40 \text{ mm}$

$$R = R_i + c_1 = 40 + 20 = 60 \text{ mm}$$

$$R_o = R_i + D = 40 + 40 = 80 \text{ mm} \text{ or } (= R + \bar{x})$$

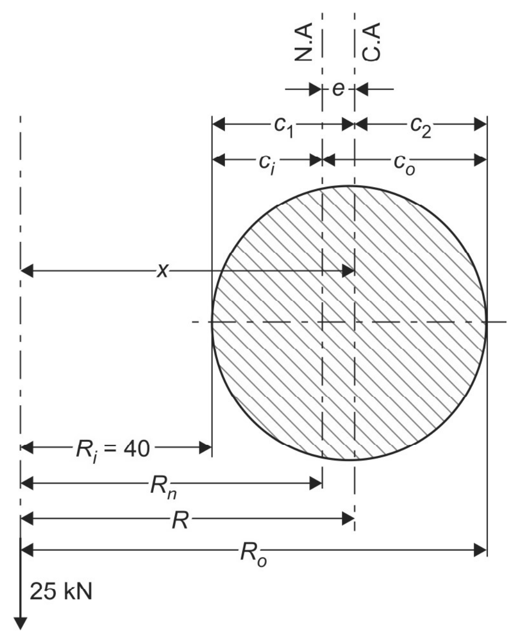

Fig. 1.21(b): Problem 16

- $$e = R - R_n$$ ... **Fig. 2-Tb 10.1/Pg 162, DHB**

$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 60 - \frac{20^2/2}{60 - \sqrt{60^2 - 20^2}}$$

$$= 1.72 \text{ mm}$$

Now, $c_i = c_1 - e = 20 - 1.72 = 18.28$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 20 + 1.72 = 21.72$ mm ... **10.1(d)/Pg 159, DHB**

Section B – B: At load line (θ = 0°, with respect to vertical)

- Moment $M_B = 0.318\, FR$ ... **10.5/Pg 160, DHB**

$$= 0.318 \times 25000 \times 60 = 477 \times 10^3 \text{ N-mm}$$

- Resultant stress in the inner fiber,

$$\sigma_i)_r = \sigma_D + \frac{-M_B c_i}{AeR_i} = \frac{F \sin\theta}{2A} + \frac{-M_B c_i}{AeR_i}$$ ... **10.8/Pg 161, DHB**

$$= \frac{25000 \times \sin(0)}{2 \times 1256.64} + \frac{-(477 \times 10^3) \times 18.28}{1256.64 \times 1.72 \times 40} = 0 - 100.85$$

$$= -100.85 \text{ MPa}$$

- Resultant stress in the outer fiber,

$$\sigma_o)_r = \sigma_D + \frac{-M_B c_o}{AeR_o} = \frac{F \sin\theta}{2A} + \frac{M_B c_o}{AeR_o}$$ ... **10.8/Pg 161, DHB**

$$= \frac{25000 \times \sin(0)}{2 \times 1256.64} + \frac{(477 \times 10^3) \times 21.72}{1256.64 \times 1.72 \times 80} = 0 + 60 = 60 \text{ MPa}$$

Section A – A: Away from load line (θ = 90°, with respect to vertical)

- Moment $M_A = -0.182\, FR$ ... **10.6/Pg 161, DHB**

$$= -0.182 \times 25000 \times 60 = -273 \times 10^3 \text{ N-mm}$$

- Resultant stress in the inner fiber,

$$\sigma_i)_r = \sigma_D + \frac{-M_A c_i}{AeR_i} = \frac{F \sin\theta}{2A} + \frac{-M_A c_i}{AeR_i}$$ ... **10.8/Pg 161, DHB**

$$= \frac{25000 \times \sin(90)}{2 \times 1256.64} + \frac{-(-273 \times 10^3) \times 18.28}{1256.64 \times 1.72 \times 40} = 9.95 + 57.68$$

$$= 67.67 \text{ MPa}$$

- Resultant stress in the outer fiber,

$$\sigma_o)_r = \sigma_D + \frac{M_A c_o}{AeR_o} = \frac{F \sin\theta}{2A} + \frac{M_A c_o}{AeR_o}$$ ... **10.8/Pg 161, DHB**

$$= \frac{25000 \times \sin(90)}{2 \times 1256.64} + \frac{(-273 \times 10^3) \times 21.72}{1256.64 \times 1.72 \times 80}$$

$$= 9.95 - 34.29 = -24.34 \text{ MPa}$$

17. A closed ring is made up of 50 mm diameter steel bar having allowable tensile stress of 200 MPa. The inner diameter of the ring is 100 mm. For a load of 30 kN, find the maximum stress in the bar and specify the location. If the ring is cut as shown in part B of Fig. 1.22(a), check whether it is safe to support the applied load.

VTU – Dec 08/Jan 09 – 10 Marks

Solution: $F = 30 \times 10^3$ N, $D = 50$ mm, $R_i = 50$ mm.

Case a: To find $\sigma_i)_r$, $\sigma_i)_r$, for ring [part A: **Fig. 1.22(a)]**

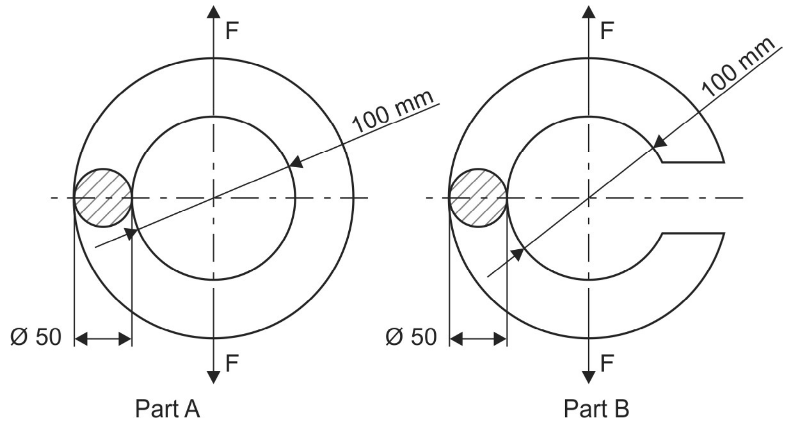

Fig. 1.22(a): Problem 17

***Case b*:** To find F, for circular beam [part B: **Fig. 1.22(a)**]

Case a: To find stresses for ring [part A: Fig. 1.22(a)]

a. To find $\bar{x}$:
$$\bar{x} = \frac{D}{2} = \frac{50}{2} = 25 \text{ mm}$$

where
$$A = \frac{\pi D^2}{4} = \frac{\pi \times 50^2}{4} = 1963.50 \text{ mm}^2$$

b. Resultant stresses:

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB,** we have
$$D = 50 \text{ mm}, c_1 = c_2 = 25 \text{ mm} = \bar{x}$$
- From **Fig. 1.22(b)**, we have
$$c_1 = c_2 = 25 \text{ mm} = \bar{x}, \qquad R_i = 50 \text{ mm}$$
$$R = R_i + c_1 = 50 + 25 = 75 \text{ mm}$$
$$R_o = R_i + D = 50 + 50 = 100 \text{ mm} \quad \text{or } (= R + \bar{x})$$

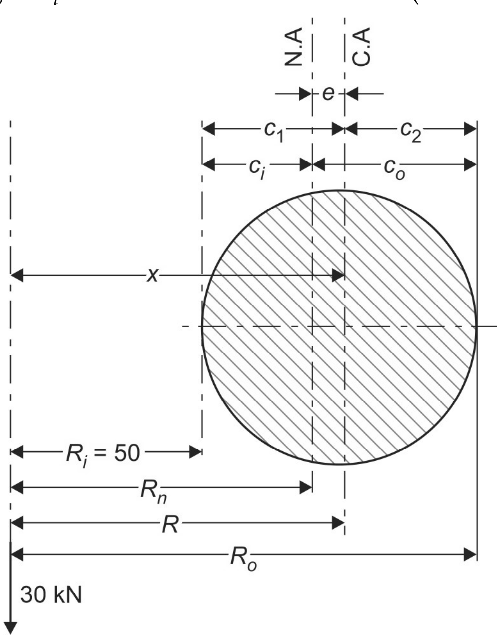

Fig. 1.22(b): Problem 17

- $e = R - R_n$ ... **Fig. 2 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 75 - \frac{25^2/2}{75 - \sqrt{75^2 - 25^2}}$$

$= 2.14$ mm

Now, $c_i = c_1 - e = 25 - 2.14 = 22.86$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 25 + 2.14 = 27.14$ mm ... **10.1(d)/Pg 159, DHB**

Section B – B: At load line (θ = 0°, with respect to vertical)

- Moment $M_B = 0.318\, FR$ ... **10.5/Pg 160, DHB**

  $= 0.318 \times 30000 \times 75 = 715.5 \times 10^3$ N-mm
- Resultant stress in the inner fiber,

$$\sigma_i)_r = \sigma_D + \frac{-M_B c_i}{AeR_i} = \frac{F\sin\theta}{2A} + \frac{-M_B c_i}{AeR_i}$$ ... **10.8/Pg 161, DHB**

$$= \frac{30000 \times \sin(0)}{2 \times 1963.50} + \frac{-(715.5 \times 10^3) \times 22.86}{1963.50 \times 2.14 \times 50} = 0 - 77.85 = -77.85 \text{ MPa}$$

- Resultant stress in the outer fiber,

$$\sigma_o)_r = \sigma_D + \frac{M_B c_o}{AeR_o} = \frac{F\sin\theta}{2A} + \frac{M_B c_o}{AeR_o}$$ ... **10.8/Pg 161, DHB**

$$= \frac{30000 \times \sin(0)}{2 \times 1963.50} + \frac{(715.5 \times 10^3) \times 27.14}{1963.50 \times 2.14 \times 100} = 0 + 46.21 = 46.21 \text{ MPa}$$

Section A – A: Away from load line (θ = 90°, with respect to vertical)

- Moment $M_A = -0.182\, FR$ ... **10.6/Pg 161, DHB**

  $= -0.182 \times 30000 \times 75$

  $= -409.5 \times 10^3$ N-mm
- Resultant stress in the inner fiber,

$$\sigma_i)_r = \sigma_D + \frac{-M_A c_i}{AeR_i} = \frac{F\sin\theta}{2A} + \frac{-M_A c_i}{AeR_i}$$ ... **10.8/Pg 161, DHB**

$$= \frac{30000 \times \sin(90)}{2 \times 1963.50} + \frac{-(-409.5 \times 10^3) \times 22.86}{1963.50 \times 2.14 \times 50} = 7.64 + 44.55 = 52.19 \text{ MPa}$$

- Resultant stress in the outer fiber,

$$\sigma_o)_r = \sigma_D + \frac{M_A c_o}{AeR_o} = \frac{F\sin\theta}{2A} + \frac{M_A c_o}{AeR_o}$$ ... **10.8/Pg 161, DHB**

$$= \frac{30000 \times \sin(90)}{2 \times 1963.50} + \frac{(-409.5 \times 10^3) \times 27.14}{1963.50 \times 2.14 \times 100} = 7.64 - 26.44 = -18.81 \text{ MPa}$$

Thus the maximum stress is 77.85 MPa, which is less than the allowable tensile stress of 200 MPa. Hence the design of ring is safe.

Case b: To find F, for circular beam [Part B: Fig. 1.22(b)]

a. Direct stress: $\sigma_D = \frac{F}{A} = \frac{F}{1963.50} = (5.09 \times 10^{-4})F$

b. Bending stresses (σ_i, σ_o):

- Moment, $M = F \cdot x = F(R_i + \bar{x}) = F(50 + 25) = 75\,F$
- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(75)F \times 22.86}{1963.50 \times 2.14 \times 50} = (8.16 \times 10^{-3})F$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(75)F \times 27.14}{1963.50 \times 2.14 \times 100} = (-4.84 \times 10^{-3})F \quad \text{... } \mathbf{10.1(c)/Pg\ 159,\ DHB}$$

c. Resultant stresses:

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = (5.09 \times 10^{-4})F + (8.16 \times 10^{-3})F$$
$$= (8.67 \times 10^{-3})F$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = (5.09 \times 10^{-4})F + (-4.84 \times 10^{-3})F$$
$$= (-4.33 \times 10^{-3})F$$

d. To find F:

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = (8.67 \times 10^{-3})F$$
$$200 = (8.67 \times 10^{-3})F$$
$$F = \frac{200}{8.67 \times 10^{-3}} = 23068.05 \text{ N}$$

Since the calculated value of F is less than the given value, it is safe to support the applied load.

18. Determine the maximum stress induced in a ring cross-section of 50 mm diameter rod subjected to a compressive load of 20 kN. The mean diameter of the ring is 100 mm.

VTU – Dec. 09/Jan. 10 – 10 Marks; [Similar: Dec. 2015/Jan. 2016 – 10 Marks]

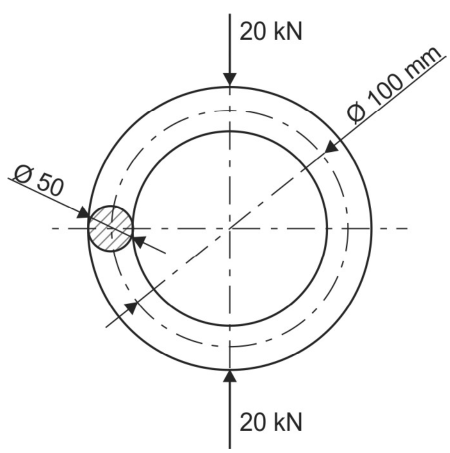

Fig. 1.23(a): Problem 18

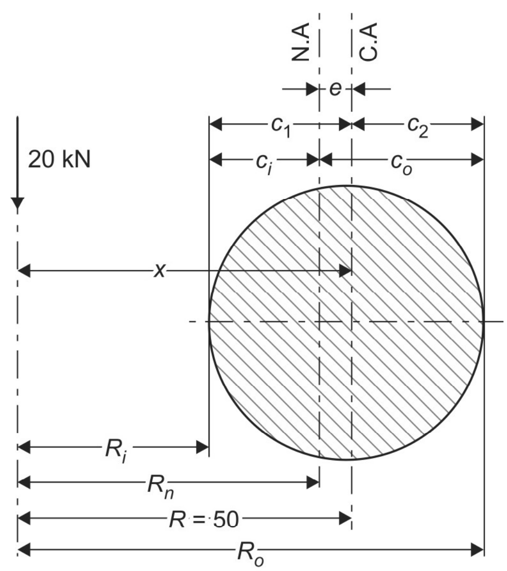

Fig. 1.23(b): Problem 18

Solution: $F = -20000$ N (compressive), $D = 50$ mm, $R = 50$ mm, $\sigma_i)_r$, $\sigma_o)_r = ?$

a. To find $\bar{x}$: $\quad \bar{x} = \frac{D}{2} = \frac{50}{2} = 25$ mm where $A = \frac{\pi D^2}{4} = \frac{\pi \times 50^2}{4} = 1963.50 \text{ mm}^2$

b. Resultant stresses:

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB,** we have

$$D = 50 \text{ mm}, c_1 = c_2 = 25 \text{ mm} = \bar{x}$$

- From **Fig. 1.21(b)**, we have

$$c_1 = c_2 = 25 \text{ mm} = \bar{x}, \quad R = 50 \text{ mm}$$
$$R_i = R - c_1 = 50 - 25 = 25 \text{ mm}$$
$$R_o = R_i + D = 25 + 50 = 75 \text{ mm} \quad \text{or } (= R + \bar{x})$$

- $e = R - R_n$ ... **Fig. 2 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 50 - \frac{25^2/2}{50 - \sqrt{50^2 - 25^2}}$$
$$= 3.35 \text{ mm}$$

Now, $c_i = c_1 - e = 25 - 3.35 = 21.65$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 25 + 3.35 = 28.35$ mm ... **10.1(d)/Pg 159, DHB**

Section B – B: At load line* (θ = 0°, *with respect to vertical)

- Moment $M_B = -0.318\, FR$ ... **10.5/Pg 160, DHB**

$$= -0.318 \times 20000 \times 50$$
$$= -318 \times 10^3 \text{ N-mm}$$

- Resultant stress in the inner fiber,

$$\sigma_i)_r = \sigma_D + \frac{-M_B c_i}{AeR_i} = \frac{F \sin\theta}{2A} + \frac{-M_B c_i}{AeR_i}$$ ... **10.8/Pg 161, DHB**

$$= \frac{(-20000) \times \sin(0)}{2 \times 1963.50} + \frac{-(-318 \times 10^3) \times 21.65}{1963.50 \times 3.35 \times 25}$$
$$= 0 + 41.87$$
$$= 41.87 \text{ MPa}$$

- Resultant stress in the outer fiber,

$$\sigma_o)_r = \sigma_D + \frac{M_B c_o}{AeR_o} = \frac{F \sin\theta}{2A} + \frac{M_B c_o}{AeR_o}$$ ... **10.8/Pg 161, DHB**

$$= \frac{(-20000) \times \sin(0)}{2 \times 1963.50} + \frac{(-318 \times 10^3) \times 28.35}{1963.50 \times 3.35 \times 75}$$
$$= 0 - 18.27$$
$$= -18.27 \text{ MPa}$$

Section A – A: Away from load line* (θ = 90°, *with respect to vertical)

- Moment $M_A = 0.182\, FR$ ... **10.6/Pg 161, DHB**

$$= 0.182 \times 20000 \times 50$$
$$= 182 \times 10^3 \text{ N-mm}$$

- Resultant stress in the inner fiber,

$$\sigma_i)_r = \sigma_D + \frac{-M_A c_i}{AeR_i} = \frac{F \sin\theta}{2A} + \frac{-M_A c_i}{AeR_i}$$ ... **10.8/Pg 161, DHB**

$$\sigma_i)_r = \frac{(-20000)\times\sin(90)}{2\times1963.50} + \frac{(-182\times10^3)\times21.65}{1963.50\times3.35\times25}$$

$$= -5.09 - 23.96$$

$$= -29.05 \text{ MPa}$$

- Resultant stress in the outer fiber,

$$\sigma_o)_r = \sigma_D + \frac{Mc_o}{AeR_o} = \frac{F\sin\theta}{2A} + \frac{Mc_o}{AeR_o} \quad \text{... 10.8/Pg 161, DHB}$$

$$= \frac{(-20000)\times\sin(90)}{2\times1963.50} + \frac{(-182\times10^3)\times28.35}{1963.50\times3.354\times75} = -5.09 + 10.46$$

$$= 5.37 \text{ MPa}$$

1.6 CHAIN LINKS

Consider a chain link as shown in **Fig. 1.24** subjected to a tensile load F. Due to symmetry, we consider only one quadrant as shown in **Fig. 1.20(b).** *The analysis is similar to closed rings.*

These are used in heavy hoisting equipment and are subjected to bending moment.

- The bending moment at section **AA**, i.e. 90° away from the point of application of the load and in all the straight parts of the link:

$$M_A = \frac{FR}{2}\left[\frac{2R-\pi R}{\pi R + l}\right]$$

... **(Eq. 1.18) 10.2(a)/Pg 159, DHB**

- The bending moment at section **BB**, i.e. at the point of application of load:

$$M_B = \frac{FR}{2}\left[\frac{2R+l}{\pi R + l}\right] \quad \text{... (Eq. 1.19), 10.2(b)/Pg 159, DHB}$$

where R = mean radius of the ring
l = length of straight portion of link
F = load

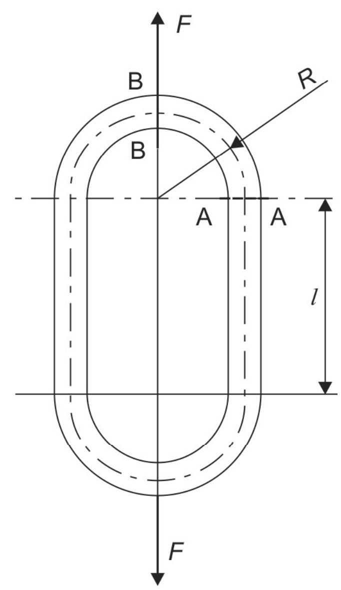

Fig. 1.24: Chain link
(Fig. 10.3/Pg 160, DHB)

19. A chain link made of 40 mm diameter rod is semicircular at each end, the mean diameter of which is 80 mm. The straight sides of the link are also 80 mm as shown in Fig. 1.25(a). If the link carries a load of 90 kN, estimate the tensile and compressive stresses in the link along the section of load line.

VTU – July/Aug. 03 – 10 Marks

Solution: $F = 90 \times 10^3$ N, D = 40 mm, mean diameter = 80 mm, mean radius R = 40 mm, l = 80 mm, $\sigma_i)_r, \sigma_o)_r = ?$

a. To find $\bar{x}$: $\quad \bar{x} = \frac{D}{2} = \frac{40}{2} = 20$ mm

where $\quad A = \frac{\pi D^2}{4} = \frac{\pi\times40^2}{4} = 1256.64 \text{ mm}^2$

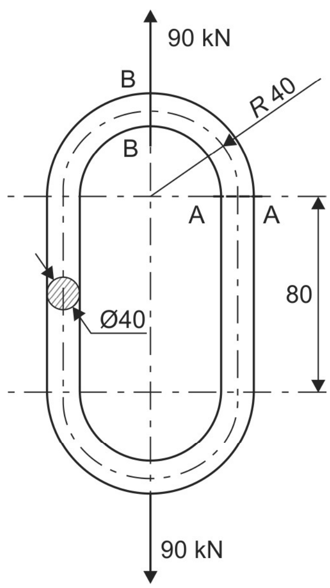

Fig. 1.25(a): Problem 19

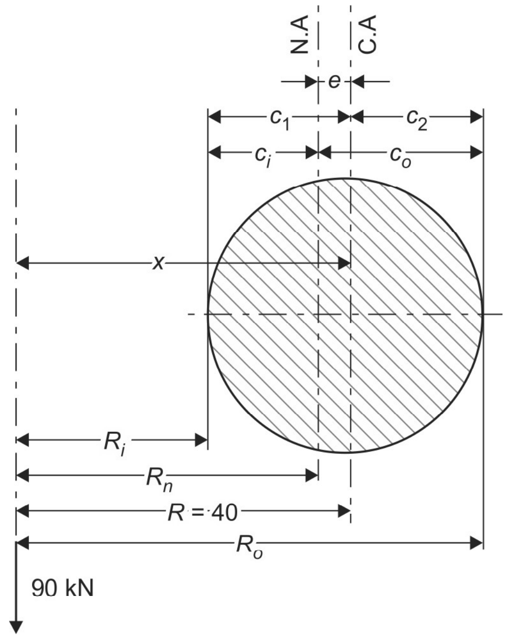

Fig. 1.25(b): Problem 19

b. Resultant stresses:

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB,** we have

$$D = 40 \text{ mm}, c_1 = c_2 = 20 \text{ mm} = \bar{x}$$

- From **Fig. 1.25(b)**, we have

$$c_1 = c_2 = 20 \text{ mm} = \bar{x}, \qquad R = 40 \text{ mm}$$

$$R_i = R - c_1 = 40 - 20 = 20 \text{ mm}$$

$$R_o = R_i + D = 20 + 40 = 60 \text{ mm} \quad \text{or } (= R + \bar{x})$$

- $e = R - R_n$ ... **Fig. 2-Tb 10.1/Pg 162, DHB**

$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 40 - \frac{20^2/2}{40 - \sqrt{40^2 - 20^2}}$$

$$= 2.68 \text{ mm}$$

Now, $c_i = c_1 - e = 20 - 2.68 = 17.32$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 20 + 2.68 = 22.68$ mm ... **10.1(d)/Pg 159, DHB**

Section B – B: At load line (θ = 0°, with respect to vertical)

- Moment $M_B = \frac{FR}{2}\left[\frac{2R + l}{\pi r + l}\right]$ ... **10.2(b)/Pg 159, DHB**

$$= \frac{90000 \times 40}{2}\left[\frac{(2 \times 40) + 80}{40\pi + 80}\right]$$

$$= 1.4 \times 10^6 \text{ N-mm}$$

- Resultant stress in the inner fiber,

$$\sigma_i)_r = \sigma_D + \frac{-M_B c_i}{AeR_i} = \frac{F \sin\theta}{2A} + \frac{-M_B c_i}{AeR_i}$$ ... **10.8/Pg 161, DHB**

$$\sigma_i)_r = \frac{90000 \times \sin(0)}{2 \times 1256.64} + \frac{-(1.4 \times 10^3) \times 17.32}{1256.64 \times 2.68 \times 20}$$
$$= 0 - 360$$
$$= -360 \text{ MPa}$$

- Resultant stress in the outer fiber,

$$\sigma_o)_r = \sigma_D + \frac{M_B c_o}{AeR_o} = \frac{F \sin\theta}{2A} + \frac{M_B c_o}{AeR_o} \qquad \textbf{... 10.8/Pg 161, DHB}$$
$$= \frac{90000 \times \sin(0)}{2 \times 1256.64} + \frac{(1.4 \times 10^6) \times 22.68}{1256.64 \times 2.68 \times 60}$$
$$= 0 + 157.14$$
$$= 157.14 \text{ MPa}$$

Section A – A: Away from load line* (θ = 90°, *with respect to vertical)

- Moment

$$M_A = \frac{FR}{2}\left[\frac{2R - \pi R}{\pi R + l}\right] \qquad \textbf{... 10.2(a)/Pg 159, DHB}$$
$$= \frac{90000 \times 40}{2}\left[\frac{(2 \times 40) - 40\pi}{40\pi + 80}\right]$$
$$= -400 \times 10^3 \text{ N-mm}$$

- Resultant stress in the inner fiber,

$$\sigma_i)_r = \sigma_D + \frac{-M_A c_i}{AeR_i} = \frac{F \sin\theta}{2A} + \frac{-M_A c_i}{AeR_i} \qquad \textbf{... 10.8/Pg 161, DHB}$$
$$= \frac{90000 \times \sin(90)}{2 \times 1256.64} + \frac{-(-400 \times 10^3) \times 17.32}{1256.64 \times 2.68 \times 20}$$
$$= 35.81 + 102.86$$
$$= 138.67 \text{ MPa}$$

- Resultant stress in the outer fiber,

$$\sigma_o)_r = \sigma_D + \frac{M_A c_o}{AeR_o} = \frac{F \sin\theta}{2A} + \frac{M_A c_o}{AeR_o} \qquad \textbf{... 10.8/Pg 161, DHB}$$
$$= \frac{90000 \times \sin(90)}{2 \times 1256.64} + \frac{(-400 \times 10^3) \times 22.68}{1256.64 \times 2.68 \times 60}$$
$$= 35.81 - 44.90$$
$$= -9.09 \text{ MPa}$$

Type II: Problems of Loads

20. A section of a C-clamp is as shown in Fig. 1.26(a). Determine the force required, if permissible stress is limited to 90 MPa.

Solution: $\sigma_{max} = 90$ MPa, $F = ?$

a. To find $\bar{x}$:

$a_1 = 20 \times 5 = 100 \text{ mm}^2$ $\qquad a_2 = 25 \times 5 = 125 \text{ mm}^2$

$x_1 = 5/2 = 2.5 \text{ mm}$ $\qquad x_2 = 5 + (25/2) = 17.5 \text{ mm}$

$A = \Sigma a = a_1 + a_2 = 100 + 125 = 225 \text{ mm}^2$

$$\bar{x} = \frac{\Sigma\, ax}{\Sigma\, a} = \frac{(100 \times 2.5) + (125 \times 17.5)}{225} = 10.83 \text{ mm}$$

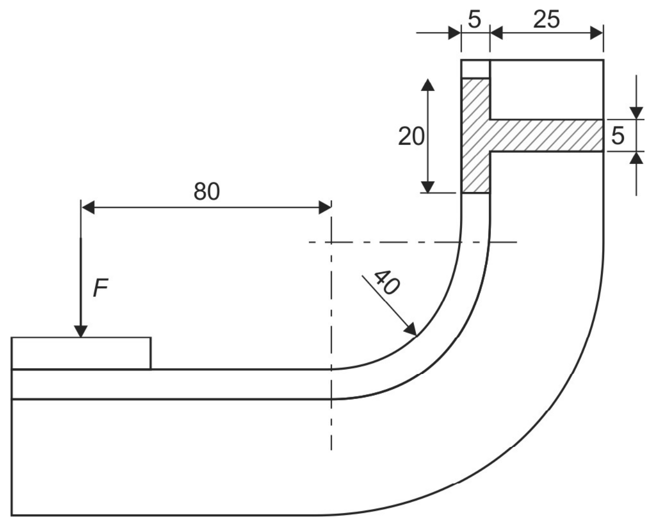

Fig. 1.26(a): Problem 20

b. Direct stress $\sigma_D = \dfrac{F}{A} = \dfrac{F}{225} = (4.45 \times 10^{-3})F$

c. Bending stresses (σ_i, σ_o)

- Comparing the given cross-section with **Fig. 8 – Tb 10.1/Pg 164, DHB**, we have

  $B = 20$ mm, $d = 5$ mm, $a = 5$ mm, $H = 5 + 25 = 30$ mm

- From **Fig. 1.26(b),** we have

$$c_1 = \bar{x} = 10.83 \text{ mm}, \quad R_i = 40 \text{ mm}$$
$$c_2 = H - c_1 = 30 - 10.83 = 19.17 \text{ mm}$$

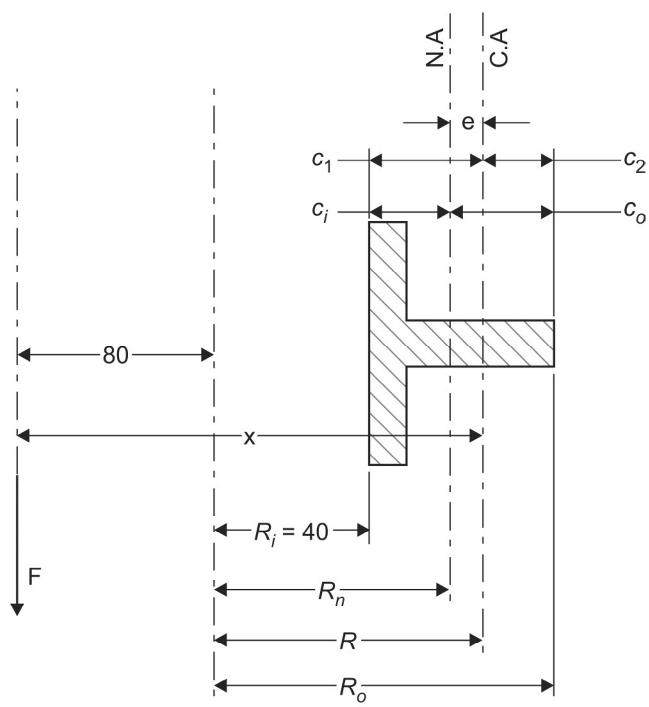

Fig. 1.26(b): Problem 20

$$R = R_i + c_1 = 40 + 10.83 = 50.83 \text{ mm}$$

$$R_o = R_i + H = 40 + 30 = 70 \text{ mm}$$

- Moment $M = F \cdot x = F(80 + R_i + \bar{x})$

$$= F \times (80 + 40 + 10.83)$$

$$= (130.83)F$$

- $e = R - R_n$ ... **Fig. 8 – Tb 10.1/Pg 164, DHB**

$$= R - \frac{A}{B \ln\left(\frac{R + d - c_1}{R - c_1}\right) + a \ln\left(\frac{R + c_2}{R + d - c_1}\right)}$$

$$= 50.83 - \frac{225}{\left[20 \ln\left(\frac{50.83 + 5 - 10.83}{50.83 - 10.83}\right)\right] + \left[5 \ln\left(\frac{50.83 + 19.17}{50.83 + 5 - 10.83}\right)\right]}$$

$$= 1.54 \text{ mm}$$

Now, $c_i = c_1 - e = 10.83 - 1.54 = 9.29$ mm ... **10.1(d)/Pg 159, DHB**

$c_0 = c_2 + e = 19.17 + 1.54 = 20.71$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(130.83)F \times 9.29}{225 \times 1.54 \times 40} = (0.0877)F$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(130.83)F \times 20.71}{225 \times 1.54 \times 70} = (-0.112)F$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = (4.45 \times 10^{-3})F + (0.0877)F = (0.0922)F$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = (4.45 \times 10^{-3})F + (-0.112)F = (-0.108)F$$

e. To find F:

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = (0.0922)F$$

$$90 = (0.0922)F$$

$$F = \frac{90}{0.0922} = 976.14 \text{ N}$$

21. A machine member has a T-shaped cross-section and is loaded as shown in Fig. 1.27(a). If the allowable compressive stress is 50 MPa, determine the largest force F which may be applied to the member safely.

VTU – Feb. 2002 – 16 Marks

Solution: $\sigma_{min} = 50$ MPa, $F = ?$

a. To find $\bar{x}$:

$a_1 = 80 \times 20 = 1600 \text{ mm}^2$ $\quad$ $a_2 = 40 \times 20 = 800 \text{ mm}^2$

$x_1 = 20/2 = 10$ mm $\quad$ $x_2 = (40/2) + 20 = 40$ mm

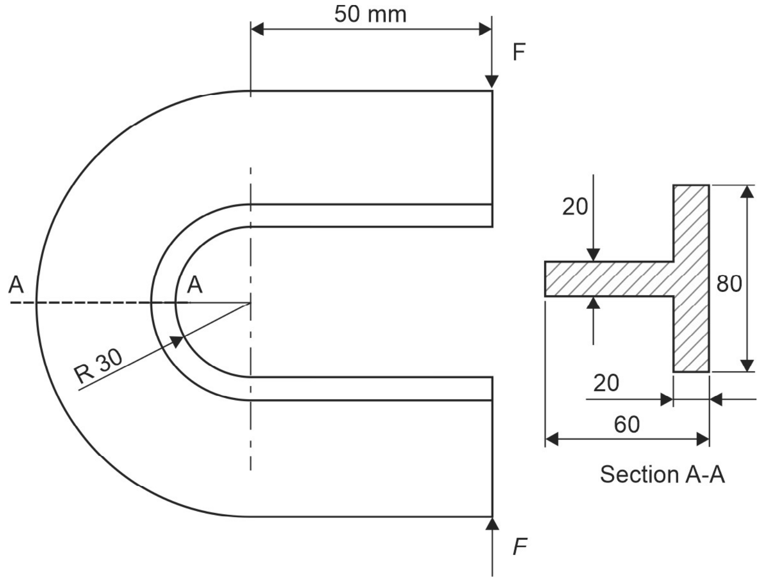

Fig. 1.27(a): Problem 21

$$A = \Sigma a = a_1 + a_2 = 1600 + 800 = 2400 \text{ mm}^2$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{(1600 \times 10) + (800 \times 40)}{2400} = 20 \text{ mm}$$

b. Direct stress $\sigma = -\frac{F}{A} = \frac{-F}{2400} = (-4.167 \times 10^{-4})F$ (compressive) ... **1.1(a)/Pg 2, DHB**

c. Bending stresses (σ_i, σ_o)

- Comparing the given cross-section with **Fig. 8 – Tb 10.1/Pg 164, DHB**, we have

$$B = 80 \text{ mm}, d = 20 \text{ mm}, a = 20 \text{ mm}, H = 60 \text{ mm}$$

- From **Fig. 1.27(b),** we have

$$c_1 = \bar{x} = 20 \text{ mm}, \quad R_i = 30 \text{ mm}$$

$$c_2 = H - c_1 = 60 - 20 = 40 \text{ mm}$$

$$R = R_i + c_1 = 30 + 20 = 50 \text{ mm}$$

$$R_o = R_i + H = 30 + 60 = 90 \text{ mm}$$

- Moment $M = F \cdot x = F(50 + R_i + \bar{x})$

$$= F \times (50 + 30 + 20)$$

$$= (100)F$$

- $e = R - R_n$ ... **Fig. 8 – Tb 10.1/Pg 164, DHB**

$$= R - \frac{A}{B \ln\left(\frac{R + d - c_1}{R - c_1}\right) + a \ln\left(\frac{R + c_2}{R + d - c_1}\right)}$$

$$= 50 - \frac{2400}{\left[80 \ln\left(\frac{50 + 20 - 20}{50 - 20}\right)\right] + \left[20 \ln\left(\frac{50 + 40}{50 + 20 - 20}\right)\right]}$$

$$= 4.39 \text{ mm}$$

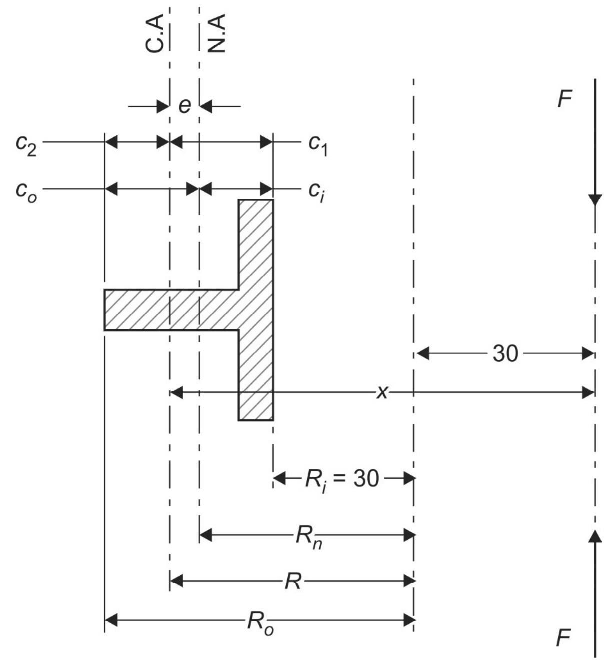

Fig. 1.27(b): Problem 21

Now, $c_i = c_1 - e = 20 - 4.39 = 15.61$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 40 + 4.39 = 44.39$ mm ... **10.1(d)/Pg 159, DHB**

Here inner fibers are in compression, while outer fibers are in tension.

- Bending stress at inner fiber,

$$\sigma_i = \frac{-Mc_i}{AeR_i} = \frac{-(100)F \times 15.61}{2400 \times 4.39 \times 30} = (-4.94 \times 10^{-3})F \quad \text{... 10.1(b)/Pg 159, DHB}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{Mc_o}{AeR_o} = \frac{(100)F \times 44.39}{2400 \times 4.39 \times 90} = (4.68 \times 10^{-3})F \quad \text{... 10.1(c)/Pg 159, DHB}$$

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = (-4.167 \times 10^{-4})F + (-4.94 \times 10^{-3})F$$
$$= (-5.36 \times 10^{-3})F$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = (-4.167 \times 10^{-4})F + (4.68 \times 10^{-3})F$$
$$= (4.26 \times 10^{-3})F$$

e. To find F:

Since minimum (compressive) stress occurs in the inner fiber

$$\sigma_i)_r = (-5.36 \times 10^{-3})F$$
$$50 = (-5.36 \times 10^{-3})F$$
$$F = \frac{50}{-5.36 \times 10^{-3}} = -9238.36 \text{ N}$$

22. The cross-section of a steel crane hook is a trapezium with an inner side of 50 mm and outer side of 25 mm. The depth of section is 64 mm. The center of curvature of the section is at a distance of 64 mm from the inner edge of the section and the line of action of the load is 50 mm from the same edge. Determine the maximum load the hook can carry if the allowable stress is limited to 60 MPa.

VTU – June/July 2013 – 10 Marks; Dec. 2011 – 16 Marks; June/July 2009 – 16 Marks; Jan/Feb. 2005 – 16 Marks [similar July/Aug. 2005 – 10 Marks; Dec. 07/Jan. 08 – 12 Marks

Solution: $\sigma_{max} = 60$ MPa, $F = ?$

Based on given data, the crane hook is as shown in **Fig. 1.28(a).**

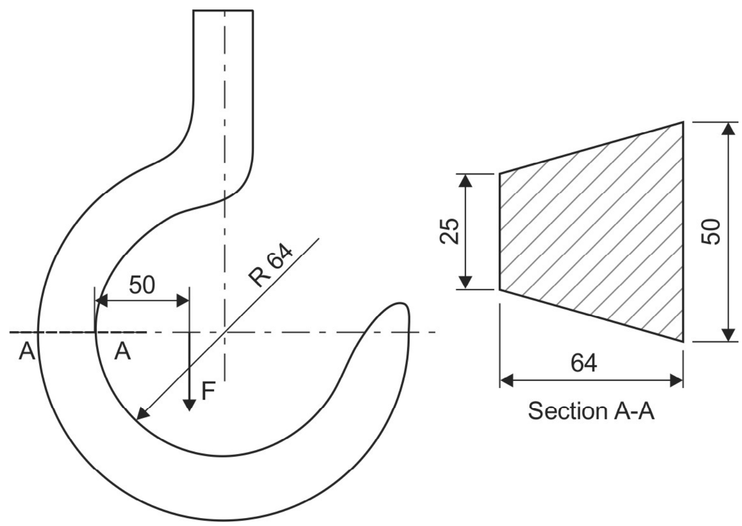

Fig. 1.28(a): Problem 22

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. f –Tb 1.3(a)/Pg 13, DHB**, we have

$b = 25$ mm, $b_1 = 50$ mm, $h = 64$ mm

$\therefore b_o = b_1 - b = 50 - 25 = 25$ mm

$$c = \frac{(3b + 2b_o)h}{3(2b + b_o)} = \frac{[(3 \times 25) + (2 \times 25)] \times 64}{3[(2 \times 25) + 25]} = 35.56 \text{ mm (from outer fiber)}$$

... **Fig. f – Tb 1.3(a)/Pg 13, DHB**

$\bar{x} = h - c = 64 - 35.56 = 28.44$ mm (from inner fiber)

b. Direct stress

$$\sigma_D = \frac{F}{A} = \frac{F}{2400} = (4.167 \times 10^{-4})\,F$$

where $$A = \frac{h(b_1 + b)}{2} = \frac{64 \times (50 + 25)}{2} = 2400 \text{ mm}^2$$

c. Bending stresses (σ_i, σ_o):

- From **Fig. 1.28(b),** we have

$c_1 = \bar{x} = 28.44$ mm, $R_i = 64$ mm $\quad c_2 = h - c_1 = 64 - 28.44 = 35.56$ mm $= c$

$R = R_i + c_1 = 64 + 28.44 = 92.44$ mm $\quad R_o = R_i + h = 64 + 64 = 128$ mm

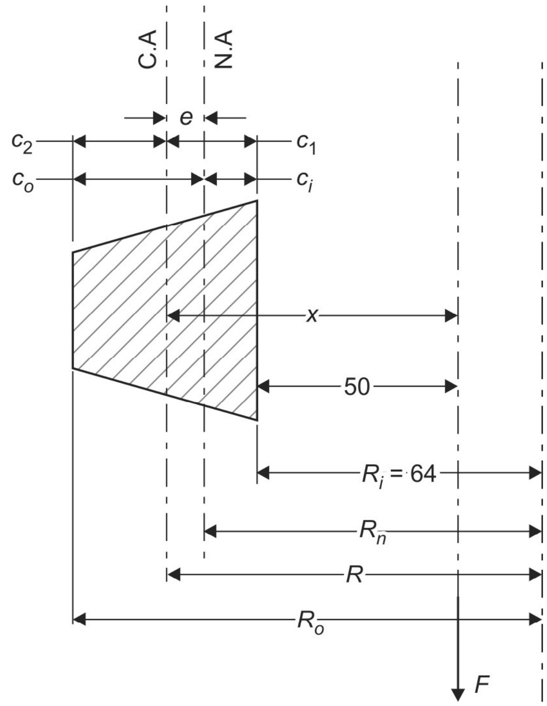

Fig. 1.28(b): Problem 22

- Moment $\quad M = F{\cdot}x = F(50 + \bar{x})$

$$= F \times (50 + 28.44)$$
$$= (78.44)F$$

- $e = R - R_n$ ... **Fig. 7 – Tb 10.1/Pg 163, DHB**

$$= R - \frac{A}{\left[\left(\frac{b_1(R+c_2) - b(R-c_1)}{h}\right)\ln\left(\frac{R+c_2}{R-c_1}\right)\right] - (b_1 - b)}$$

$$= 92.44 - \frac{2400}{\left[\left(\frac{50\times(92.44+35.56) - 25\times(92.44-28.44)}{64}\right)\times\ln\left(\frac{92.44+35.56}{92.44-28.44}\right)\right] - (50-25)}$$

$$= 3.51 \text{ mm}$$

Now, $\quad c_i = c_1 - e = 28.44 - 3.51 = 24.93$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 35.56 + 3.51 = 39.07$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(78.44)F\times 24.93}{2400\times 3.51\times 64} = (3.63\times 10^{-3})F \quad \text{... \textbf{10.1(b)/Pg 159, DHB}}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(78.44)F\times 39.07}{2400\times 3.51\times 128} = (-2.84\times 10^{-3})F \quad \text{... \textbf{10.2(c)/Pg 159, DHB}}$$

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = (4.167 \times 10^{-4})F + (3.63 \times 10^{-3})F$$
$$= (4.05 \times 10^{-3})F$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = (4.167 \times 10^{-4})F + (-2.84 \times 10^{-3})F$$
$$= (-2.42 \times 10^{-3})F$$

e. To find F:

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = (4.05 \times 10^{-3})F$$
$$60 = (4.05 \times 10^{-3})F$$
$$F = \frac{60}{4.05 \times 10^{-3}} = 14.81 \text{ kN}$$

23. Determine the safe load F that the frame of a punch press shown in Fig. 1.29(a) can carry considering the cross-section along A-A for an allowable tensile stress of 100 MPa. What is the stress at the outer fiber for the above load? What will be the stress at the inner fiber, if the beam is a straight beam for the above load?

VTU – July/Aug. 2004 – 16 Marks

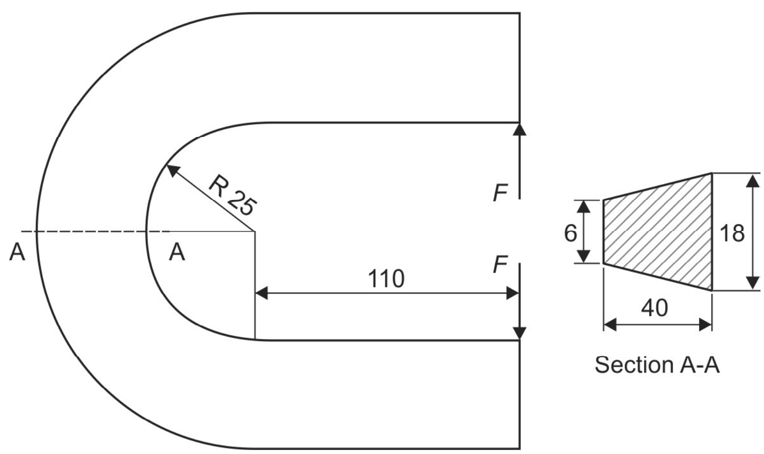

Fig. 1.29(a): Problem 23

Solution: Case 1: Curved beam: $\sigma_{max} = 100$ MPa, $F = ?$ $\sigma_o)_r = ?$;
Case 2: Straight beam: $\sigma_i)_r = ?$

Case 1: Curved beam:

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. f – Tb 1.3(a)/Pg 13, DHB**, we have

$$b = 6 \text{ mm}, b_1 = 18 \text{ mm}, h = 40 \text{ mm}$$
$$\therefore \quad b_o = b_1 - b = 18 - 6 = 12 \text{ mm}$$

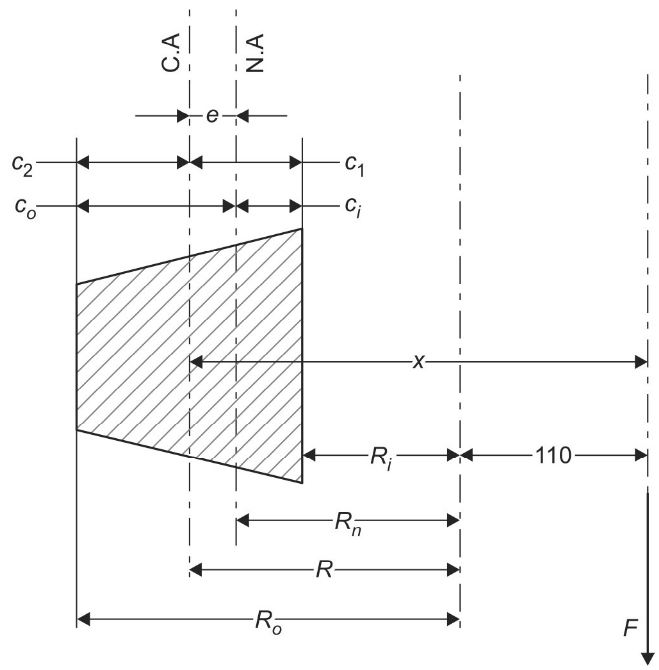

Fig. 1.29(b): Problem 23

$$c = \frac{(3b + 2b_o)h}{3(2b + b_o)} = \frac{[(3 \times 6) + (2 \times 12)] \times 40}{3[(2 \times 6) + 12]} = 23.33 \text{ mm (from outer fiber)}$$

... **Fig. f – Tb 1.3(a)/Pg 13, DHB**

$$\bar{x} = h - c = 40 - 23.33 = 16.67 \text{ mm (from inner fiber)}$$

b. Direct stress

$$\sigma_D = \frac{F}{A} = \frac{F}{480} = (2.08 \times 10^{-3})\, F$$

... **1.1(a)/Pg 2, DHB**

where $$A = \frac{h(b_1 + b)}{2} = \frac{40 \times (18 + 6)}{2} = 480 \text{ mm}^2$$

c. Bending stresses (σ_i, σ_o):

- From **Fig. 1.29(b),** we have
  $c_1 = \bar{x} = 16.67$ mm, $R_i = 25$ mm $\quad c_2 = h - c_1 = 40 - 16.67 = 23.33$ mm $= c$
  $R = R_i + c_1 = 25 + 16.67 = 41.67$ mm $\quad R_o = R_i + h = 25 + 40 = 65$ mm
- Moment $M = F \cdot x = F(110 + R_i + \bar{x})$
  $= F \times (110 + 25 + 16.67)$
  $= (151.67)F$
- $e = R - R_n$ ... **Fig. 7 – Tb 10.1/Pg 163, DHB**

$$= R - \frac{A}{\left[\left(\frac{b_1(R + c_2) - b(R - c_1)}{h}\right)\ln\left(\frac{R + c_2}{R - c_1}\right)\right] - (b_1 - b)}$$

$$= 41.67 - \frac{480}{\left[\left(\frac{18 \times (41.67 + 23.33) - 6 \times (41.67 - 16.67)}{40}\right) \times \ln\left(\frac{41.67 + 23.33}{41.67 - 16.67}\right)\right] - (18 - 6)}$$

$= 2.85$ mm

Now, $c_i = c_1 - e = 16.67 - 2.85 = 13.82$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 23.33 + 2.85 = 26.18$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(151.67)F \times 13.82}{480 \times 2.85 \times 25} = (61.29 \times 10^{-3})F$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(151.67)F \times 26.18}{480 \times 2.85 \times 65} = (-44.65 \times 10^{-3})F$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = (2.08 \times 10^{-3})F + (61.29 \times 10^{-3})F$$
$$= (63.37 \times 10^{-3})F$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = (2.08 \times 10^{-3})F + (-44.65 \times 10^{-3})F$$
$$= (-42.57 \times 10^{-3})F$$

e. To find F:

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = (63.37 \times 10^{-3})F$$
$$100 = (63.37 \times 10^{-3})F$$
$$F = \frac{100}{63.37 \times 10^{-3}} = 1578.03 \text{ N}$$

f. Resultant stress in the outer most fiber,

$$\sigma_o)_r = (-42.57 \times 10^{-3}) \times 1578.03 = -67.17 \text{ MPa}$$

Case 2: Straight beam:

- We know that

$$\frac{M}{I} = \frac{\sigma}{c} = \frac{E}{R}$$

$\Rightarrow$ $$\sigma = \sigma_b \frac{Mc}{I}$$ ... **1.1(b)/Pg 2, DHB**

But $$I = \frac{(6b^2 + 6bb_o + b_o^2)h^3}{36(2b + b_o)}$$ ... **Fig. f – Tb 1.3(a)/Pg 13, DHB**

$$= \frac{(6 \times 6^2 + 6 \times 6 \times 12 + 12^2)40^3}{36 \times (2 \times 6 + 12)}$$

$$I = 58666.67 \text{ mm}^4$$

Also $c_1 = \bar{x} = 16.67$ mm, $c_2 = h - c_1 = c = 40 - 16.67 = 23.33$ mm

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_1}{I} = \frac{(151.67 \times 1578.03) \times 16.67}{58666.67} = 68 \text{ MPa}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_2}{I} = \frac{-(151.67 \times 1578.03) \times 23.33}{58666.67} = -95.18 \text{ MPa}$$

g. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = (2.08 \times 10^{-3}) \times 1578.03 + 68 = 71.28 \text{ MPa}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = (2.08 \times 10^{-3}) \times 1578.03 - 95.18 = -91.89 \text{ MPa}$$

24. Crane hook of trapezoidal cross-section with an inner side of 120 mm and outer side of 60 mm. The depth of section is 90 mm. The center of curvature is at a distance of 120 mm from the inner edge of the section and the line of action of load is at a distance of 135 mm from the inner edge. Determine the safe load that the hook can carry if it is made of steel having an allowable stress of 90 MPa.

VTU – Dec. 2016/Jan. 2017 – 10 Marks

Solution: $\sigma_{max} = 100$ MPa, $F = ?$

Proceeding on similar lines to problem 23, we have

- $c = 50$ mm
- $\bar{x} = h - c = c_1 = 40$ mm
- Direct stress, $\sigma_D = (1.23 \times 10^{-4})F$
- $M = (175)F$
- $e = 4$ mm, $c_i = 36$ mm, $c_o = 54$ mm
- Bending stress at inner fiber, $\sigma_i = (1.62 \times 10^{-3})F$
- Bending stress at outer fiber, $\sigma_o = (-1.39 \times 10^{-3})F$
- Resultant stress in the inner most fiber, $\sigma_i)_r = (1.74 \times 10^{-3})F$
- Resultant stress in the outer most fiber, $\sigma_o)_r = (-1.27 \times 10^{-3})F$
- Load $F = 51.61$ kN

25. Determine a safe value for load *F* for a machine element loaded as shown in Fig. 1.30(a), limiting the maximum normal stress induced on the cross-section X–X to 120 MPa.

VTU – Jan/Feb 2006–10 Marks

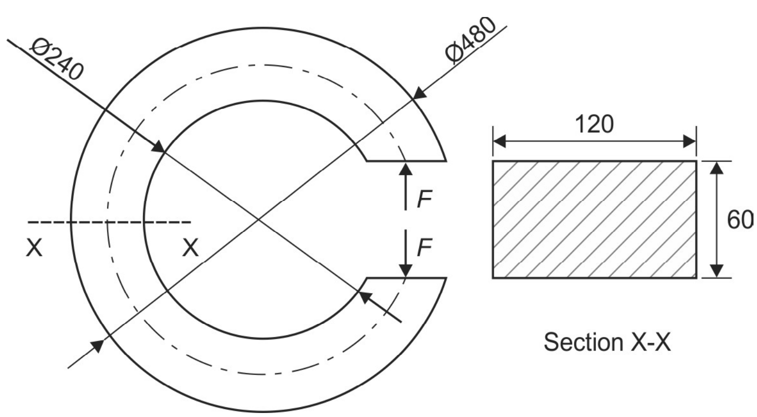

Fig. 1.30(a): Problem 25

Solution: $\sigma_{max} = 120$ MPa, $F = ?$

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. 1 – Tb 10.1/Pg 162, DHB**, we have

$$b = 60 \text{ mm}, h = 120 \text{ mm}$$

$$\bar{x} = \frac{h}{2} = \frac{120}{2} = 60 \text{ mm}$$

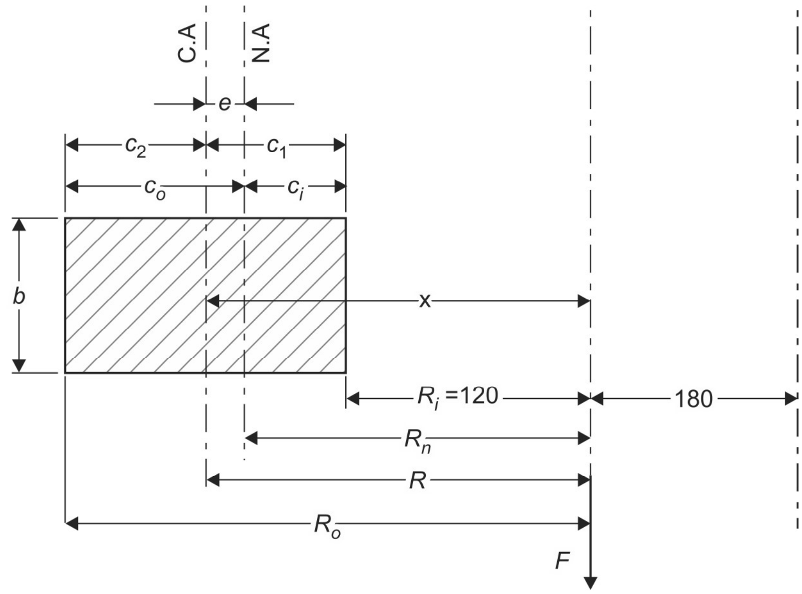

Fig. 1.30(b): Problem 25

b. Direct stress $\sigma_D = \dfrac{F}{A} = \dfrac{F}{120 \times 60} = (1.39 \times 10^{-4})\,F$ ... **1.1(a)/Pg 2, DHB**

c. Bending stresses (σ_i, σ_o):

- From **Fig. 1.30(b),** we have

  $c_1 = c_2 = 60 \text{ mm} = \bar{x}$

  $R_i = 120 \text{ mm}, \quad R_o = 240 \text{ mm}$

  $R = R_i + c_1 = 120 + 60 = 180 \text{ mm} \Rightarrow D = 360 \text{ mm}$

- Moment $M = F{\cdot}x = F \times D = F \times 360 = (360)\,F$
- $e = R - R_n$ ... **Fig. 1 – Tb 10.1/Pg 162, DHB**

$$= R - \frac{h}{\ln\left(\dfrac{R+c}{R-c}\right)} = 180 - \frac{120}{\ln\left(\dfrac{180+60}{180-60}\right)}$$

$= 6.88$ mm

Now, $c_i = c_1 - e = 60 - 6.88 = 53.12$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 60 + 6.88 = 66.88$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(360)F \times 53.12}{(120 \times 60) \times 6.88 \times 120} = (3.22 \times 10^{-3})\,F$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(360)F \times 66.88}{(120 \times 60) \times 6.88 \times 240} = (-2.03 \times 10^{-3})\,F$$

... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = (1.39 \times 10^{-4})F + (3.22 \times 10^{-3})F$$
$$= (3.36 \times 10^{-3})F$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = (1.39 \times 10^{-4})F + (-2.03 \times 10^{-3})F$$
$$= (-1.89 \times 10^{-3})F$$

e. To find F:

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = (3.36 \times 10^{-3})F$$
$$120 = (3.36 \times 10^{-3})F$$
$$F = \frac{120}{3.36 \times 10^{-3}} = 35.72 \text{ kN}$$

26. A cast iron frame of a small punch is as shown in Fig. 1.31(a). Determine the force that will produce a maximum tensile stress of 60 MPa at (a) Section A–A, (b) Section B–B. Also find the corresponding compressive stress in the section.

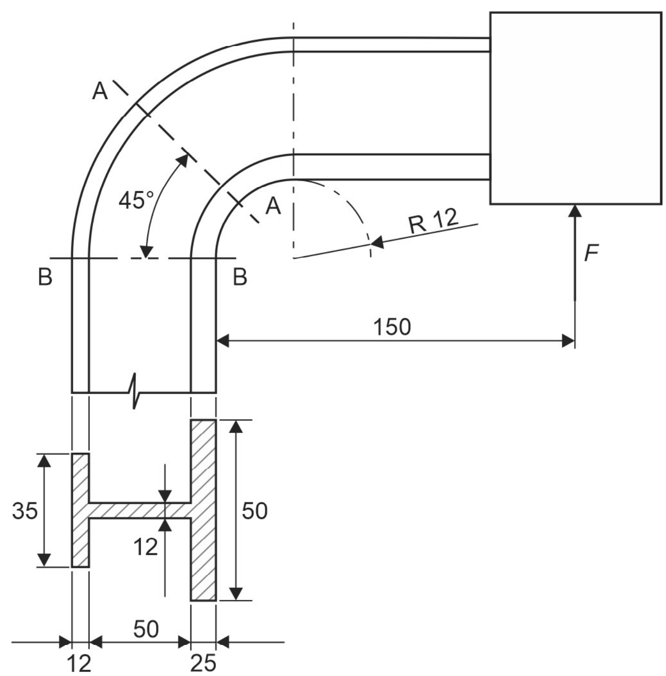

Fig. 1.31(a): Problem 26

Solution: $R_i = 12$ mm, $\sigma_{max} = 60$ MPa, $F = ?$

a. To find $\bar{x}$:

$a_1 = 50 \times 25 = 1250 \text{ mm}^2$ $\quad a_2 = 12 \times 50 = 600 \text{ mm}^2$ $\quad a_3 = 35 \times 12 = 420 \text{ mm}^2$

$x_1 = 25/2 = 12.5$ mm $\quad x_2 = (50/2) + 25 = 50$ mm $\quad x_3 = (12/2) + 75 = 81$ mm

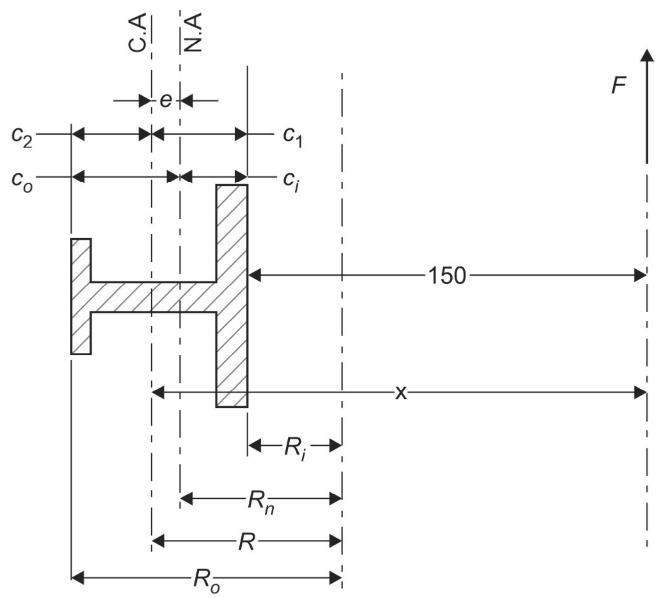

Fig. 1.31(b): Problem 26

$$A = \Sigma a = a_1 + a_2 + a_3 = 2270 \text{ mm}^2$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{(1250 \times 12.5) + (600 \times 50) + (420 \times 81)}{(1250 + 600 + 420)} = 35.1 \text{ mm}$$

b. Direct stress @ A–A $\sigma_D = \dfrac{F\cos\theta}{A} = \dfrac{F \times \cos(45)}{2270} = \left(3.12 \times 10^{-4}\right)F$ (at Section A–A)

@ B–B $\sigma_D = \dfrac{F}{A} = \dfrac{F}{2270} = \left(4.41 \times 10^{-4}\right)F$ (at Section B–B)

c. Bending stresses (σ_i, σ_o)

- Comparing the given cross-section with **Fig. 10 – Tb 10.1/Pg 164, DHB,** we have

  $B = 50$ mm, $d = 25$ mm, $d_1 = 12$ mm, $b_1 = 35$ mm, $b_2 = 12$ mm

  $H = 25 + 50 + 12 = 87$ mm

- From **Fig. 1.31(b)**, we have

  $c_1 = \bar{x} = 35.1$ mm $\quad c_2 = H - c_1 = 87 - 35.1 = 51.9$ mm, $\quad R_i = 12$ mm

  $R = R_i + c_1 = 12 + 35.1 = 47.1$ mm

  $R_o = R_i + H = 12 + 87 = 99$ mm

- $e = R - R_n$ ... **Fig. 10 – Tb 10.1/Pg 164, DHB**

$$= R - \frac{A}{B\ln\left(\dfrac{R+d-c_1}{R-c_1}\right) + b_2\ln\left(\dfrac{R+c_2-d_1}{R+d-c_1}\right) + b_1\ln\left(\dfrac{R+c_2}{R+c_2-d_1}\right)}$$

$$= 47.1 - \frac{2270}{\left[50\ln\left(\dfrac{47.1+25-35.1}{47.1-35.1}\right)\right] + \left[12\ln\left(\dfrac{47.1+51.9-12}{47.1+25-35.1}\right)\right] + \left[35\ln\left(\dfrac{47.1+51.9}{47.1+51.9-12}\right)\right]}$$

$= 15.17$ mm

Now $\quad c_i = c_1 - e = 35.1 - 15.17 = 19.93$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 51.9 + 15.17 = 67.07$ mm ... **10.1(d)/Pg 159, DHB**

Section A–A:

- Moment $M = F \cdot x = F[(150 - R_i) + R\cos\theta]$
  $= F\{(150 - 12) + [47.1 \times \cos(45)]\}$
  $= (171.30)\ F$
- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(171.30)F \times 19.93}{2270 \times 15.17 \times 12} = \left(8.26 \times 10^{-3}\right)F \qquad \text{... 10.1(b)/Pg 159, DHB}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(171.30)F \times 67.07}{2270 \times 15.17 \times 99} = \left(-3.37 \times 10^{-3}\right)F \qquad \text{... 10.1(c)/Pg 159, DHB}$$

d. To find F:

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = (3.12 \times 10^{-4})F + (8.26 \times 10^{-3})F$$
$$60 = (8.572 \times 10^{-3})F$$
$$F = 6999.53 \text{ N}$$

e. To find σ_0:

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = (3.12 \times 10^{-4})F + (-3.37 \times 10^{-3})F$$
$$= -(3.078 \times 10^{-3}) \times 6999.53$$
$$= -21.40 \text{ MPa}$$

Section B–B:

- Moment $M = F \cdot x = F[(150 + \bar{x})$
  $= F(150 + 35.1)$
  $= (185.1)\ F$
- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(185.1)F \times 19.93}{2270 \times 15.17 \times 12} = \left(8.93 \times 10^{-3}\right)F \qquad \text{... 10.1(b)/Pg 159, DHB}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(185.1)F \times 67.07}{2270 \times 15.17 \times 99} = \left(-3.64 \times 10^{-3}\right)F \qquad \text{... 10.1(c)/Pg 159, DHB}$$

f. To find F:

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = (4.41 \times 10^{-4})F + (8.93 \times 10^{-3})F$$
$$60 = (9.371 \times 10^{-3})F$$
$$F = 6402.73 \text{ N}$$

g. To find σ_o:

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = (4.41 \times 10^{-4})F + (-3.64 \times 10^{-3})F$$
$$= -(3.2 \times 10^{-3}) \times 6402.73$$
$$= -20.48 \text{ MPa}$$

Note: Solution at section B–B may also be obtained by substituting $\theta = 0°$ in formula of section A–A.

Type III: Problems on Dimensions

27. Taking a permissible stress in the material as 100 MPa, estimate the thickness '*t*' for the cross-section shown in Fig. 1.32(a). Determine the maximum stress induced in the material of the member shown considering curved beam effect. By how much the factor of safety is reduced if the ultimate strength of the material is 440 MPa.

VTU – July/August 2002 – 20 Marks

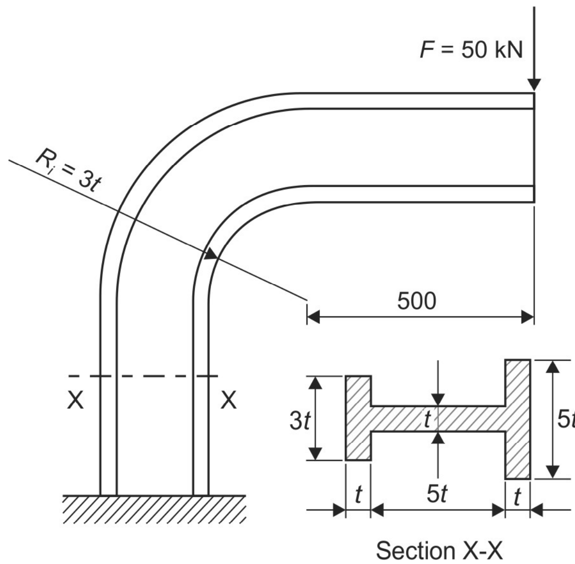

Fig. 1.32(a): Problem 27

Solution: $\sigma_{min} = -100$ MPa, $F = -50 \times 10^3$ N, $t = ?$, $\sigma_{max} = ?$, $n = ?$

a. To find $\bar{x}$:

$a_1 = 5t \times t = 5t^2$ $\quad a_2 = 5t \times t = 5t^2$ $\quad a_3 = 3t \times t = 3t^2$

$x_1 = t/2 = 0.5t$ $\quad x_2 = (5t/2) + t = 3.5t$ $\quad x_3 = (t/2) + 5t + t = 6.5t$

$A = \Sigma a = a_1 + a_2 + a_3 = 13t^2$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{(5t^2 \times 0.5t) + (5t^2 \times 3.5t) + (3t^2 \times 6.5t)}{(5t^2 + 5t^2 + 3t^2)} = 3.04t$$

b. Direct stress $\sigma_D = \dfrac{F}{A} = \dfrac{-50 \times 10^3}{13t^2} = \dfrac{-3846.15}{t^2}$ MPa (compressive)

... 1.1(a)/Pg 2, DHB

c. Bending stresses (σ_i, σ_o):

- Comparing the given cross-section with **Fig. 10 – Tb 10.1/Pg 164, DHB**, we have

$$B = 5t,\ b_1 = 3t,\ d = d_1 = t,\ b_2 = t,\ H = t + 5t + t = 7t$$

- From **Fig. 1.32(b)**, we have

$$c_1 = \bar{x} = 3.04t \qquad R_i = 3t$$
$$c_2 = H - c_1 = 7t - 3.04t = 3.96t$$
$$R = c_1 + R_i = 3.04t + 3t = 6.04t$$
$$R_o = R_i + H = 3t + 7t = 10t$$

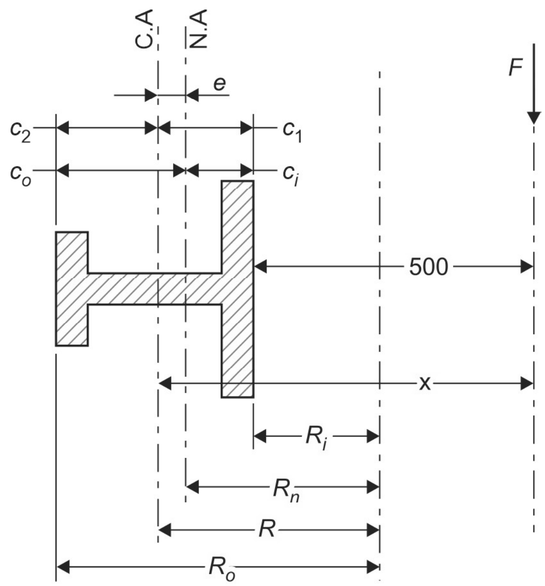

Fig. 1.32(b): Problem 27

- Moment $\quad M = F\cdot x = F(500 + R_i + \bar{x})$

$$= 50 \times 10^3 \times [500 + 3t + (3.04)t]$$
$$= 25 \times 10^6 + (302 \times 10^3)t$$

- $e = R - R_n$ ... **Fig. 10 – Tb. 10.1/Pg 164, DHB**

$$= R - \frac{A}{B\ln\left(\frac{R+d-c_1}{R-c_1}\right) + b_2 \ln\left(\frac{R+c_2-d_1}{R+d-c_1}\right) + b_1 \ln\left(\frac{R+c_2}{R+c_2-d_1}\right)}$$

$$= 6.04t - \frac{13t^2}{\left[5t\ln\left(\frac{6.04t+t-3.04t}{6.04t-3.04t}\right)\right] + \left[t\ln\left(\frac{6.04t+3.96t-t}{6.04t+t-3.04t}\right)\right] + \left[3t\ln\left(\frac{6.04t+3.96t}{6.04t+3.96t-t}\right)\right]}$$

$= 0.973t$ mm

Now, $c_i = c_1 - e = 3.04t - 0.973t = (2.067)t$ ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 3.96t + 0.973t = (4.933)t$ ... **10.1(d)/Pg 159, DHB**

Here inner fibers are in compression while outer fibers are in tension.

- Bending stress at inner fiber,

$$\sigma_i = \frac{-Mc_i}{AeR_i} = \frac{-[25\times 10^6 + (302\times 10^3)t]\times 2.067t}{13t^2 \times 0.973t \times 3t}$$ ... **10.1(b)/Pg 159, DHB**

$$= \frac{-51.68\times 10^6 - (624234)t}{37.95t^3}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{Mc_o}{AeR_o} = \frac{[25\times 10^6 + (302\times 10^3)t]\times 4.933t}{13t^2 \times 0.973t \times 10t}$$ ... **10.1(c)/Pg 159, DHB**

$$= \frac{123.33\times 10^6 + (1.49\times 10^6)t}{126.49t^3}$$

d. Resultant stresses:
- Resultant stress in the inner most fiber,
$$\sigma_i)_r = \sigma_D + \sigma_i$$
- Resultant stress in the outer most fiber,
$$\sigma_o)_r = \sigma_D + \sigma_o$$

e. To find t:
- Since minimum stress occurs in the inner fiber
$$\sigma_i)_r = \sigma_D + \sigma_i$$
$$-100 = \frac{-3846.15}{t^2} + \frac{-51.68\times10^6 - (624234)t}{37.95t^3}$$
$$-3795t^3 = (-146 \times 10^3)t - 51.68 \times 10^6 - (624234)t$$
$$= (-770.2 \times 10^3)t - 51.68 \times 10^6$$
$$3795t^3 - (770.2 \times 10^3)t - 51.68 \times 10^6 = 0$$
$$t = 26.70 \text{ mm} \approx 27 \text{ mm}$$
- Since maximum stress occurs in the outer fiber
$$\sigma_o)_r = \sigma_D + \sigma_o$$
$$= \frac{-3846.15}{27^2} + \frac{123.33\times10^6 + (1.49\times10^6)\times27}{126.49\times27^3}$$
$$= 60.42 \text{ MPa} = \sigma_{max}$$

Factor of safety $n = \dfrac{400}{60.42} = 6.62$

28. An offset bar is loaded as shown in Fig. 1.33(a). Neglecting the weight of the bar, determine the maximum offset 'L' if allowable stress in tension is limited to 60 MPa.

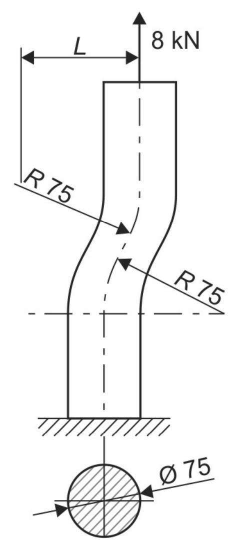

Fig. 1.33(a): Problem 28

Solution: $\sigma_{max} = 60$ MPa, $D = 75$ mm, $F = 8000$ N, $L = ?$

a. To find $\bar{x}$: $\quad \bar{x} = \dfrac{D}{2} = \dfrac{75}{2} = 37.5$ mm

b. Direct stress $\quad \sigma_D = \dfrac{F}{A} = \dfrac{8000}{4417.8} = 1.81$ MPa

... **1.1(a)/Pg 2, DHB**

where $\quad A = \dfrac{\pi D^2}{4} = \dfrac{\pi\times75^2}{4} = 4417.86 \text{ mm}^2$

c. Bending stresses (σ_i, σ_o)
- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB**, we have
$$D = 70 \text{ mm}, c_1 = c_2 = 37.5 \text{ mm} = \bar{x}$$
- From **Fig. 1.33(b)**, we have

$c_1 = c_2 = 37.5 \text{ mm} = \bar{x}$ $\qquad R = 75$ mm (data)

$R_i = R - c_1 = 75 - 37.5 = 37.5$ mm $\qquad R_o = R_i + D = 37.5 + 75 = 112.5$ mm
- Moment $\quad M = F{\cdot}R = F{\cdot}L = (8000)L$
- $e = R - R_n$ ... **Fig. 2 – Tb. 10.1/Pg 162, DHB**
$$= R = \frac{c^2/2}{R-\sqrt{R^2-c^2}} = 75 - \frac{37.5^2/2}{75-\sqrt{75^2-37.5^2}}$$
$$= 5.02 \text{ mm}$$

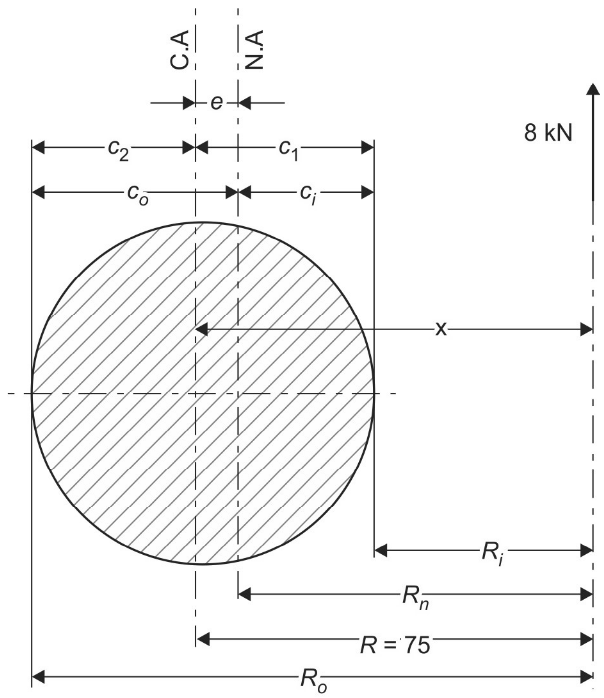

Fig. 1.33(b): Problem 28

Now, $c_i = c_1 - e = 37.5 - 5.02 = 32.48$ mm ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 37.5 + 5.02 = 42.52$ mm ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(8000\,L) \times 32.48}{4417.86 \times 5.02 \times 37.5} = (0.3124)L$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(8000\,L) \times 42.52}{4417.86 \times 5.02 \times 112.5} = (-0.1363)L$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = 1.81 + (0.3124)\,L$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = 1.81 + (-0.1363)\,L$$

e. To find L:

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = 1.81 + (0.3124)\,L$$

$$60 = 1.81 + (0.3124)\,L$$

$$58.91 = (0.3124)\,L$$

$$L = 186.27 \text{ mm}$$

29. Determine the dimensions of the curved bar as shown in Fig. 1.34(a). Assume σ_{yt} = 400 MPa and factor of safety as 3.5.

VTU – June/July 2011 – 12 Marks; June/July 2016 – 10 Marks

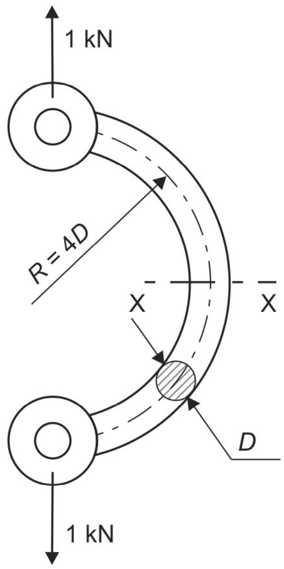

Fig. 1.34(a): Problem 29

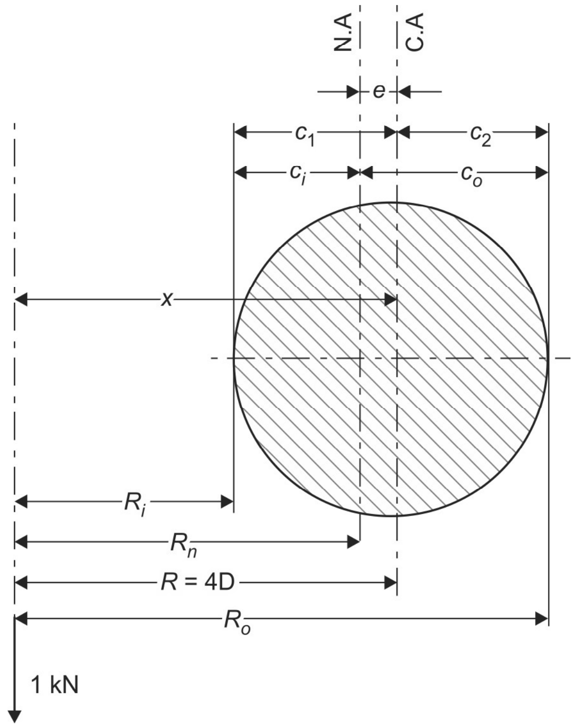

Fig. 1.34(b): Problem 29

Solution: σ_{yt} = 400 MPa, n = 3.5, F = 1000 N, $R = 4D$, D = ?

Here, $$\sigma = \frac{\sigma_{yt}}{n} = \frac{400}{3.5} = 114.29 \text{ MPa}$$

a. To find $\bar{x}$: $$\bar{x} = \frac{D}{2} = 0.5D$$

b. Direct stress $$\sigma_D = \frac{F}{A} = \frac{1000}{0.7854D^2} = \frac{1273.24}{D^2} \quad \text{... 1.1(a)/Pg 2, DHB}$$

where $$A = \frac{\pi D^2}{4} = 0.78534D^2$$

c. Bending stresses (σ_i, σ_0)

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB**, we have
$$D = D,\ c_1 = c_2 = 0.5D = \bar{x}$$
- From **Fig. 1.34(b)**, we have
$$c_1 = c_2 = 0.5D = \bar{x} \qquad R = 4D$$
$$R_i = R - c_1 = 4D - 0.5D = 3.5D$$
$$R_o = R_i + D = 3.5D + D = 4.5D$$
- Moment $M = F{\cdot}x = F{\cdot}R = 1000 \times 4D = (4000)D$
- $e = R - R_n$ ... **Fig. 2 – Tb 10.1/Pg 162 DHB**
$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 4D - \frac{(0.5D)^2/2}{4D - \sqrt{(4D)^2 - (0.5D)^2}}$$
$$= 0.0157D$$

Now, $c_i = c_1 - e = 0.5D - 0.0157D = 0.4843D$ ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 0.5D + 0.0157D = 0.5157D$ ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(4000D)\times 0.4843D}{0.7854D^2 \times 0.0157D \times 3.5D} = \frac{44.88\times 10^3}{D^2} \quad \text{... 10.1(b)/Pg 159, DHB}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(4000D)\times 0.5157D}{0.7854D^2 \times 0.0157D \times 4.5D} = \frac{-37.18\times 10^3}{D^2} \quad \text{... 10.1(c)/Pg 159, DHB}$$

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = \frac{1273.24}{D^2} + \frac{44.88\times 10^3}{D^2}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = \frac{1273.24}{D^2} + \frac{-37.18\times 10^3}{D^2}$$

e. To find *L:*

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = \frac{1273.24}{D^2} + \frac{44.88\times 10^3}{D^2}$$

$$114.29 = \frac{1273.24}{D^2} + \frac{44.88\times 10^3}{D^2}$$

$$D = 20.10 \text{ mm}$$

30. A link of S-shape made of steel bar is shown in Fig. 1.35(a). It is made of steel 45C8 with σ_{yt} = 380 MPa and factor of safety as 4.5. Calculate the dimensions of the link.

VTU – June/July 2013 – 10 Marks

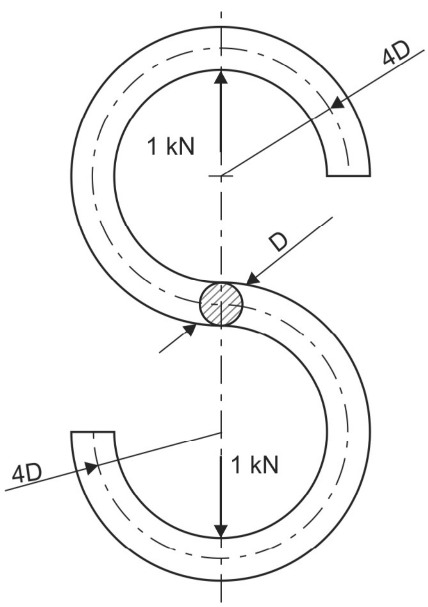

Fig. 1.35(a): Problem 30

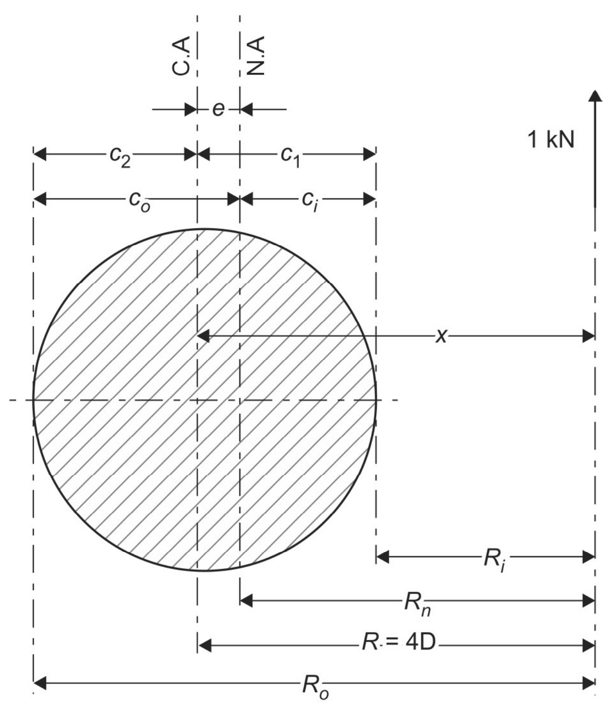

Fig. 1.35(b): Problem 30

Solution: $\sigma_{yt} = 380$ MPa, $n = 4.5$, $F = 1000$ N, $R = 4D$, $D = ?$

Here, $$\sigma = \frac{\sigma_{yt}}{n} = \frac{380}{4.5} = 84.85 \text{ MPa}$$

a. To find $\bar{x}$: $$\bar{x} = \frac{D}{2} = 0.5D$$

b. Direct stress $$\sigma_D = \frac{F}{A} = \frac{1000}{0.7854D^2} = \frac{1273.24}{D^2}$$ ... **1.1(a)/Pg 2, DHB**

where $$A = \frac{\pi D^2}{4} = 0.7854D^2$$

c. Bending stresses (σ_i, σ_0)

- Comparing the given cross-section with **Fig. 2 – Tb 10.1/Pg 162, DHB**, we have
$$D = D,\ c_1 = c_2 = 0.5D = \bar{x}$$
- From **Fig. 1.35(b)**, we have
$$c_1 = c_2 = 0.5D = \bar{x} \qquad R = 4D \qquad R_i = R - c_1 = 4D - 0.5D = 3.5D$$
$$R_o = R_i + D = 3.5D + D = 4.5D$$
- Moment $M = F{\cdot}x = F{\cdot}R = 1000 \times 4D = (4000)D$
- $e = R - R_n$ ... **Fig. 2 – Tb 10.1/Pg 162 DHB**
$$= R - \frac{c^2/2}{R - \sqrt{R^2 - c^2}} = 4D - \frac{(0.5D)^2 / 2}{4D - \sqrt{(4D)^2 - (0.5D)^2}}$$
$$= 0.0157D$$

Now, $$c_i = c_1 - e = 0.5D - 0.0157D = 0.4843D$$ ... **10.1(d)/Pg 159, DHB**
$$c_o = c_2 + e = 0.5D + 0.0157D = 0.5157D$$ ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,
$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(4000D) \times 0.4843D}{0.7854D^2 \times 0.0157D \times 3.5D} = \frac{44.88 \times 10^3}{D^2}$$ ... **10.1(b)/Pg 59, DHB**
- Bending stress at outer fiber,
$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(4000D) \times 0.5157D}{0.7854D^2 \times 0.0157D \times 4.5D} = \frac{-37.18 \times 10^3}{D^2}$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,
$$\sigma_i)_r = \sigma_D + \sigma_i = \frac{1273.24}{D^2} + \frac{44.88 \times 10^3}{D^2}$$
- Resultant stress in the outer most fiber,
$$\sigma_o)_r = \sigma_D + \sigma_o = \frac{1273.24}{D^2} + \frac{-37.18 \times 10^3}{D^2}$$

e. To find D:
Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = \frac{1273.24}{D^2} + \frac{44.88 \times 10^3}{D^2}$$

$$84.45 = \frac{1273.24}{D^2} + \frac{44.88 \times 10^3}{D^2}$$

$$D = 23.38 \text{ mm}$$

31. A crane hook of trapezoidal section is shown in Fig. 1.36(a). Through the center of curvature, a load of 60 kN is applied on the hook. Determine the dimensions of the section, if the maximum stress is not to exceed 80 MPa.

VTU – Dec. 2013/Jan. 2014 – 15 Marks

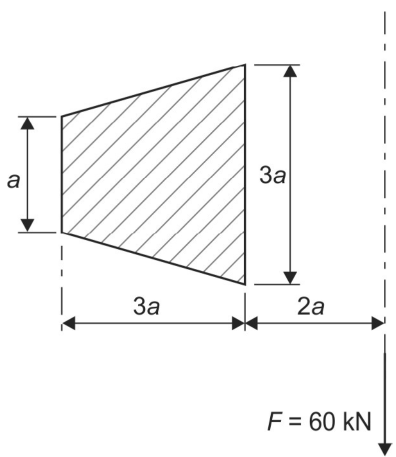

Fig. 1.36(a): Problem 31

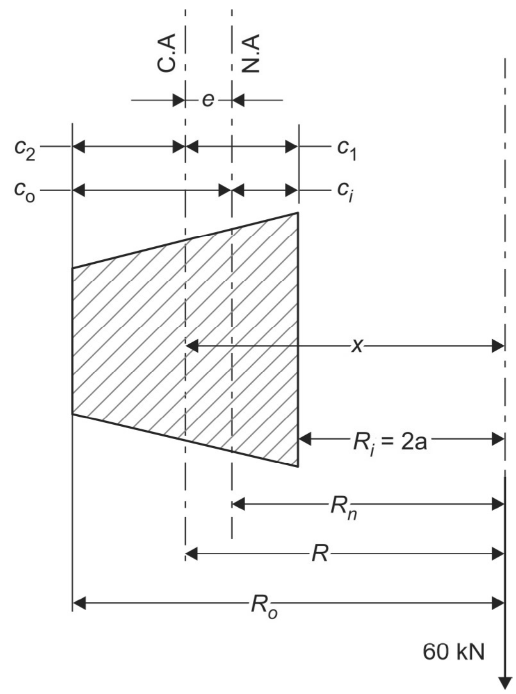

Fig. 1.36(b): Problem 31

Solution: $F = 60 \times 10^3$ N, $\sigma_{max} = 80$ MPa, $a = ?$

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. f – Tb 1.3(a)/Pg 13, DHB**, we have

$$b = a,\ b_1 = 3a,\ h = 3a,\ R_i = 2a$$

$$\therefore \quad b_o = b_1 - b = 3a - a = 2a$$

$$c = \frac{(3b + 2b_o)h}{3(2b + b_o)} = \frac{[(3 \times a) + (2 \times 2a)] \times 3a}{3 \times [(2 \times a) + 2a]} = 1.75a \text{ (from outer fiber)}$$

... **Fig. f – Tb. 1.3(a)/Pg 13, DHB**

$$\bar{x} = h - c = 3a - 1.75a = 1.25a \text{ (from inner fiber)}$$

b. Direct stress $\quad \sigma_D = \dfrac{F}{A} = \dfrac{60 \times 10^3}{6a^2} = \dfrac{10 \times 10^3}{a^2}$ ... **1.1(a)/Pg 2, DHB**

where $\quad A = \dfrac{h(b_1 + b)}{2} = \dfrac{3a \times (3a + a)}{2} = 6a^2$

c. Bending stresses (σ_i, σ_o)

- From **Fig. 1.36(b)**, we have

$$c_1 = \bar{x} = 1.25a \qquad R_i = 2a \qquad c_2 = h - c_1 = 3a - 1.25a = 1.75a = c$$

$$R = R_i + c_1 = 2a + 1.25a = 3.25a$$

$$R_o = R_i + h = 2a + 3a = 5a$$

- Moment

$$M = F \cdot x = F(R_i + \bar{x}) = 60 \times 10^3 \times (2a + 1.25a) = (1.95 \times 10^3)a$$

- $e = R - R_n$ ... **Fig. 7 – Tb 10.1/Pg 163 DHB**

$$= R - \frac{A}{\left[\left(\frac{b_1(R+c_2) - b(R-c_1)}{h}\right)\ln\left(\frac{R+c_2}{R-c_1}\right)\right] - (b_1 - b)}$$

$$= 3.25a - \frac{6a^2}{\left[\left(\frac{3a\times(3.25a+1.75a) - a\times(3.25a-1.25a)}{3a}\right)\times\ln\left(\frac{3.25a+1.75a}{3.25a-1.25a}\right)\right] - (3a-a)}$$

$$= (0.21)a$$

Now, $c_i = c_1 - e = 1.25a - 0.21a = (1.04)a$ ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 1.75a + 0.21a = (1.96)a$ ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(195\times10^3)\times(1.04)a}{6a\times(0.21)a\times 2a} = \frac{80.48\times10^3}{a^2}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(195\times10^3)\times(1.96)a}{6a\times(0.21)a\times 5a} = \frac{-60.67\times10^3}{a^2}$$ ... **10.1(c)/Pg 159, DHB**

d. Resultant stresses

- Resultant stress in the inner most fiber,

$$\sigma_i)_r = \sigma_D + \sigma_i = \frac{10\times10^3}{a^2} + \frac{80.48\times10^3}{a^2} = \frac{90.48\times10^3}{a^2}$$

- Resultant stress in the outer most fiber,

$$\sigma_o)_r = \sigma_D + \sigma_o = \frac{10\times10^3}{a^2} + \frac{(-60.67\times10^3)}{a^2} = \frac{-50.67\times10^3}{a^2}$$

e. To find *a:*

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = \frac{90.48\times10^3}{a^2}$$

$$80 = \frac{90.48\times10^3}{a^2}$$

$$a = 33.63 \text{ mm} \approx 34 \text{ mm}$$

Thus the dimensions of the section are:

$c_1 = \bar{x} = 1.25 \times 34 = 42.5$ mm $\quad R_i = 2 \times 34 = 68$ mm

$c_2 = 1.75 \times 34 = 59.5$ mm $\quad R = 3.25 \times 34 = 110.5$ mm

$R_o = 5 \times 34 = 170$ mm

32. The U-section frame is to resist a straightening load of 125 kN as shown in Fig. 1.37(a). The material of the frame has a permissible stress of 65 MPa. Determine the dimensions of the frame.

VTU – June/July 2014 – 12 Marks

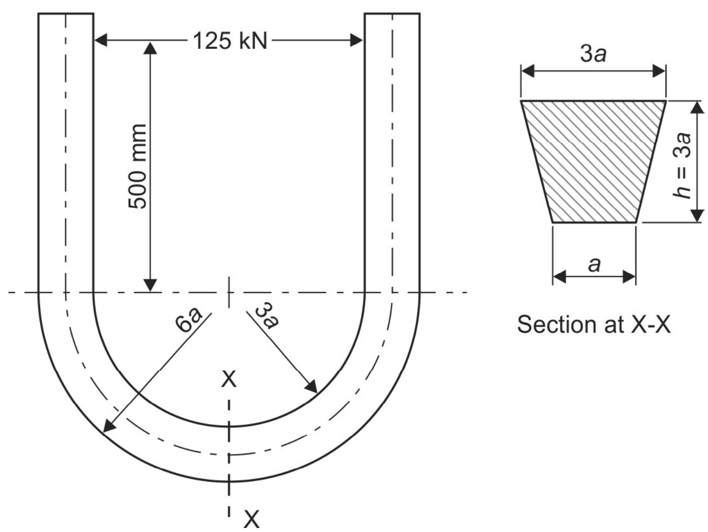

Fig. 1.37(a): Problem 32

Solution: $F = 125 \times 10^3$ N, $\sigma_{max} = 65$ MPa, a = ?

a. To find $\bar{x}$:

- Comparing the given cross-section with **Fig. f – Tb 1.3(a)/Pg 13, DHB**, we have

$$b = a,\ b_1 = 3a,\ h = 3a,\ R_i = 3a,\ R_o = 6a$$

$$\therefore \quad b_o = b_1 - b = 3a - a = 2a$$

$$c = \frac{(3b + 2b_o)h}{3(2b + b_o)} = \frac{[(3 \times a) + (2 \times 2a)] \times 3a}{3 \times [(2 \times a) + 2a]} = 1.75a \text{ (from outer fiber)}$$

... **Fig. f – Tb 1.3(a)/Pg 13, DHB**

$$\bar{x} = h - c = 3a - 1.75a = 1.25a \text{ (from inner fiber)}$$

b. Direct stress $\quad \sigma_D = \dfrac{F}{A} = \dfrac{125 \times 10^3}{6a^2} = \dfrac{20.83 \times 10^3}{a^2}$ ... **1.1(a)/Pg 2, DHB**

where $\quad A = \dfrac{h(b_1 + b)}{2} = \dfrac{3a \times (3a + a)}{2} = 6a^2$

c. Bending stresses (σ_i, σ_o)

- From **Fig. 1.36(b)**, we have

$c_1 = \bar{x} = 1.25a \quad R_i = 3a \quad c_2 = h - c_1 = 3a - 1.25a = 1.75a = c$

$R = R_i + c_1 = 3a + 1.25a = 4.25a$

$R_o = R_i + h = 3a + 3a = 6a$

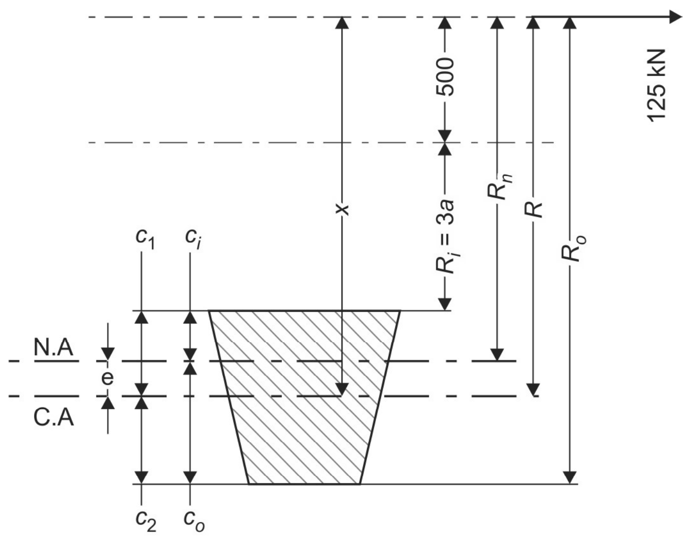

Fig. 1.37(b): Problem 32

- Moment $M = F \cdot x = F(\text{offset} + R_i + \bar{x})$
  $= 125 \times 10^3 \times [500 + 3a + 1.25a]$
  $= 62.5 \times 10^6 + (531250)a$
- $e = R - R_n$ ... **Fig. 7 – Tb 10.1/Pg 163 DHB**

$$= R - \frac{A}{\left[\left(\frac{b_1(R+c_2) - b(R-c_1)}{h}\right)\ln\left(\frac{R+c_2}{R-c_1}\right)\right] - (b_1 - b)}$$

$$= 4.25a - \frac{6a^2}{\left[\left(\frac{3a \times (4.25a + 1.75a) - a \times (4.25a - 1.25a)}{3a}\right) \times \ln\left(\frac{4.25a + 1.75a}{4.25a - 1.25a}\right)\right] - (3a - a)}$$

$= (0.16)a$

Now, $c_i = c_1 - e = 1.25a - 0.16a = (1.09)a$ ... **10.1(d)/Pg 159, DHB**

$c_o = c_2 + e = 1.75a + 0.16a = (1.91)a$ ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{[62.5 \times 10^6 + (531250)a] \times (1.09)a}{6a \times (0.16)a \times 3a}$$ ... **10.1(b)/Pg 159, DHB**

$$= \frac{68.13 \times 10^6 + (579.06 \times 10^3)a}{2.88a^3}$$

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-[62.5 \times 10^6 + (531250)a] \times (1.91)a}{6a \times (0.16)a \times 6a}$$ ... **10.1(c)/Pg 159, DHB**

$$= \frac{-119.38 \times 10^6 - (101468.75)a}{5.76a^3}$$

d. Resultant stresses

- Resultant stress in the inner most fiber, $\sigma_{i})_r = \sigma_D + \sigma_i$
- Resultant stress in the outer most fiber, $\sigma_{o})_r = \sigma_D + \sigma_o$

e. To find *a:*

Since maximum stress occurs in the inner fiber,

$$\sigma_i)_r = \frac{20.83\times10^3}{a^2} + \frac{68.13\times10^6 + (579.06\times10^3)a}{2.88a^3}$$

$$65 = \frac{20.83\times10^3}{a^2} + \frac{68.13\times10^6 + (579.06\times10^3)a}{2.88a^3}$$

$$187.2a^3 = (60\times10^3)a + 68.13\times10^6 + (579.06\times10^3)a$$

$$0 = 187.2a^3 - (639.06\times10^3)a - 68.13\times10^6$$

$$a = 87.13 \text{ mm}$$

Thus the dimensions of the section are:

$c_1 = \bar{x} = 1.25 \times 87.13 = 108.91$ mm $\quad c_2 = 1.75 \times 87.13 = 152.48$ mm

$R_i = 3 \times 87.13 = 261.39$ mm $\quad R = 4.25 \times 87.13 = 370.30$ mm

$R_o = 6 \times 87.13 = 522.78$ mm

1.7 CONDITION FOR BENDING STRESS AT EXTREME FIBERS TO BE NUMERICALLY EQUAL

For a curved beam subjected to bending, the extreme fiber stresses are

Bending stress at inner fiber, $\sigma_i = \dfrac{Mc_i}{AeR_i}$

Bending stress at outer fiber, $\sigma_o = \dfrac{Mc_o}{AeR_o}$

For extreme fibers to be numerically equal, we have

$$\sigma_i = \sigma_o$$

$$\frac{Mc_i}{AeR_i} = \frac{Mc_o}{AeR_o}$$

$$\frac{c_i}{R_i} = \frac{c_o}{R_o} \qquad \text{... (Eq. 1.20)}$$

But $\quad c_i = c_1 - e = R_n - R_i$

and $\quad c_o = c_2 + e = R_o - R_n$

(Eq. 1.20) yields...

$$\frac{R_n - R_i}{R_i} = \frac{R_o - R_n}{R_o}$$

$$\frac{R_n}{R_i} - 1 = 1 - \frac{R_n}{R_o}$$

$$\frac{R_n}{R_i} + \frac{R_n}{R_o} = 2$$

$$\frac{(R_o + R_i)R_n}{R_iR_o} = 2$$

$$R_n = \frac{2R_iR_o}{(R_o + R_i)} \qquad \text{... (Eq. 1.21)}$$

33. Determine the width of larger side of a T-section shown in Fig. 1.38 for extreme fiber stresses in bending to be numerically equal.

Solution: $R_i = 60$ mm, $R_o = 120$ mm, $B = ?$

- Since extreme bending stresses are numerically equal, we have

$$R_n = \frac{2R_iR_o}{(R_o + R_i)}$$

$$= \frac{2 \times 60 \times 120}{(60 + 120)}$$

$$R_n = 80 \text{ mm}$$

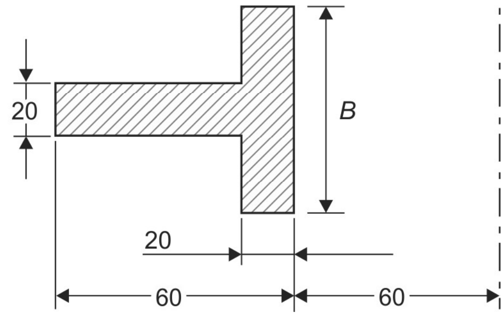

Fig. 1.38: Problem 33

- Comparing the given cross-section with **Fig. 8 – Tb 10.1/Pg 164, DHB**, we have

$$B = ?, a = d = 20 \text{ mm}, H = 60 \text{ mm}$$

- From **Fig. 1.38,** we have

$$R_i = R - c_1 = 60 \text{ mm}, R_o = R + c_2 = 120 \text{ mm}$$

- $e = R - R_n$ ... **Eq. 8 – Tb. 10.1/Pg 164, DHB**

  Since $e = 0$ for stresses to be numerically equal, we have , $R = R_n$

$$R_n = \frac{A}{B\ln\left(\frac{R + d - c_1}{R - c_1}\right) + a\ln\left(\frac{R + c_2}{R + d - c_1}\right)}$$

Here, $A = 20B + (40 \times 20)$, $R + d - c_1 = R_i + d = 60 + 20 = 80$ mm

$$80 = \frac{20B + (40 \times 20)}{\left[B\ln\left(\frac{80}{60}\right)\right] + \left[20\ln\left(\frac{120}{80}\right)\right]}$$

$$80[(0.287)B + 8.109] = (20)B + 800$$

$$(22.96)B + 648.72 = (20)B + 800$$

$$(2.96)B = 151.28$$

$$B = 51.11 \text{ mm}$$

34. Determine the value of steam thickness '*t*' in the T-cross-section of a curved beam shown in Fig. 1.39 such that the normal stresses due to bending at the extreme inner and outer fibers are numerically equal.

VTU – June/July 09 – 15 Marks; Jan./Feb. 05 – 10 Marks

Solution: $R_i = 150$ mm, $R_o = 290$ mm, $B = 100$ mm, $t = ?$

- Since extreme bending stresses are numerically equal, we have

$$R_n = \frac{2R_iR_o}{(R_o + R_i)}$$

$$= \frac{2 \times 150 \times 290}{(290 + 150)}$$

$$R_n = 197.73 \text{ mm}$$

- Comparing the given cross-section with **Fig. 8 – Tb 10.1/Pg 164, DHB**, we have

$$B = 100 \text{ mm}, a = t = ?, d = 40 \text{ mm}, H = 140 \text{ mm}$$

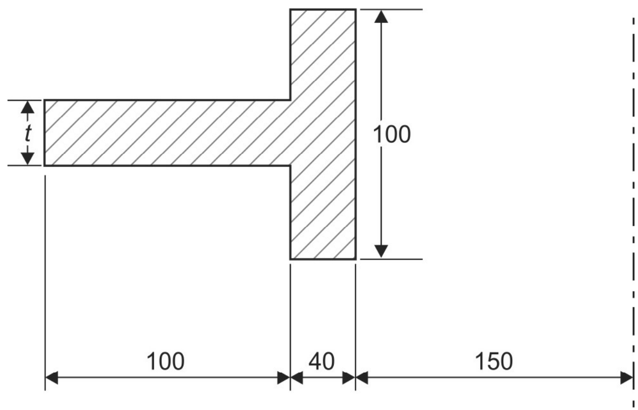

Fig. 1.39: Problem 34

- From **Fig. 1.39,** we have

$$R_i = R - c_1 = 150 \text{ mm}, R_o = R + c_2 = 290 \text{ mm}$$

- $e = R - R_n$ ... **Fig. 8 – Tb. 10.1/Pg 164, DHB**

  Since $e = 0$ for stresses to be numerically equal, we have , $R = R_n$

$$R_n = \frac{A}{B\ln\left(\frac{R+d-c_1}{R-c_1}\right) + a\ln\left(\frac{R+c_2}{R+d-c_1}\right)}$$

Here, $A = (100 \times 40) + 100t$, $R + d - c_1 = R_i + d = 150 + 40 = 190$ mm

$$197.73 = \frac{(100 \times 40) + 100t}{\left[100 \times \ln\left(\frac{190}{150}\right)\right] + \left[t \ln\left(\frac{290}{190}\right)\right]}$$

$$197.73[23.638 + (0.423)t] = 4000 + (100)t$$

$$4673.94 + (83.61)t = 4000 + (100)t$$

$$673.94 = 16.39t$$

$$t = 41.22 \text{ mm} = a$$

35. Determine the dimensions of a unsymmetrical I-beam having circular center line and subjected to pure bending in the plane of unsymmetry, such that the extreme fiber stresses due to bending are numerically equal. The dimensions are: R_i = 75 mm, $R_i + d$ = 100 mm, R_o = 175 mm, b_2 = 25 mm, $b_1 + B$ = 125 mm, web length = 50 mm.

VTU – [similar: Dec. 2014/Jan. 2015 – 10 Marks

Solution:

- Since extreme bending stresses are numerically equal, we have

$$R_n = \frac{2R_iR_o}{(R_o + R_i)}$$

$$= \frac{2 \times 75 \times 175}{(175 + 75)}$$

$$= 105 \text{ mm}$$

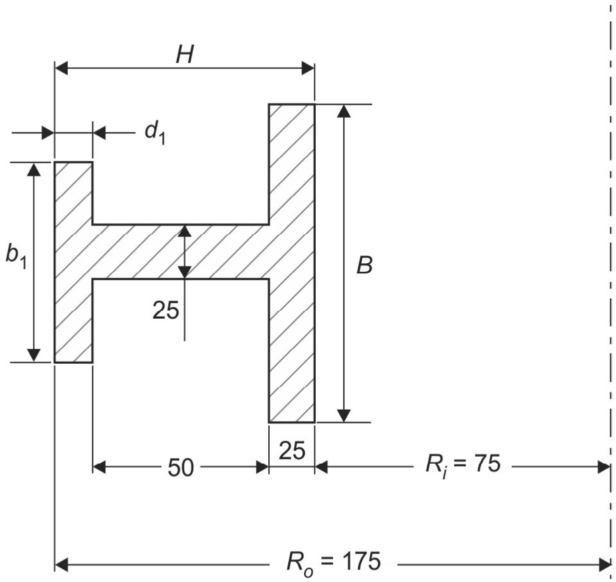

Fig. 1.40: Problem 35

- Comparing the given cross-section and data with **Fig. 10 – Tb 10.1/Pg 164, DHB**, we have

$$R_i = 75 \text{ mm}, \quad R_o = 175 \text{ mm}. \quad R_i + d = 100 \text{ mm}$$
$$b_2 = 25 \text{ mm}, \quad b_1 + B = 125 \text{ mm}, \quad \text{web length} = 50 \text{ mm}$$

- From **Fig. 1.40,** we have

$$R_i = R - c_1 = 75 \text{ mm}, \quad R_o = R + c_2 = R_i + H = 175 \text{ mm}$$
$$H = R_o - R_i = 175 - 75 = 100 \text{ mm}, \quad B = B$$
$$d = 100 - R_i = 100 - 75 = 25 \text{ mm (using data)}$$
$$b_1 = b_1, \quad d_1 = H - d + 50 = 100 - 25 + 50 = 25 \text{ mm}$$
$$b_2 = 25 \text{ mm} \quad \text{web length} = 50 \text{ mm}$$

- $e = R - R_n$ ... **Fig. 8 – Tb 10.1/Pg 164, DHB**

Since $e = 0$ for stresses to be numerically equal, we have , $R = R_n$

$$R_n = \frac{A}{B\ln\left(\frac{R+d-c_1}{R-c_1}\right) + b_2 \ln\left(\frac{R+c_2-d_1}{R+d-c_1}\right) + b_1 \ln\left(\frac{R+c_2}{R+c_2-d_1}\right)}$$

Here, $R + d - c_1 = R_i + d = 75 + 25 = 100$ mm

$$R + c_2 - d_1 = R_o - d_1 = 175 - 25 = 150 \text{ mm}$$
$$A = (b_1 d_1 + 50 b_2 + B \cdot d) = 25 b_1 + (50 \times 25) + 25B$$
$$= 25(b_1 + \text{B}) + 1250 = (25 \times 125) + 1250$$
$$= 4375 \text{ mm}^2$$

$$105 = \frac{4375}{\left[B\ln\left(\frac{100}{75}\right)\right] + \left[25 \times \ln\left(\frac{150}{100}\right)\right] + \left[b_1 \ln\left(\frac{175}{150}\right)\right]}$$

$$105[0.287)B + 10.14 + (0.154)b_1] = 4375$$
$$(0.287)B + 10.14 + (0.154)b_1 = 41.67$$
$$(0.287)B + (0.154)b_1 = 31.53$$

$$(0.287)(125 - b_1) + (0.154)b_1 = 31.53 \qquad [\text{since } b_1 + B = 125 \text{ mm, hence, } B = 125 - b_1]$$

$$35.88 - (0.287)b_1 + (0.154)b_1 = 31.53$$

$$4.35 = (0.133)b_1$$

$$b_1 = 32.71 \text{ mm}$$

and $$B = 125 - 32.71 = 92.29 \text{ mm}$$

36. The section of a crane hook is a trapezium having the following dimensions. Width on inner side = b_1, width on outer side = 40 mm, depth of cross-section = 80 mm. The center of curvature is at a distance of 150 mm from inner edge. Determine the value of b_1, if extreme fiber stresses are numerically equal.

Solution: $b = 40$ mm, $h = 80$ mm, $b_1 = ?$, $R_i = 150$ mm.

Based on given data, the crane hook is as shown in **Fig. 1.41**.

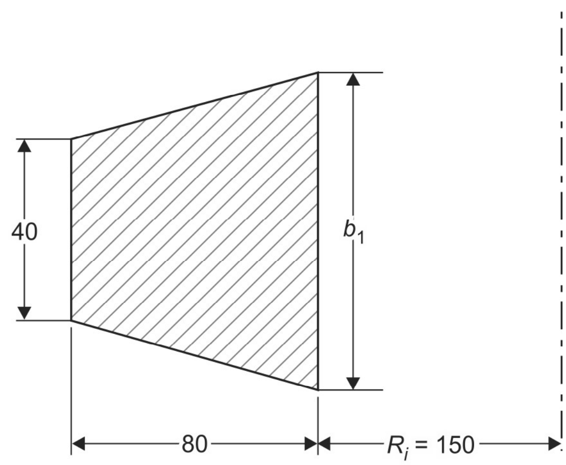

Fig. 1.41: Problem 36

- Since extreme bending stresses are numerically equal, we have

$$R_n = \frac{2R_iR_o}{(R_o + R_i)}$$

$$= \frac{2 \times 150 \times 230}{(230 + 150)}$$

$$R_n = 181.58 \text{ mm}$$

- Comparing the given cross-section with **Fig. 7 – Tb 10.1/Pg 163, DHB**, we have

$$b = 40 \text{ mm}, \quad h = 80 \text{ mm}, \quad b_1 = ?$$

- From **Fig. 1.41,** we have

$$R_i = R - c_1 = 150 \text{ mm}$$

$$R_o = R + c_2 = R_i + h = 150 + 80 = 230 \text{ mm}$$

- $e = R - R_n$ ... **Fig. 7 – Tb 10.1/Pg 163, DHB**

Since $e = 0$ for stresses to be numerically equal, we have , $R = R_n$

$$R_n = \frac{A}{\left[\left(\frac{b_1(R + c_2) - b(R - c_1)}{h}\right)\ln\left(\frac{R + c_2}{R - c_1}\right)\right] - (b_1 - b)}$$

Here, $$A = \frac{h(b_1 + b)}{2} = \frac{80 \times (b_1 + 40)}{2} = 40 \times (b_1 + 40)$$

$$181.58 = \frac{40 \times (b_1 + 40)}{\left[\left(\frac{230b_1 - (40 \times 150)}{80}\right) \times \ln\left(\frac{230}{150}\right)\right] - (b_1 - 40)}$$

$$= \frac{40b_1 + 1600}{[1.23b_1 - 32.06] - b_1 + 40}$$

$$= \frac{40b_1 + 1600}{0.23b_1 + 7.94}$$

$$41.76b_1 + 1441.75 = 40b_1 + 1600$$

$$1.76b_1 = 158.25$$

$$b_1 = 89.91 \text{ mm} \approx 90 \text{ mm}$$

37. Determine the value of 't' in the cross-section of a curved machine member shown in Fig. 1.42, so that the normal stresses due to bending at extreme fibers are numerically equal. Also determine the normal stresses so induced at extreme fibers due to bending moment of 10 kN-m.

VTU – Dec 2010 – 12 Marks; June/July 2015 – 10 Marks

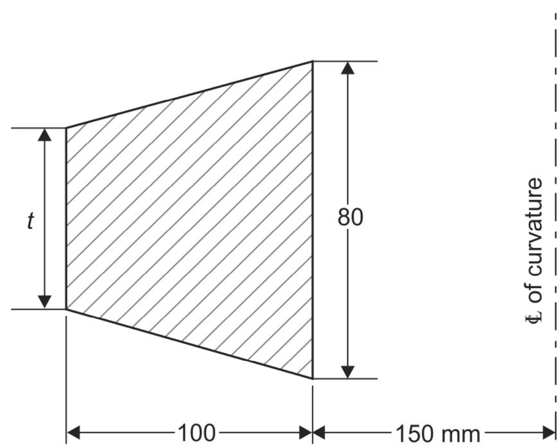

Fig. 1.42: Problem 37

Solution: t = ?, $M = 10 \times 10^6$ N-mm, σ_i, σ_o = ?

Case a: To find t

- Since extreme bending stresses are numerically equal, we have

$$R_n = \frac{2R_iR_o}{(R_o + R_i)}$$

$$= \frac{2 \times 150 \times 250}{(250 + 150)}$$

$$= 187.5 \text{ mm}$$

- Comparing the given cross-section with **Fig. 7 – Tb 10.1/Pg 163, DHB**, we have

$$b = t, h = 100 \text{ mm}, b_1 = 80 \text{ mm}$$

- From **Fig. 1.42,** we have

$$R_i = R - c_1 = 150 \text{ mm}$$

$$R_o = R + c_2 = R_i + h = 150 + 100 = 250 \text{ mm}$$

- $e = R - R_n$ ... **Fig. 7 – Tb 10.1/Pg 163, DHB**

  Since $e = 0$ for stresses to be numerically equal, we have $R = R_n$

$$R_n = \frac{A}{\left[\left(\frac{b_1(R+c_2)-b(R-c_1)}{h}\right)\ln\left(\frac{R+c_2}{R-c_1}\right)\right]-(b_1-b)}$$

Here, $$A = \frac{h}{2}(b_1+b) = \frac{100\times(80+t)}{2} = 50\times(80+t)$$

$$187.5 = \frac{50\times(80+t)}{\left[\left(\frac{(80\times 250)-150t}{100}\right)\times\ln\left(\frac{250}{150}\right)\right]-(80-t)}$$

$$= \frac{4000+50t}{[102.17-0.776t]-80+t}$$

$$= \frac{4000+50t}{22.17+0.234t}$$

$$4156.88 + 43.88t = 4000 + 50t$$

$$156.88 = 6.12t$$

$$t = 25.63 \text{ mm} = b$$

Case b: Bending stresses (σ_i, σ_o)

$\therefore$ $$b_o = b_1 - b = 80 - 25.63 = 54.37 \text{ mm}$$

$$c = \frac{(3b+2b_o)h}{3(2b+b_o)}$$

$$= \frac{[(3\times 25.63)+(2\times 54.37)]\times 100}{3\times[(2\times 25.63)+54.37]} = 58.58 \text{ mm (from outer fiber)}$$

... **Fig. f – Tb 1.3(a)/Pg 13, DHB**

$$\bar{x} = h - c = 100 - 58.58 = 41.42 \text{ mm}$$

Thus $$c_1 = \bar{x} = 41.42 \text{ mm}, \quad c_2 = h - c_1 = c = 58.58 \text{ mm}$$

$$A = \frac{100\times(80+t)}{2} = 50\times(80+25.63) = 5281.5 \text{ mm}^2$$

but $$R = R_i + c_1 = 150 + 41.42 = 191.42 \text{ mm}$$

eccentricity $$e = R - R_n = 191.42 - 187.5 = 3.92 \text{ mm}$$

Now $$c_i = c_1 - e = 41.42 - 3.92 = 37.5 \text{ mm}$$ ... **10.1(d)/Pg 159, DHB**

$$c_o = c_2 + e = 58.58 + 3.92 = 62.5 \text{ mm}$$ ... **10.1(d)/Pg 159, DHB**

- Bending stress at inner fiber,

$$\sigma_i = \frac{Mc_i}{AeR_i} = \frac{(10\times 10^6)\times 37.5}{5281.5\times 3.92\times 150} = 120.75 \text{ MPa}$$ ... **10.1(b)/Pg 159, DHB**

- Bending stress at outer fiber,

$$\sigma_o = \frac{-Mc_o}{AeR_o} = \frac{-(10\times 10^6)\times 62.5}{5281.5\times 3.92\times 250} = -120.75 \text{ MPa}$$ ... **10.1(c)/Pg 159, DHB**

VTU QUESTION PAPERS

Feb. 2002 (ME6T2)

1. a. Discuss the stress distribution pattern in curved beams when compared to straight beams with sketches. **(04 Marks)**

b. A machine member has a T-shaped cross-section and is loaded as shown in **Fig. M1.1**. If the allowable compressive stress is 50 MPa, determine the largest force '*P*' which may be applied to the member safely. **(16 Marks)**

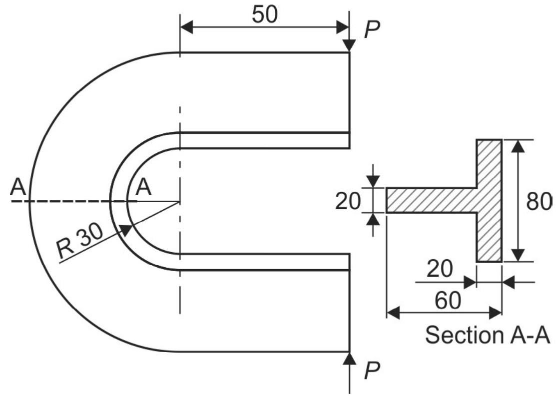

Fig. M1.1

July/August 2002 (ME6T2)

2. Taking a permissible stress in the material as 100 MPa, estimate the thickness '*t*' for the cross section shown in **Fig. M1.2**. Determine the maximum stress induced in the material of the member shown considering curved beam effect. By how much the factor of safety is reduced if the ultimate strength of the material is 440 MPa. **(20 Marks)**

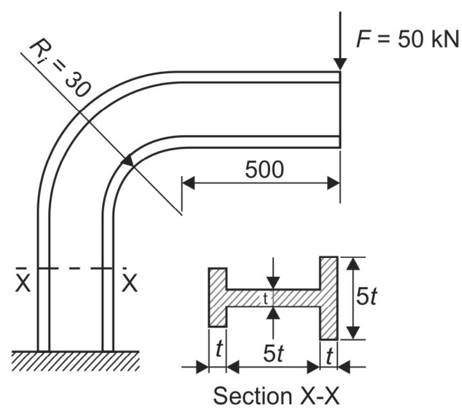

Fig. M1.2

Jan./Feb. 2003 (ME6T2)

3. a. List the main differences between straight and curved beams. **(05 Marks)**

b. A crane hook has a section which, for purpose of analysis, is considered as trapezoidal. The dimensions are shown in **Fig. M1.3**. Determine the maximum stresses and the location. **(15 Marks)**

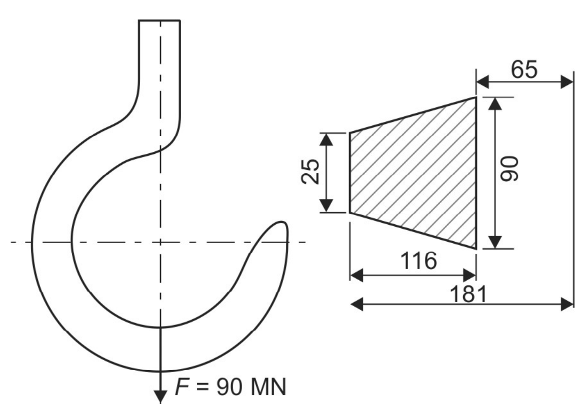

Fig. M1.3

July/August 2003 (ME6T2)

4. a. Using usual notations, prove the moment of resistance M of a curved beam of initial radius R_1 when bent to radius R_2 by uniform bending moment is $M = EAh^2\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$. **(10 Marks)**

b. A chain link made of 40 mm diameter rod is semicircular at each end, the mean diameter of which is 80 mm. The straight sides of the link are also 80 mm. If the link carries a load of 90 kN, estimate the tensile and compressive stresses in the link along the section of load line. **(10 Marks)**

July/Aug. 2004 (ME6T2)

5. a. Discuss the stress distribution pattern in curved and straight beams with appropriate sketches. **(04 Marks)**

b. Determine the safe load 'F' that the frame of a punch press shown in **Fig. M1.4** can carry considering the cross-section along section A-A for an allowable tensile stress of 100 MPa. What is the stress at the outer fiber for the above load? What will be the stress at the inner fiber, if the beam is a straight beam for the above load? **(16 Marks)**

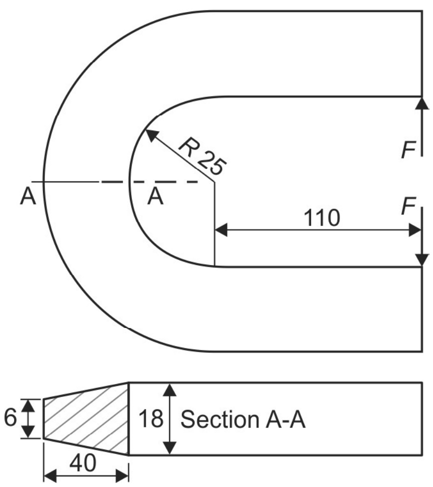

Fig. M1.4

Jan/Feb. 2005 (ME6T2)

6. a. Give the differences between a straight beam and a curved beam. **(04 Marks)**

b. The cross-section of steel crane hook is a trapezium with an inner side of 50 mm and outer side of 25 mm. The depth of the section is 64 mm. The center of curvature of the section is at a distance of 64 mm from the inner edge of the section and the line of action of load is 50 mm from the same edge. Determine the maximum load the hook can carry if the allowable strength is limited to 60 MPa.

(16 Marks)

Jan/ Feb. 2005 (AU53)

7. Determine the value of t in the cross-section of a curved beam as shown in **Fig. M1.5**

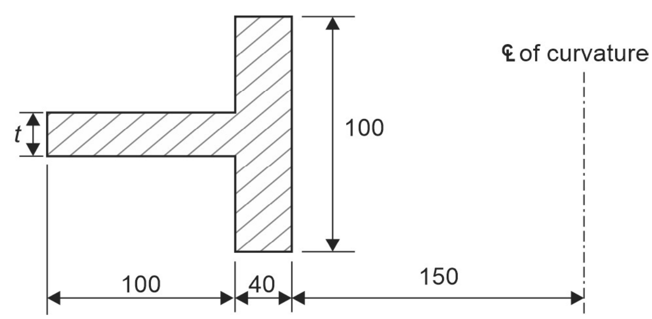

Fig. M1.5

such that the normal stresses due to bending at the extreme fibers are numerically equal. **(10 Marks)**

July/Aug. 2005 (AU53)

8. The section of a crane hook is a trapezium, whose inner and outer sides are 80 mm and 40 mm respectively and has a depth of 100 mm. The center of curvature of the section is at a distance of 120 mm from the inner side of the section and the load line is 110 mm from the same point. Find the maximum load the hook can carry if the maximum stress is not to exceed 70 MPa. **(10 Marks)**

Jan./Feb. 2006 (AU53)

9. Determine a safe value of load P for a machine element loaded as shown in **Fig. M1.6**, limiting the maximum normal stress induced on the cross-section X-X to 120 MPa. **(10 Marks)**

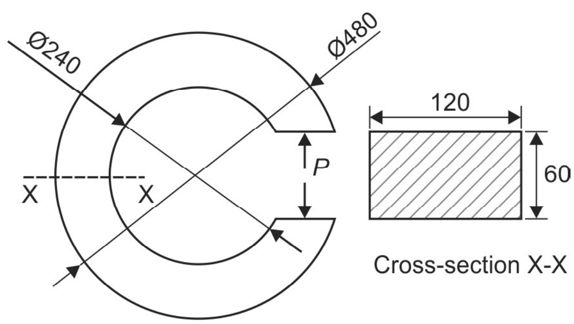

Fig. M1.6

July 2006 (AU53)

10. a. Explain why curved beams have to be analyzed for stresses specially when we already have straight beam equations for determining the stresses? **(04 Marks)**

b. The beam shown in **Fig. M1.7** is subjected to a load of 50 kN. Determine the stresses at the inner and outer fibers. Plot the stress distribution. **(16 Marks)**

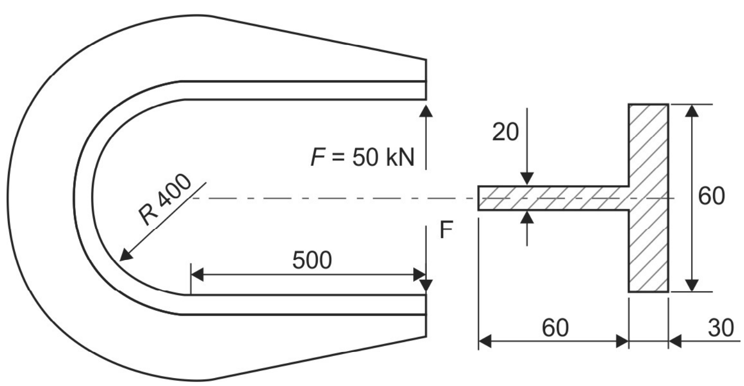

Fig. M1.7

Dec. 06/Jan. 07 (AU53)

11. Compute the combined stresses at the inner and outer fibres in the critical section of a crane hook which is required to lift loads up to 25 kN. The hook has trapezoidal cross-section with parallel sides 60 mm and 30 mm, the distance between them being 90 mm. The inner radius of the hook is 100 mm. The load line is nearer to the inner surface of the hook by 25 mm than the center of curvature at the critical section. What will be the stresses at the inner and outer fibre, if the beam is treated as straight beam for the given load? **(12 Marks)**

July 2007 (AU53)

12. Calculate the stresses at the points A and B for a circular beam as shown in **Fig. M1.8.** The circular beam is subjected to a compressive load of 6 kN. **(10 Marks)**

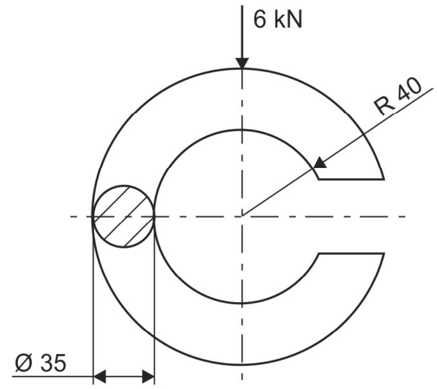

Fig. M1.8

Dec. 07/Jan. 08 ((AU53)

13. The section of a crane hook is trapezoidal, whose inner and outer sides are 90 mm and 25 mm respectively and has a depth of 116 mm. The center of curvature of the section is at a distance of 65 mm from the inner side of the section and the load line passes through the center of curvature. Find the maximum load the hook can carry, if the maximum stress is not to exceed 70 MPa. **(12 Marks)**

Dec. 07/Jan. 08 (ME6T2)

14. a. Derive an expression for normal stresses due to bending, across the cross-section of a curved beam. **(08 Marks)**

b. Compute the stresses in critical section of a crane hook which is required to lift loads up to 50 kN. The hook has trapezoidal cross-section with parallel sides 100 mm and 60 mm. The distance between them is 120 mm. Inner radius of the hook is 150 mm. **(12 Marks)**

June/July 08 (AU53)

15. a. Differentiate between a straight beam and a curved beam with stress distribution in each of the beam. **(04 Marks)**

b. **Fig. M1.9** shows a 100 kN crane hook with a trapezoidal section. Determine stress in the outer, inner, Cg and also at the neutral fiber and draw the stress distribution across the section AB. **(16 Marks)**

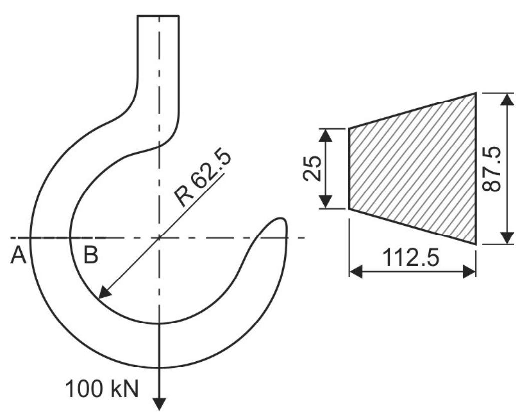

Fig. M1.9

Dec. 08/Jan. 09 (ME6T2)

16. a. Derive an expression for normal stress due to bending, across the section of a curved beam. **(08 Marks)**

b. Compute the stresses in critical section of a crane hook which is required to lift loads up to 50 kN. The hook has trapezoidal cross section with parallel sides 100 mm and 60 mm. The distance between them is 120 mm. Inner radius of the hook is 150 mm. **(12 Marks)**

June/July 2009 (ME6T2)

17. a. Give the differences between a straight beam and a curved beam. **(04 Marks)**

b. The cross-section of a steel crane hook is a trapezium with an inner side of 50 mm and outer side of 25 mm. The depth of section is 64 mm. The center of curvature of the section is at a distance of 64 mm from the inner edge of the section and the line of action of the load is 50 mm from the same edge. Determine the maximum load the hook can carry if the allowable stress is limited to 60 MPa. **(16 Marks)**

Dec. 08/Jan. 09 (AU53)

18. A closed ring is made up of 50 mm diameter steel bar having allowable tensile stress of 200 MPa. The inner diameter of the ring is 100 mm. For a load of 30 kN, find the maximum stress in the bar and specify the location. If the ring is cut as shown in **part B** of **Fig. M1.10**, check whether it is safe to support the applied load. **(10 Marks)**

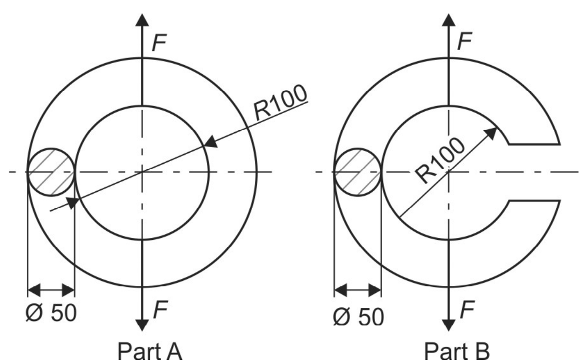

Fig. M1.10

June/July 2009 (AU53)

19. The horizontal cross section of a crane hook is an isosceles triangle of 120 mm deep, the inner width being 90 mm. The hook carries a load of 50 kN. Inner radius of curvature is 100 mm. The line of action of load passes through the center line of curvature. Determine the stress at the extreme fibres. **(12 Marks)**

June/July 2009 (06ME61)

20. a. What are the assumptions made in finding stress distribution for a curved flexural member? Also state two major differences between a straight beam and a curved beam. **(05 Marks)**

b. Determine the value of thickness *t* in the T-cross-section of a curved beam shown in **Fig. M1.11** such that the normal stresses due to bending at the extreme inner and outer fibers are numerically equal. **(15 Marks)**

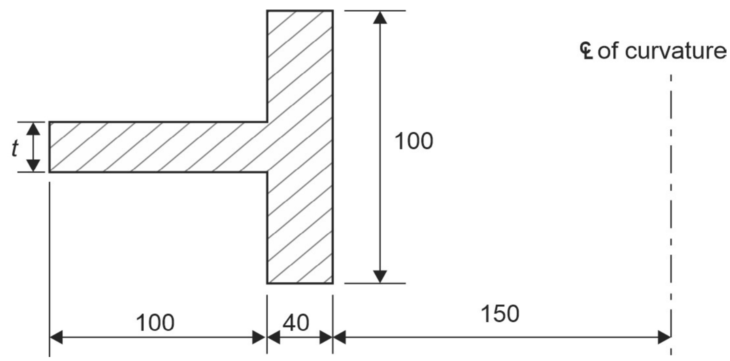

Fig. M1.11

Dec. 09/Jan. 10 (06ME61)

21. Determine the maximum stress induced in a ring cross-section of 50 mm diameter rod subjected to a compressive load of 20 kN. The mean diameter of the ring is 100 mm. **(10 Marks)**

May/June 2010 (AU53)

22. The frame of a punch press is shown in **Fig. M1.12**. Find the stresses at the inner and outer fibers at section X-X of the frame, if $W = 5000$ N. **(10 Marks)**

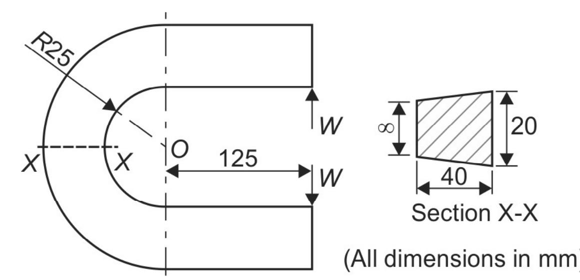

Fig. M1.12

Dec. 2010 (AU53)

23. A central horizontal section of a crane hook is a symmetrical trapezium of 100 mm deep, the inner width being 60 mm and the outer width being 40 mm. The crane hook carries a load of 25 kN. The inner radius of the hook is 75 mm. The load line is nearer to the inner surface of the hook by 25 mm than the center of curvature at the critical section. Determine the extreme fiber stresses. **(10 Marks)**

May/June 2010 (06ME61)

24. a. Derive an expression for extreme fiber stresses in a curved beam subjected to pure bending. **(08 Marks)**

b. Determine the combined stresses at the inner and outer fibers at the critical section of a crane hook which is required to lift loads up to 50 kN. The hook has trapezoidal cross-section with inner and outer sides of 90 mm and 40 mm respectively. Depth is 120 mm. The center of curvature of the section is at a distance of 100 mm from the inner side of the section and the load line passes through the center of curvature. Also, determine the factor of safety according to max shear stress theory if $\tau_{all} = 80$ MPa. **(12 Marks)**

Dec. 2010 (06ME61)

25. a. Derive an expression for normal stresses due to bending at the extreme fibers on the cross-section of a curved machine member. **(08 Marks)**

b. Determine the value of t in the cross-section of a curved machine member shown in **Fig. M1.13**, so that the normal stresses due to bending at extreme fibers are

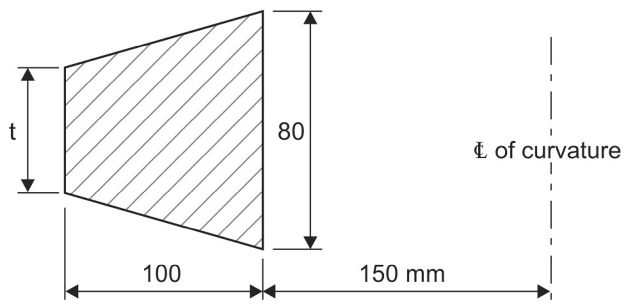

Fig. M1.13

numerically equal. Also determine the normal stresses so induced at extreme fibers due to bending moment of 10 kN-m. **(12 Marks)**

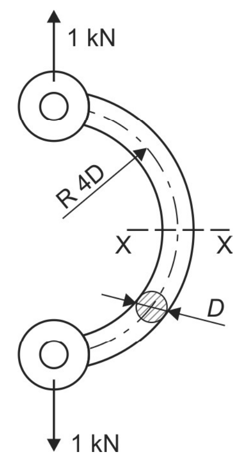

Fig. M1.14

June/July 2011 (06ME61)

26. Determine the dimensions of the curved bar as shown in **Fig. M1.14**. Assume $\sigma_{yt} = 400$ N/mm^2 and FOS = 3.5. **(12 Marks)**

Dec. 2011 (06ME61)

27. a. Give the differences between a straight beam and a curved beam. **(04 Marks)**

b. The cross-section of a steel crane hook is a trapezium with an inner side of 50 mm and outer side of 25 mm. The depth of the section is 64 mm. The center of curvature of the section is at a distance of 64 mm from the inner edge of the section and the line of action of the load is 50 mm from the same edge. Determine the maximum load the hook can carry if the allowable strength is limited to 60 MPa. **(16 Marks)**

June 2012 (06ME61)

28. a. Compare the stresses due to bending moment applied in a straight beam and a curved beam. **(05 Marks)**

b. The parallel sides of a trapezoidal cross section of a crank hook of capacity 50 kN are 100 mm and 60 mm. The depth of the section is 120 mm. The radius of curvature of inner fiber is 150 mm as shown in **Fig. M1.15.** Determine the stresses at the extreme members of the cross section of the crane hook. **(15 Marks)**

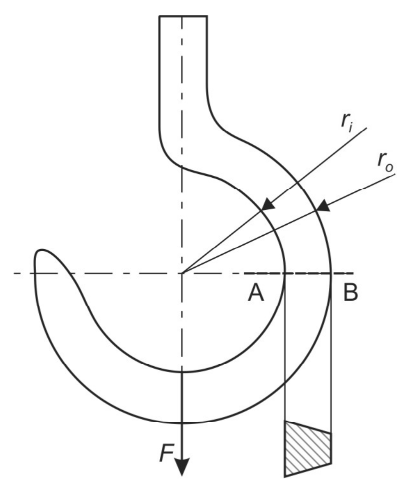

Fig. M1.15

Dec. 2012 (06ME61)

29. a. Differentiate between a straight beam and a curved beam. **(04 Marks)**

b. Compute the combined stresses at the inner and outer fibres in a critical cross-section of a crane hook which is required to lift loads up to 25 kN. The hook has trapezoidal cross section with parallel sides 60 mm and 30 mm, the distance between them being 90 mm. The inner radius of the hook is 100 mm. The load line is nearer to the inner surface of the hook by 25 mm than the center of curvature at the critical section. What will be the stresses at the inner and outer fiber, if the beam is treated as straight beam for the given load? **(16 Marks)**

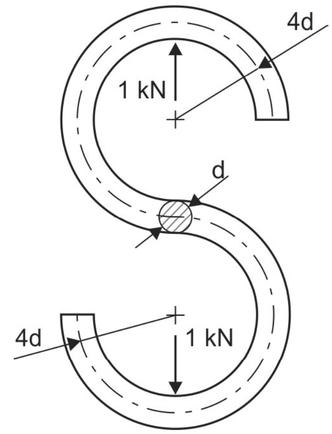

Fig. M1.16

June/July 2013 (AU53)

30. A link of S-shape made of steel bar is shown in **Fig. M1.16**. It is made of steel 45C8 with $\sigma_{yt} = 380$ MPa and factor of safety is 4.5. Calculate the dimensions of the link. **(10 Marks)**

June/July 2013 (06ME61)

31. a. Clearly state five assumptions used in determining the stress distribution in a curved flexural member. **(05 Marks)**

b. **Fig. M1.17** shows a frame of a punching machine and its various dimensions. Determine the combined stress at the inner and outer fibers. Also find the maximum shear stress and its location. Take the force as 85 kN. **(15 Marks)**

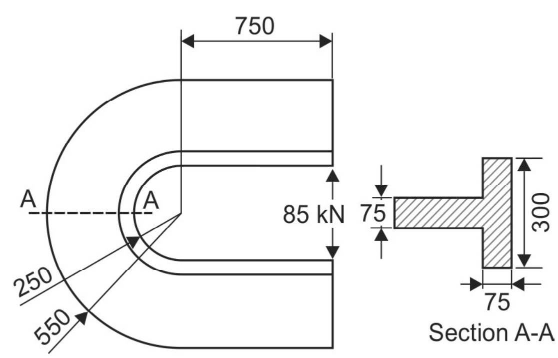

Fig. M1.17

June/July 2013 (10ME62)

32. The cross-section of a steel crane hook is a trapezium with an inner side of 50 mm and outer side of 25 mm. The depth of the section is 64 mm. The center of curvature of the section is at a distance of 64 mm from the inner edge of the section and the line of action of the load is 50 mm from the same edge. Determine the maximum load the hook can carry if the allowable strength is limited to 60 MPa. **(10 Marks)**

Dec. 2013/Jan. 2014 (06ME61)

33. a. Differentiate between a straight beam and a curved beam with suitable examples. **(05 Marks)**

b. A crane hook of trapezoidal section is shown in **Fig. M1.18**. Through the center of curvature, a load of 60 kN is applied on the hook. Determine the dimensions of the section, if the maximum stress is not to exceed 80 MPa. **(15 Marks)**

Dec. 2013/Jan. 2014 (10ME62)

34. A curved link mechanism made from a round steel bar is shown in **Fig. M1.19**. The material for the link is plain carbon steel 30C8 with an allowable yield strength of 400 MPa. Determine the factor of safety. **(10 Marks)**

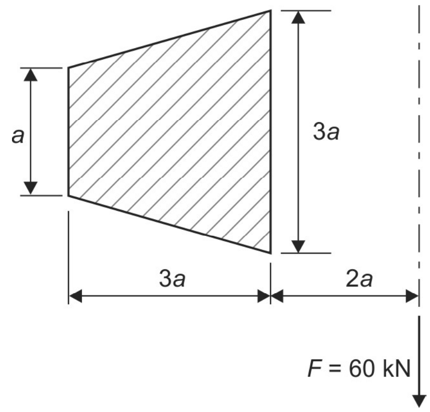

Fig. M1.18

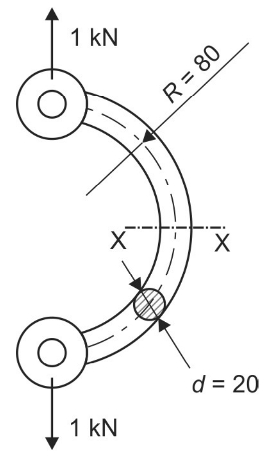

Fig. M1.19

Dec. 2013/Jan. 2014 (AU53)

35. A curved machine member is loaded as shown in **Fig. M1.20**. Determine the maximum tensile stress induced and locate that point. **(10 Marks)**

June/July 2014 (06ME61)

36. The U-section frame is to resist a straightening load of 125 kN as shown in **Fig. M1.21**. The material of the frame has a permissible stress of 65 MPa. Determine the dimensions of the frame. **(12 Marks)**

June/July 2014 (10ME62)

37. Determine the maximum tensile stress and maximum shear stress of the component shown in **Fig. M1.22** and indicate the location. **(10 Marks)**

Dec. 2014/Jan. 2015 (06ME61)

38. a. Derive expressions for extreme fiber stresses in a curved beam subjected to pure bending moment. **(08 Marks)**

b. Determine the combined stresses in the inner and outer fibers at the critical section in a crane hook which is required to lift loads up to 50 kN. The hook has

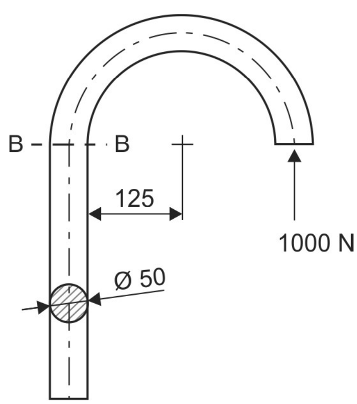

Fig. M1.20

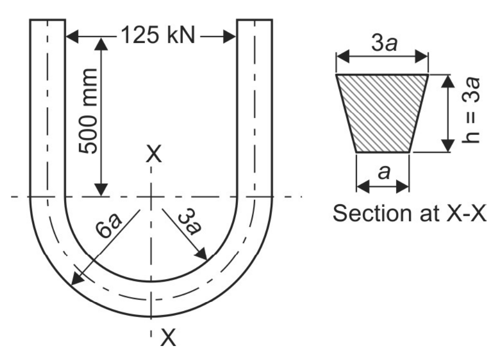

Fig. M1.21

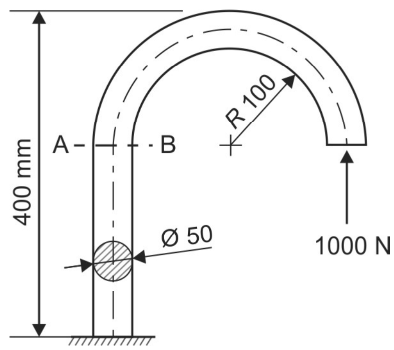

Fig. M1.22

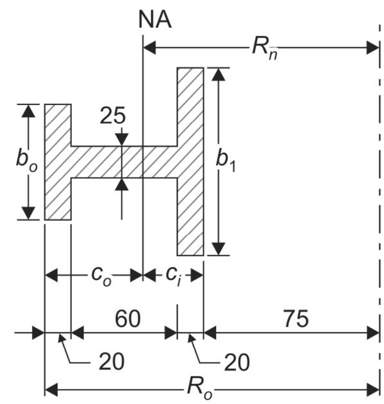

Fig. M1.23

trapezoidal section with inner and outer sides of 90 mm and 40 mm respectively, depth is 120 mm. The center of curvature of the section is at a distance of 100 mm from the inner side of the section and the load line passes through the center of curvature. Also determine the factor of safety according to maximum shear stress theory. **(12 Marks)**

Dec. 2014/Jan. 2015 (10ME62)

39. Determine the dimensions of I-section shown in **Fig. M1.23** in which the maximum fiber stresses are numerically equal in pure bending, given $b_i + b_o = 120$ mm. **(10 Marks)**

June/July 2015 (06ME61)

40. a. Find an expression for bending stress produced in curved beam, subjected to a bending moment M. Enumerate the assumptions. **(10 Marks)**

b. A curved beam of rectangular cross-section of width 20 mm and depth 40 mm is subjected to a pure bending moment of 600 N-m. The mean radius of curvature is 50 mm. Determine the location of neutral axis, maximum and minimum stress, ratio of maximum to minimum stress. **(10 Marks)**

June/July 2015 (10ME62)

41. Determine the value of t in the cross-section of a curved machine member shown in **Fig. M1.24**, so that the normal stresses due to bending at extreme fibers are numerically equal. Also determine the normal stresses so induced at extreme fibers due to bending moment of 10 kN-m. **(10 Marks)**

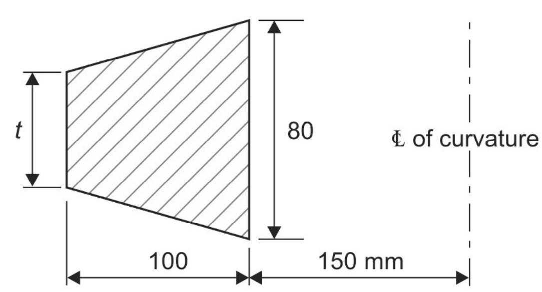

Fig. M1.24

Dec. 2015/Jan. 2016 (06ME61)

42. a. List the differences between a straight beam and a curved beam. **(04 Marks)**

b. A crane hook having trapezoidal cross-section is shown in **Fig. M1.25**. It is made of plain carbon steel (σ_{yt} = 380 MPa). Assuming a factor of safety as 3.5, determine the load carrying capacity of the hook. **(16 Marks)**

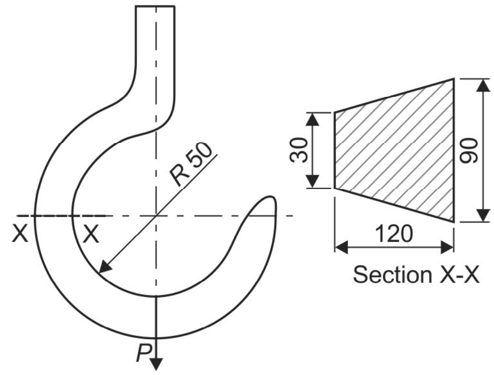

Fig. M1.25

Dec. 2015/Jan. 2016 (10ME62)

43. A ring is made from a 75 mm diameter bar. The inside diameter of the ring is 200 mm. For the load shown in **Fig. M1.26**, calculate the maximum shear stress in the ring and specify its location. **(10 Marks)**

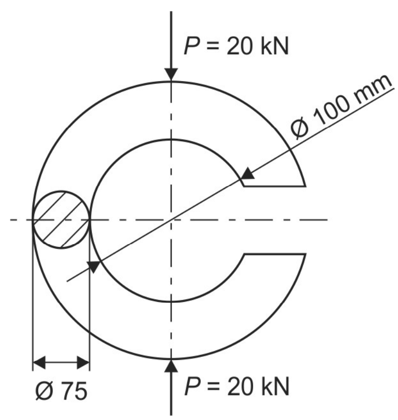

Fig. M1.26

June/July 2016 (10ME62)

44. Determine the dimensions of the curved bar shown in **Fig. M1.27**. Assume σ_{yt} = 400 MN/m^2 and FOS = 3.5. **(10 Marks)**

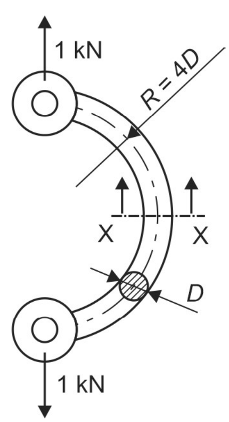

Fig. M1.27

Dec. 2016/Jan. 2017 (10ME62)

45. A crane hook of trapezoidal cross-section has an inner side of 120 mm and outer side of 60 mm. The depth of the section is 90 mm. The center of curvature is at a distance of 120 mm from the inner edge of the section and the line of action of load is at a distance of 135 mm from the inner edge. Determine the safe load that the hook can carry if it is made of steel having an allowable stress of 90 MPa. **(10 Marks)**

June/July 2017 (06ME61)

46. a. Define a curved beam and mention its applications. Also differentiate between a straight beam and a curved beam. **(06 Marks)**

b. A crane hook of trapezoidal cross section whose inner and outer sides are 60 mm and 30 mm has a depth of 64 mm. The center of curvature is at a distance of 90 mm from the inside of the beam. Determine the maximum tensile, compressive and shear stresses induced in the hook when its lifting capacity is 60 kN. **(14 Marks)**

June/July 2017 (10ME62)

47. a. Give the differences between a straight beam and a curved beam. **(04 Marks)**

b. Compute the combined stresses at the inner and outer fibres in the critical section of a crane hook which is required to lift loads up to 25 kN. The hook has trapezoidal cross-section with parallel sides 60 mm and 30 mm, the distance between them being 90 mm. The inner radius of the hook is 100 mm. The load line is nearer to the inner surface of the hook by 25 mm than the center of curvature at the critical section. What will be the stresses at the inner and outer fibre, if the beam is treated as straight beam for the given load? **(16 Marks)**

Dec. 2017/Jan. 2018 (06ME/AU61)

48. a. What are the assumptions made in stress analysis of curved beams? **(04 Marks)**

b. List out the differences between straight and curved beams. **(04 Marks)**

c. Compute the maximum stress at inner and outer fibers in the critical section of a crane hook which is required to lift a load up to 25 kN. The hook has trapezoidal section with parallel sides 60 mm and 30 mm, the distance between them being 90 mm. The inner radius of the hook is 100 mm, the load line is nearer to the inner surface of the hook by 25 mm than the center of curvature at the critical section. **(12 Marks)**

Dec. 2017/Jan. 2018 (10ME62)

49. A chain link made up of 40 mm diameter rod is semi-circle at each end. The mean diameter of which is 80 mm. The straight side of the link is also 80 mm. If the link carries a load of 90 kN, estimate the tensile and compressive stresses in the link along the section of the load line. Also find the stresses at a section 90° away from the load line. **(15 Marks)**

June/July 2018 (06ME61)

50. a. Compare the stresses due to bending moment applied on a straight beam and a curved beam. **(05 Marks)**

b. The parallel sides of a trapezoidal cross-section of a crane hook of capacity 50 kN are 100 mm and 60 mm, the depth of the section being 120 mm. The radius of curvature of the inner fiber is 150 mm as shown in **Fig. M1.28.** Determine the stresses at the extreme fibers of the cross-section of the hook. **(15 Marks)**

June/July 2018 (10ME62)

51. Plot a stress distribution diagram about section A-B for the hook shown in **Fig. M1.29.** **(10 Marks)**

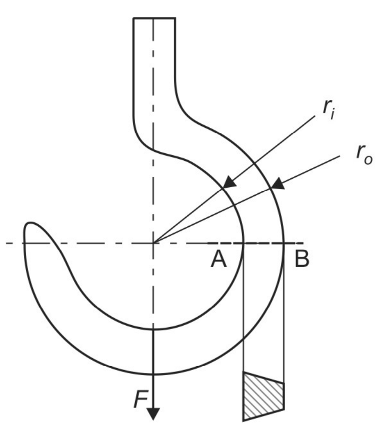

Fig. M1.28

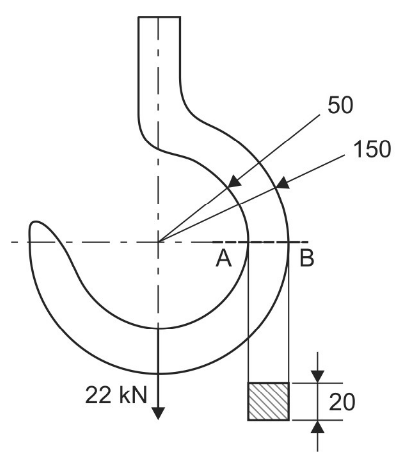

Fig. M1.29

June/July 2018 (15ME64)

52. a. Differentiate between a straight beam and a curved beam. **(04 Marks)**

b. The C-frame of a 100 kN capacity press is shown in **Fig. M1.30**. The material of the frame is grey cast iron FG 200 and the factor of safety is 3. Determine the dimensions of the frame. **(12 Marks)**

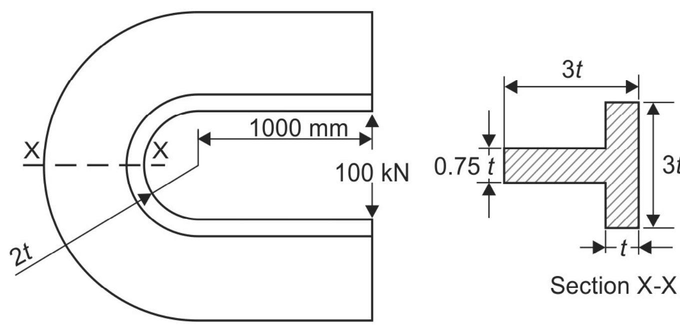

Fig. M1.30

Dec. 2018/Jan. 2019 (10ME62)

53. The cross-section of a steel crane hook is a trapezium with inner side 120 mm and outer side 60 mm. The depth of the section is 90 mm. The load line is 15 mm away from the center of curvature. Determine the safe load that the hook can carry, if the allowable stress is 90 MPa. ʻ **(10 Marks)**

Dec. 2018/Jan. 2019 (15ME64)

54. a. List the assumptions made in obtaining stress equation in a curved beam. **(06 Marks)**

b. Compute the combined stresses at the inner and outer fibres in the critical section of a crane hook which is required to lift loads up to 25 kN. The hook has trapezoidal cross-section with parallel sides 60 mm and 30 mm, the distance between them being 90 mm. The inner radius of the hook is 100 mm. The load line is nearer to the inner surface of the hook by 25 mm than the center of curvature at the critical section. What will be the stresses at the inner and outer fibre, if the beam is treated as straight beam for the given load? **(10 Marks)**

June/July 2019 (10ME62)

55. Determine the value of b_1 of a unsymmetrical I-beam cross-section of a curved beam as shown in **Fig. M1.31** such that the extreme fiber bending stresses are numerically equal. **(12 Marks)**

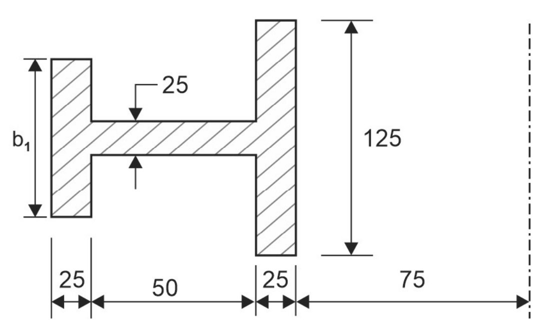

Fig. M1.31

June/July 2019 (15ME64)

56. a. Differentiate between a straight beam and a curved beam. **(04 Marks)**

b. A closed ring is made up of 50 mm diameter steel bar having allowable tensile stress of 200 MPa. The inner diameter of the ring is 100 mm. For a load of 30 kN, find the maximum stress in the bar and specify the location. If the ring is cut as shown in **part B** of **Fig. M1.32**, check whether it is safe to support the applied load. **(12 Marks)**

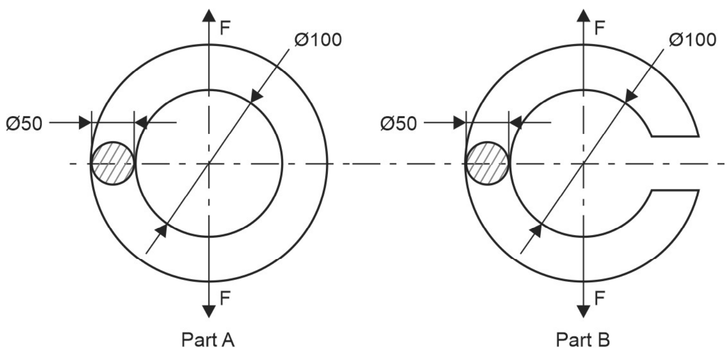

Fig. M1.32

Dec. 2019/Jan. 2020 (10ME62)

57. a. Differentiate between a straight and a curved beam. **(04 Marks)**

b. A small hand operated punching machine has a circular cross-section of 15 mm diameter and is loaded as shown in **Fig. M1.33.** Taking the permissible stress in tension as 55 MPa for the material, determine the largest allowable distance *a* from the line of action of 220 N forces to the plane contacting the centre of curvature of the punch. **(10 Marks)**

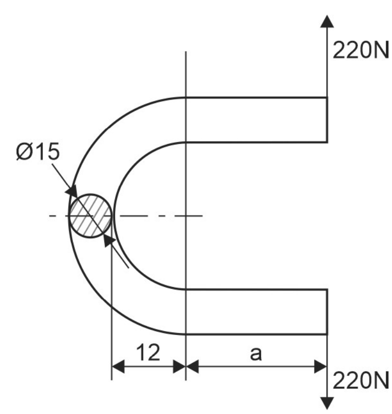

Fig. M1.33

Dec. 2019/Jan. 2020 (15ME64)

58. a. List differences between a curved beam and a straight beam. **(04 Marks)**

b. A chain link is made of 16 mm diameter steel rod. The mean radius of the semi-circular end is 50 mm and length of straight portion of the link is 80 mm. Determine the maximum tensile and compressive stresses when the link is subjected to a pull of 5 kN. **(12 Marks)**

Aug./Sept. 2020 (10ME62)

59. The horizontal cross-section of a crane hook is an isosceles triangle of 120 mm deep, the inner width being 90 mm. The hook carries a load of 50 kN. Inner radius of curvature is 100 mm. The line of action of load passes through the center line of curvature. Determine the stresses at the extreme fibers. **(10 Marks)**

Aug./Sept. 2020 (15ME64)

60. a. Write the differences between a straight beam and a curved beam. **(06 Marks)**

b. The cross-section of a curved link is a symmetrical trapezium 50 mm deep. The inner width and outer width are 50 mm and 25 mm respectively. Find the maximum stress when the link carries a load of 15 kN which passes through the center of curvature of link. The internal radius of the link is 50 mm. **(10 Marks)**

Jan./Feb. 2021 (10ME62)

61. A crane hook of trapezoidal cross-section whose inner and outer sides are 60 mm and 30 mm has a depth of 64 mm. The center of curvature is at a distance of 90 mm from the inside of the beam. Determine the maximum tensile and compressive stresses induced in the hook when its lifting capacity is 60 kN. **(12 Marks)**

Jan./Feb. 2021 (15ME64)

62. a. Determine the dimensions of an I-section as shown in **Fig. M1.34** in which the maximum stresses are numerically equal in pure bending.
Given $b_i + b_o = 120$ mm **(12 Marks)**

b. Discuss the differences between a straight and curved beam. **(04 Marks)**

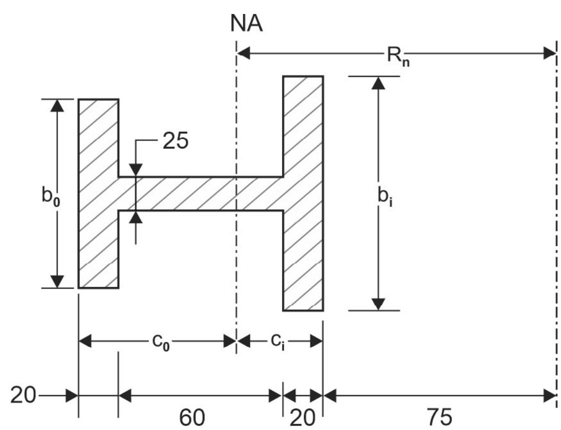

Fig. M1.34