

PROBLEMS IN ELECTRICAL ENGINEERING

Inductance	henry	H
Permeability	H/m	
Volt-amperes	VA	
Reactive volt-amperes	VA (reactive)	VA _r
Watt-hours	Wh	
Ampere-hours	Ah	
Volt-ampere-hours	VAh	
Reactance	Ω	
Susceptance	\mathcal{S}	
Impedance	Ω	
Admittance	\mathcal{S}	

Miscellaneous Units

Time	second	s, sec
Frequency	hertz	Hz
Radial frequency	radians/sec	rad/s
Linear speed	m/s	
Rotational speed	rev per sec	r.p.s.
Mechanical power	horse-power	h.p.
Luminous intensity	candela	cd
Luminous flux	lumen	lm
Illumination	lux, or lm/m ²	lx
Heat	calorie	cal
Temperature	°Centigrade	°C

Miscellaneous Conversions

1 inch	= 2.54 cm	1 foot-pound	= 1.36 J
1 foot	= 0.305 m	1 pound-foot	= 1.36 Nm
1 mile	= 1.61 km	1 slug-foot ²	= 1.36 kg m ²
1 pound	= 0.454 kg	1 pound/inch ²	= 0.69 N/cm ²
1 slug	= 14.62 kg	1 horse-power	= 746 W
1 ton	= 1 017 kg	1 calorie	= 4.18 J
1 metric tonne	= 1 000 kg	1 B.Th.U.	= 253 cal
1 gallon	= 4.54 litre	1 candle	= 1 cd
1 mile per hour	= 0.447 m/s	1 foot-candle	= 10.75 lx
1 cycle/second	= 1 Hz	1 gilbert	= 10/4 π AT
1 dyne	= 10 ⁻⁵ N	1 oersted	= 10 ³ /4 π AT/m
1 pound force	= 4.44 N	1 maxwell	= 10 ⁻⁸ Wb
1 kg force	= 9.81 N	1 gauss	= 10 ⁻⁴ Wb/m ²
1 erg	= 10 ⁻⁷ J	°K	= °C + 273

(°C + 40) × 9/5 = °F + 40

Dimensional Prefixes

G	giga-	(10 ⁹)		m	milli-	(10 ⁻³)
M	mega-	(10 ⁶)		μ	micro-	(10 ⁻⁶)
k	kilo-	(10 ³)		n	nano-	(10 ⁻⁹)
d	deci-	(10 ⁻¹)		p	pico-	(10 ⁻¹²)
c	centi-	(10 ⁻²)				

INTRODUCTION

Abbreviations

Electromotive force	e.m.f.
Potential difference	p.d.
Alternating voltage	a.v.
Alternating current	a.c.
Direct voltage	d.v.
Direct current	d.c.
Root-mean-square	r.m.s.
High voltage	h.v.; high tension: h.t.
Low voltage	l.v.; low tension: l.t.

Rationalized M.K.S. System : Constants

Free-space velocity of propagation of electromagnetic waves :

$$c = 1/\sqrt{(\mu_0 \epsilon_0)} = 2.998 \cdot 10^8 \simeq 3 \cdot 10^8 \text{ m/s.}$$

Free-space permeability (magnetic space-constant) :

$$\mu_0 = 4\pi/10^7 \text{ V-s/m}^2 \text{ per A/m, or H/m.}$$

Free-space permittivity (electric space-constant) :

$$\epsilon_0 = 8.854/10^{12} \simeq 1/(36\pi \cdot 10^9) \text{ A-s/m}^2 \text{ per V/m, or F/m.}$$

Absolute permeability and permittivity :

$$\mu = \mu_r \mu_0 \text{ and } \epsilon = \epsilon_r \epsilon_0$$

where μ_r and ϵ_r are respectively the relative permeability and permittivity, numerically equal to the values of permeability and permittivity in other systems of units.

Physical Constants

COPPER:—

Resistance of a standard annealed copper wire at 20°C is 1/58 Ω for 1 m length and 1 mm² cross-section or 0.0173 $\mu\Omega$ -m (1.73 $\mu\Omega$ -cm).

Resistance of hard-drawn copper is taken as 102% of that of standard annealed copper.

Resistance-temperature coefficient : The resistance R_t of a wire at temperature t °C is given by

$$R_t = R_0 (1 + \alpha t + \beta t^2)$$

where R_0 is the resistance at 0°C and $\alpha = 0.00426 = 1/234.5$ per °C at 0°C ; $\beta = 0.00000112$ and is neglected except where stated otherwise.

Specific gravity : 8.9.

Specific heat : 0.095.

Electrochemical equivalent : 0.000828.

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ALUMINIUM:—

Resistance of a standard aluminium wire, 1 m long and 1 mm² cross-section, is 0.0286 Ω at 20° C.

Resistance-temperature coefficient : $\alpha = 0.00880$ per °C at 0° C.

Specific gravity : 2.7.

STEEL:—

Specific gravity of armature steel plates : 7.8 ; of transformer steel plates : 7.5.

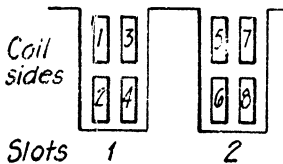
Specific heat : 0.16.

To allow for insulation of stampings, assume : net iron length = 0.9 × gross iron length.

Miscellaneous Constants

e	= 2.718 3	g	= 9.81 m/s ²
j	= $\sqrt{-1} = 1/90^\circ$		
$/\pm\theta$	= phase-angle $\pm\theta$ measured from reference axis		
electron charge	= 1.60×10^{-19} C		
electron mass	= 9.1×10^{-31} kg		
proton mass	= 1.66×10^{-27} kg		
electron charge/mass ratio	= 1.76×10^{11} C/kg		
Boltzmann's constant K	= 1.38×10^{-23} J/°K		

Commutator Windings



The system adopted for numbering coil-sides and slots in a 2-layer commutator winding is shown in the accompanying sketch. Upper coil-sides are numbered odd and lower coil-sides even.

SYMBOLS

- p = number of pole-pairs.
- a = number of pairs of parallel paths.
- S = number of slots.
- C = number of coils or commutator sectors.
- u = number of coil-sides per slot.
- y_b = back-pitch measured in coil-sides, or coil-span measured in slot-pitches.
- y_f = front-pitch measured in coil-sides.
- y_c = commutator pitch measured in sectors.
- Z = number of conductors.
- N_{ph} = number of phases.
- y_{eq} = equipotential pitch measured in sectors.
- y_{ph} = phase-pitch measured in sectors.

INTRODUCTION

Standard Curves

Reference is made in the text to the curves on pp. 6–11, included for use in problems involving magnetic circuits, hysteresis, and the characteristics of motors, generators, valves and transistors.

Where designs are needed for specific purposes, appropriate curves should be obtained.

Sign Convention for Electronic Amplifiers

The voltage or current gain of an amplifier is given as positive when no phase reversal takes place and as negative when the output signal is in anti-phase with the input signal.

The input impedance of an amplifier is given as positive when an increase in input current is accompanied by an increase in the voltage across the input terminals.

The output impedance of an amplifier is given as positive when an increase in output current is accompanied by a decrease in the voltage across the output terminals.

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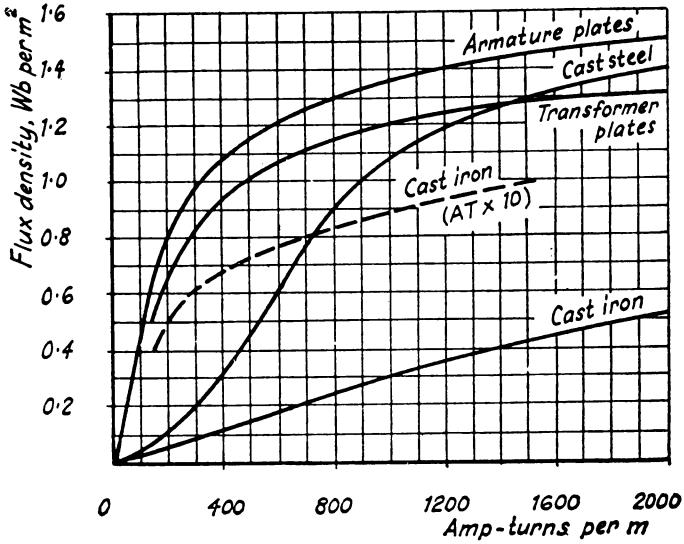


FIG. 1.—Magnetization curves, range 0–1.6 Wb/m^2 .

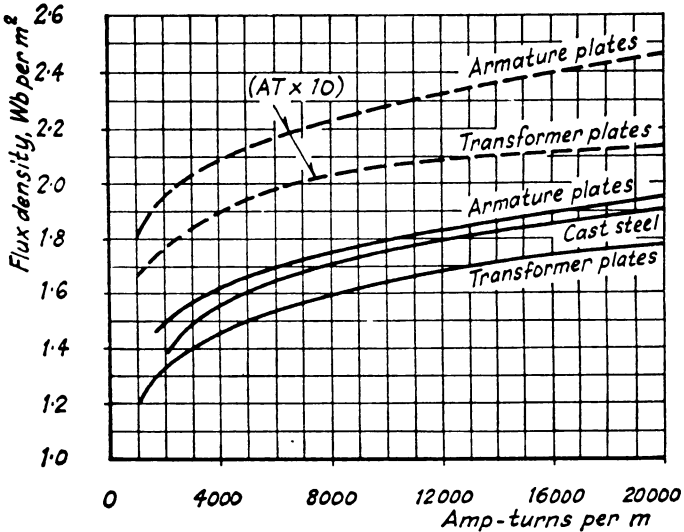


FIG. 2.—Magnetization curves, range 1–2.6 Wb/m^2 .

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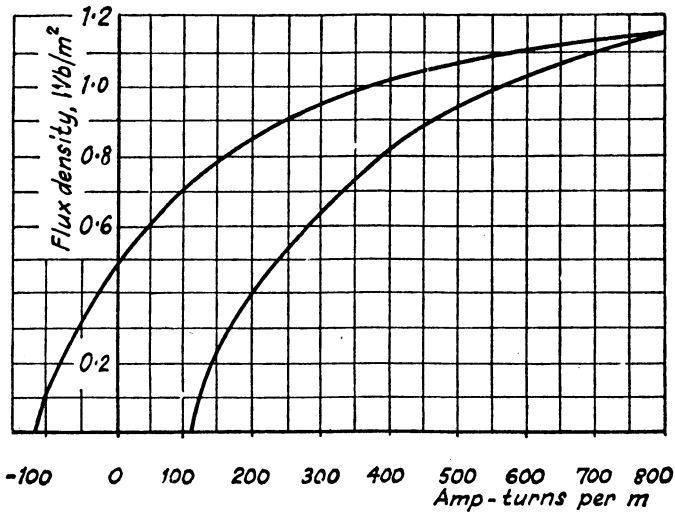


FIG. 3.—Hysteresis loop for transformer steel plates.

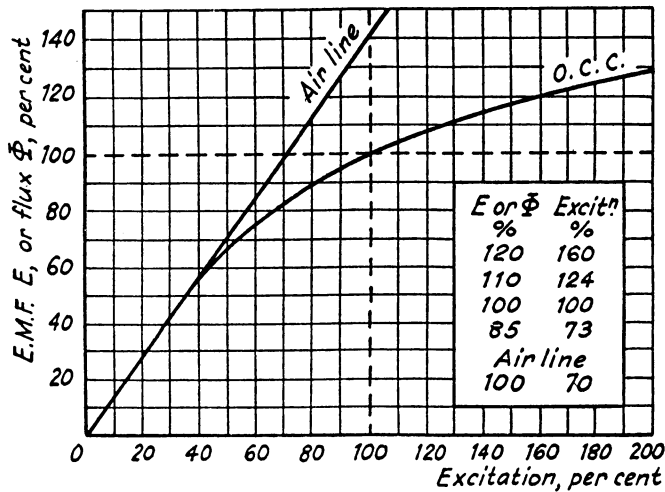


FIG. 4.—Saturation or magnetization curve or open-circuit characteristic for a generator or motor.

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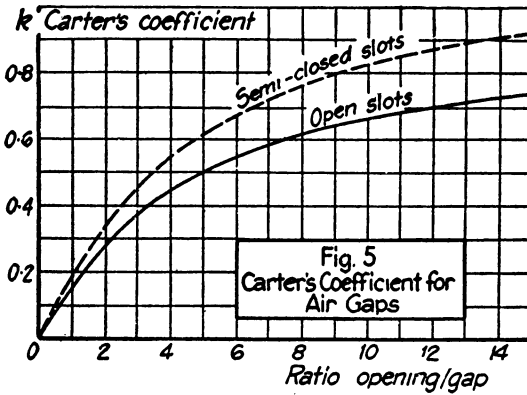


FIG. 5.—Carter's coefficient for air-gaps with slotted armatures.

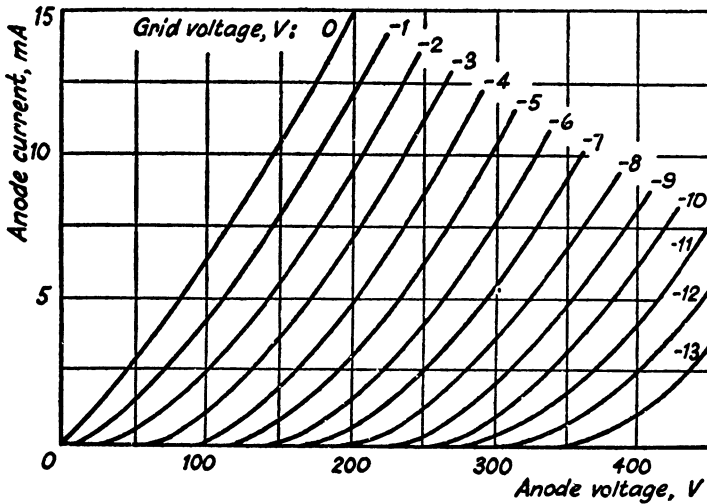


FIG. 6.—Thermionic Valve Characteristics.

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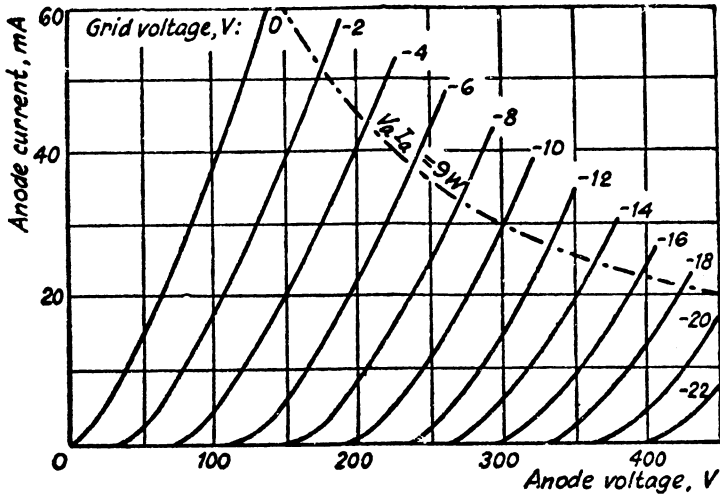


FIG. 7.—Thermionic Valve Characteristics.

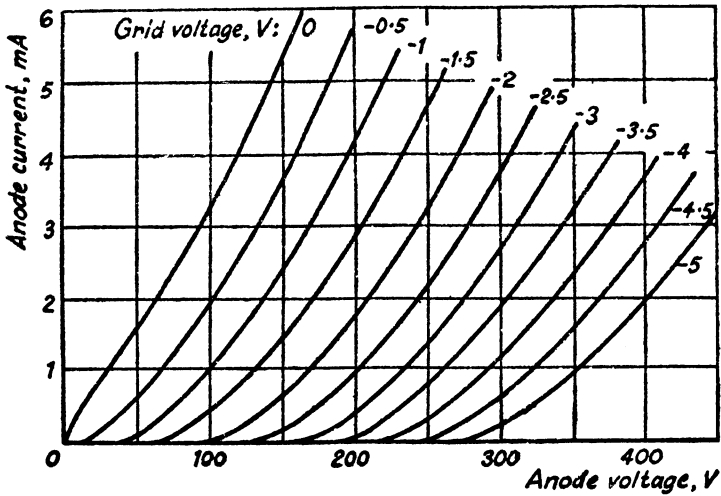


FIG. 8.—Thermionic Valve Characteristics.

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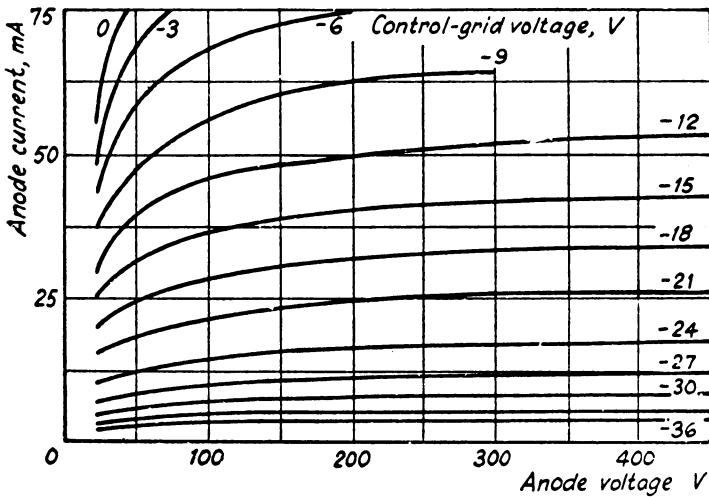


FIG. 9.—Thermionic Valve Characteristics.

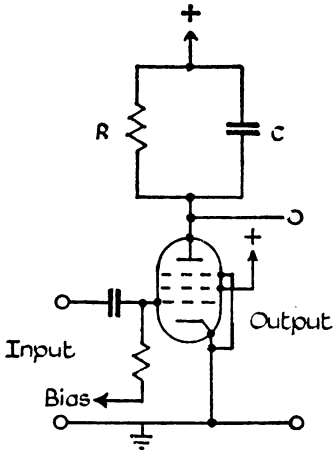


FIG. 10.—Typical pentode amplifier with resistive load R and shunt capacitive load C . Mutual conductance of valve = g_m . Slope resistance high compared to R .

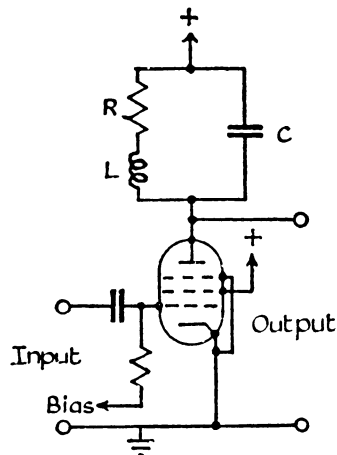


FIG. 11.—Typical pentode amplifier with resistive load R , shunt capacitive load C and shunt peaking inductance L . Mutual conductance of valve = g_m . Slope resistance high compared to R .

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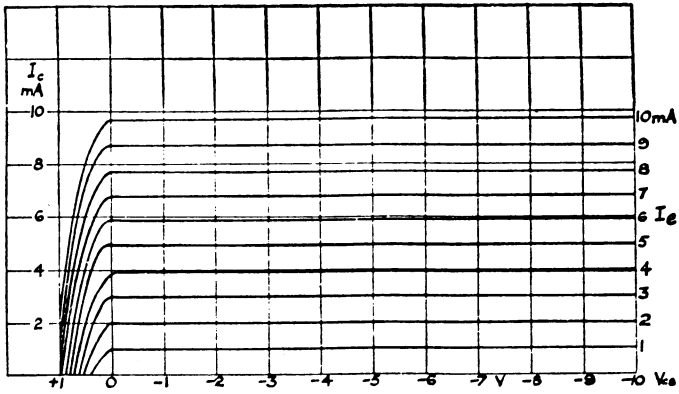


FIG. 12.—Transistor Characteristics for common base connexion.

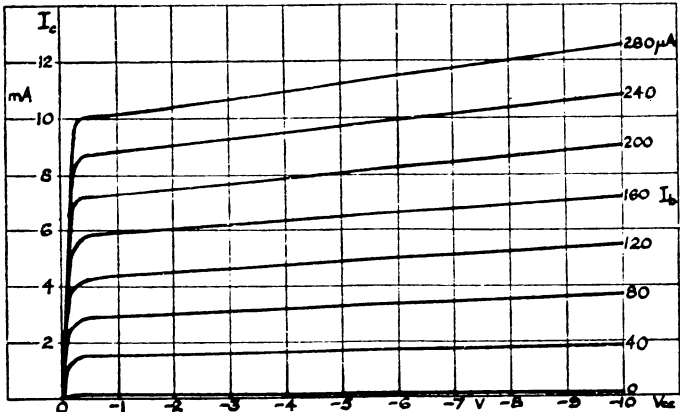


FIG. 13.—Transistor Characteristics for common emitter connexion.

CHAPTER I

ELECTRO-MECHANICAL CONVERSIONS AND DIMENSIONS

Electro-mechanical Conversions

- 1 What must be the horse-power of an engine to drive, by means of a belt, a generator supplying 7 000 lamps each taking 0.5 A at 250 V? The line drop is 5 V and the efficiency of the generator is 95%. There is a 2.5% loss in the belt drive.
[1 292 h.p.]
- 2 Find the current and energy taken by a crane motor when raising 1 000 kg through 15 m in 10 sec, when the supply is (a) 400 V, d.c.; (b) 400 V, 3-phase. Gear efficiency, 0.6; motor efficiency, 0.85; induction-motor power factor, 0.85.
[(a) 72.3 A, 0.08 kWh; (b) 49.2 A, 0.08 kWh.]
- 3 An electric hoist makes 10 double journeys per hour. In each journey a load of 6 000 kg is raised to a height of 60 m in $1\frac{1}{2}$ min and the hoist returns empty in $1\frac{1}{4}$ min. The hoist cage weighs 500 kg and has a balance weight of 3 000 kg. The efficiency of the hoist is 80% and of the driving motor 88%. Calculate (a) the electrical energy absorbed per journey, (b) the hourly consumption in kilowatt-hours, (c) the horse-power of the motor.
[(a) 1.4 kWh; (b) 14 kWh; (c) 38.4 h.p.]
- 4 An output of 480 000 kg of material per 6-hour shift is required by an endless-rope haulage from a dip 900 m long, gradient 1 in 8, the haulage speed being 5 km per hr. The motor has to exert a constant pull of 5 000 N on the rope to overcome all friction losses. Assuming an efficiency of 0.75, calculate the motor h.p.
[56.8 h.p.]
- 5 A shaft (mean diameter, 15 cm) is being turned in a lathe at 25 r.p.m. Taking the feed at 8 mm, the tangential force of the tool against the job at 18 000 N, and the overall efficiency of motor and gearing as 0.7, find the kWh consumed by the driving motor in a traverse of 1 m.
[1.12 kWh.]
- 6 Calculate the current taken by a 500-V, d.c. motor, efficiency 80%, driving a fan, efficiency 70%, which delivers 1 400 m³ of air per min at a water gauge of 15 cm.
[123 A.]
- 7 Find the current taken by a 480-V, d.c. motor driving a pump to raise 14 000 litres of water per minute to a height of 27 m. Motor efficiency 0.9; pump efficiency 0.75. Allow a head of 3 m for pipe friction.
[212 A.]

CONVERSIONS

- 8** Find the h.p. rating of a pump to supply 365×10^6 litres of water per annum to a storage tank 5 m in diameter. An automatic control is to start the pump at 80 N/cm^2 pressure and shut it off at 40 N/cm^2 pressure. Each operation takes 2 hours. Find also the annual cost with energy at 1.5d per kWh. Assume the efficiencies of pump and motor to be 0.6 and 0.9 respectively. [22 h.p., £410.]
- 9** A boiler feed pump delivers 185 000 kg of water per hour at 200 N/cm^2 . The pump efficiency is 0.85. Calculate the line current when the pump is driven by (a) a 500-V, d.c. motor, efficiency 0.9; (b) a 500-V, 3-phase induction motor, efficiency 0.9, power factor 0.9. [(a) 215 A; (b) 138 A.]
- 10** A fully-charged car battery can give 100 Wh per charge under starting conditions. At each start, the engine has to be cranked at 60 r.p.m. for 10 sec against a resisting torque of 67.5 Nm. Allowing an overall efficiency for motor and gearing of 25%, estimate the number of starts per battery charge. [21*.]
- 11** Calculate the available continuous power of a hydro-electric station supplied from a catchment area A of 150 km^2 with an annual rainfall F of 125 cm and an effective head H of 300 m. Assume yield factor k , to allow for run-off and loss by evaporation, to be 50%, and efficiency η of plant to be 80%. If the load factor be 33%, what should be the rating of generators installed? Show that average power = $0.312 \eta k A F H$ kW. [7 MW; 21 MW.]
- 12** Find the head in metres of a hydro-electric generating station in which the surface of a reservoir $4\,000 \text{ m}^2$ in extent falls by 30 cm when 75 kWh is developed in the turbine. The efficiency of the turbine is 70%. [32.8 m.]
- 13** A hydro-electric generating station is supplied from a reservoir of capacity $6 \times 10^9 \text{ m}^3$ at a head of 150 m. (a) What is the total available energy in kilowatt-hours if the hydraulic efficiency be 0.8 and the electrical efficiency 0.9? (b) Find the fall in the reservoir level after a load of 12 MW has been supplied for 3 hr if the area of the reservoir is 2.5 km^2 . (c) If the reservoir be supplied by a river at the rate of 1.2 m^3 per sec, what does this flow represent in kilowatts, in kilowatt-hours per day, and in kilowatt-hours per annum?
 [(a) $1.76 \times 10^6 \text{ kWh}$; (b) 4.9 cm; (c) 1 270 kW;
 80 400 kWh; $11.1 \times 10^6 \text{ kWh}$.]
- 14** A current of 80 A flows for 1 hr in a resistance across which there is a voltage of 2 V. Determine the velocity with which a weight of 1 000 kg must move in order that its kinetic energy shall be equal in amount to the energy dissipated in the resistance. [34 m per sec.]

I. CONVERSIONS AND DIMENSIONS

- 15 Find the amount of electrical energy expended in raising the temperature of 50 litres of water by 75°C . To what height could a weight of 5 000 kg be raised with the same expenditure of energy? Assume that the heater has an efficiency of 90% and the hoist an efficiency of 70%. [4.85 kWh; 249 m.]
- 16 An 80-V battery drives a 3 000-kg truck at 20 km/h on the level against a frictional resistance of 525 N. The overall efficiency is 0.6. Find the hourly discharge of the battery in ampere-hours. If the same power be taken from the battery on an up gradient of 1%, find the speed. [61 Ah; 12.8 km/h.]
- 17 A train weighing 250 000 kg is running at 50 km/h; the frictional resistance is 50 N per 1 000 kg and the gearing efficiency is 0.8. Find the kW input to the motors of efficiency 0.9 (a) on the level, (b) on a 1 in 100 gradient, (c) when on the level at this speed and accelerating at 0.15 m/s^2 , (d) as (c) when on 1% gradient. [(a) 242; (b) 716; (c) 968; (d) 1 440 kW.]
- 18 A 30 000-kg car has a frictional resistance of 2 700 N. Find the energy needed to drive the car up a 5% gradient, 150 m long, in 2 min. Gear efficiency 0.75, motor efficiency 0.9. Calculate the current for a line voltage of 550 V. [1.08 kWh; 58.7 A.]

Electro-thermal Conversions

- 19 A generating station has an overall thermal efficiency of 15.5%, and 0.80 kg of coal is burnt for every kilowatt-hour measured on the switchboard. Determine the calorific value of the fuel in kilo-calories per kilogram. [6 950.]
- 20 A small generating plant uses producer gas with a calorific value of $1\,200\text{ kcal/m}^3$. Find the volume of gas required per hour to generate 100 kW. The overall efficiency of the plant is 20%. [360 m^3 .]
- 21 A gas fire consumes 1.4 m^3 of gas per hour, the calorific value of the gas being $4\,500\text{ kcal/m}^3$. If the efficiency of this fire is 50%, calculate the loading of an electric fire to give the same heat output, and compare the hourly cost. Gas costs 5d per m^3 and electricity 1.8d per unit. [8.66 kW; gas 7d; electricity 6.6d.]
- 22 Calculate the thermal efficiencies of (a) a 2-litre, 2 000 W electric kettle, and (b) a 2-litre gas kettle, from the following data. The temperature of 1 litre of water is raised from 15°C to 95°C in $3\frac{1}{4}$ min in the electric kettle, and for the same temperature rise in the gas kettle 0.06 m^3 of gas is used. Calculate also the cost in each case, with electricity at 1.7d per unit and gas at 3d per m^3 . The gas has a calorific value of $4\,500\text{ kcal/m}^3$. [(a) 85.7%, 0.184d; (b) 29.6%, 0.18d.]

CONVERSIONS

- 23** A well-lagged 450-litre tank with its heater weighs 90 kg and has a specific heat of 0.1. Find the kWh needed to raise the temperature of a full tank of (a) water, (b) oil, from 10° to 50° C. Take specific heat of oil as 0.5 and its specific gravity as 0.9. Ignore tank losses. [(a) 21.3; (b) 9.8 kWh.]
- 24** The water-cooled brake-pulley of an electric motor contains 1 litre of water at 100° C. If the input to the motor is 25 A at 250 V, find the time taken to boil off all the water. The efficiency of the motor is 80% and the latent heat of steam is 538 cal per g. [7.5 min.]
- 25** Estimate the loading necessary for an electric water heater to raise the temperature of 3.4 litres of water from 15° to 95° C in 10 min. Take the heat loss in radiation during this period as 60 000 J, and the water equivalent of the heater as 130 g. Determine approximately the efficiency of the operation. How long would the heater take to raise the temperature of the water if its loading were halved? [2 070 W; 91.5%; 21 min.]
- 26** The heater element of an electric kettle has a constant resistance of 100 Ω , and the applied voltage is 250 V. Calculate the time taken to raise the temperature of 1.135 litres of water from 15° to 90° C, assuming that 85% of the power input to the kettle is usefully employed. If the water equivalent of the kettle is 100 g, find how long it will take to raise a second 1.135 litres of water through the same temperature range immediately after the first. What is the efficiency of the second operation? [11.1 min.; 10.2 min.; 93%.]
- 27** The daily cycle of a thermal-storage electric heating installation begins at 6 a.m., when 100 000 kg of water is stored at 95° C. Until 5 p.m., 5 000 kg of water per hr is supplied for radiators at an average temperature of 75° C returning at 40° C. For washing, etc., a total of 25 000 kg at an average temperature of 72° C is supplied during the day and is replaced by water at 12° C. The electrical supply to the boiler, which has an efficiency of 97%, is available between 12 midnight and 4 a.m. Estimate the kW loading of the boiler, and the temperature of the water at 5 p.m. Neglect all losses. [1 030 kW; 60.8° C.]
- 28** A cubic tank of 1-m side is filled to 90% capacity five times daily. The water is heated from 10° to 55° C. The loss per m² of tank surface per 1° C temperature difference is 17 W for unlagged tank and 7 W for lagged tank. Find the loading in each case and the corresponding efficiency. [14.4 kW, 68%; 11.7 kW, 84%.]
- 29** A 2-kg electric iron has an initial rate of temperature rise of 33.5° C per min. (a) Estimate its loading in watts. (b) If the total heat dissipation be 3.0 W per °C for the whole surface,

I. CONVERSIONS AND DIMENSIONS

- estimate the final steady temperature rise of the iron. Specific heat, 0.16. [(a) 750 W; (b) 250° C.]
- 30 The star-connected elements of a 3-phase resistance oven are each wound with 560 cm of nickel-chrome strip 0.25 mm × 0.5 cm. If the strip temperature is to be 1 000° C and the charge temperature 650° C, estimate the kW loading of the oven. Estimate also the strip temperature when the charge is 20° C. Take the radiating efficiency as 0.6 and the emissivity as 0.9. [9.85 kW; 903° C.]
- 31 A 250-V, 1-kW single-element electric fire is to employ a nickel-chrome resistance wire operating at 1 000° C. Estimate a suitable diameter and length for the wire. Take the radiating efficiency = 1, the emissivity = 0.9, and the resistivity of the wire at 1 000° C = $42.4 \times 10^{-8} \Omega\text{-cm}$. [0.272 mm; 860 cm.]
- 32 What must be the useful rating of a tin-smelting furnace in order to smelt 50 kg of tin per hour? Smelting temperature of tin = 235° C; specific heat = 0.055; latent heat of liquefaction = 13.31 kcal per kg. Take the initial temperature of the metal as 15° C. [1.47 kW.]
- 33 A 3-phase arc furnace has the following characteristics: current, 7 500 A; arc voltage, 90 V; reactance and resistance of transformer and electrodes together (excluding arc), 0.003 Ω and 0.002 Ω respectively. Find (a) the power factor, (b) the electrical efficiency, (c) the kVA taken from the supply, and (d) the time taken to melt 5 000 kg of steel if the overall efficiency is 50%. Take the specific heat and latent heat of fusion of steel as 0.12 and 8.9 cal/g respectively, and the melting point of steel as 1 370° C. Initial temperature, 15.5° C. [(a) 0.978; (b) 85.7%; (c) 2 415 kVA; (d) 50.5 min.]
- 34 A room measures 3 m × 4 m × 4.75 m. The air in it has to be renewed every 30 min and maintained at a temperature 10° C above that of the incoming air. Find the necessary rating of an electric heater, ignoring losses. Find the rating if the heat loss from walls, etc., is 182 kcal/°C/hr. (For air: density, 1.28 kg/m³; specific heat, 0.24.) [0.41 kW; 2.52 kW.]

Dimensions

- 35 The magnetic pull F of an electromagnet is given by $F \propto B^a A^b \mu^c$ where B = flux-density in the air-gap; A = cross-sectional area of the gap; and μ = the permeability of the air. Determine the values of a , b and c . [$a = 2$, $b = 1$, $c = -1$.]

DIMENSIONS

- 36** Given that the energy stored per unit area in a parallel-plate condenser depends upon (i) the permittivity ϵ of the dielectric; (ii) the distance d between the plates; (iii) the applied potential difference V ; determine how each factor influences the stored energy W .

$$\left[W \propto \frac{\epsilon V^2}{d} \right]$$

- 37** Is the following equation dimensionally correct for the current in a particular circuit containing mutual-inductance M , self-inductances L_1 and L_2 , and resistances R_1 and R_2 ?

$$I = V\omega M / \{(\omega^2 M^2 + R_2)^2 + \omega^2 L_1 L_2 R_1^2\}^{1/4}.$$

If not, suggest any necessary corrections. [R_2 should read R_2^2 .]

CHAPTER II

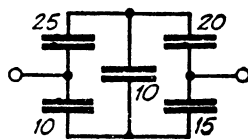
ELECTRIC FIELDS

Capacitance and Permittivity

- 1 Calculate the capacitance of an air-insulated capacitor with 13 plates each 10 cm square, the distance between plates being 2 mm. [530 pF.]
- 2 Find the equation of the voltage to be applied to a $50\text{-}\mu\text{F}$ capacitor initially uncharged to produce a steady current of 10 mA. If charging ceases after 10 sec, calculate (a) capacitor voltage, (b) charge, (c) stored energy. [(a) 2 000 V; (b) 0.1 C; (c) 100 J.]
- 3 Find the capacitance of a spherical conductor 18 cm in diameter (a) when isolated in air, (b) when totally enclosed in a conducting spherical shell 20 cm in diameter and air dielectric. [(a) 10 pF; (b) 100 pF.]
- 4 A bakelite-filled insulator, 1 m long, consists of coaxial cylinders, the inside and outside diameters of the outer and inner cylinders being 6 cm and 5.5 cm respectively. Calculate (a) the capacitance (relative permittivity = 4); (b) the charge in micro-coulombs at a potential difference of 25 kV. [(a) 2 560 pF; (b) $64.0\ \mu\text{C}$.]
- 5 Two capacitors A and B are placed in (a) series, (b) parallel. $C_A = 100\ \mu\text{F}$, $C_B = 50\ \mu\text{F}$. Find the current and the maximum stored energy in the circuit when a 240-V, 50-Hz voltage is applied. How is the voltage distributed in case (a)? [(a) 2.5 A, 1.92 J, $V_A = 80\ \text{V}$, $V_B = 160\ \text{V}$; (b) 11.3 A, 8.63 J.]

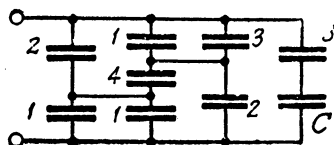
- 6 Determine the capacitance of the combination shown between terminals. The figures shown are capacitances in microfarads.

[17.3 μF .]



- 7 The figures in the network refer to capacitance in microfarads. If the combined capacitance is $5\ \mu\text{F}$, find C .

[21 μF .]



- 8 A plate capacitor A is charged to $40\ \mu\text{C}$ at 100 V. It is then connected to a similar type capacitor B of 4 times the plate

CAPACITANCE

ELECTRIC FIELD

- area of A . Find the charge on each capacitor and the loss in energy. [A , $8 \mu\text{C}$; B , $32 \mu\text{C}$. 0.0016 J .]
- 9 An air capacitor consisting of 2 parallel square plates of 50-cm side is charged to a p.d. of 250 V when the plates are 1 mm apart. Find the work done in separating the plates from 1 to 3 mm. Assume perfect insulation. [$137.5 \mu\text{J}$.]
- 10 A plate capacitor has three similar parallel plates, the outside two being metallicly connected. Find the ratio of the capacitance when the inner plate is midway between the outers to the capacitance when the inner plate is three times as near one plate as the other. [$\frac{3}{4}$.]
- 11 A capacitor is composed of two plates separated by a sheet of insulating material 3 mm thick and of relative permittivity 4. The distance between the plates is increased to allow of the insertion of a second sheet 5 mm thick and of relative permittivity ϵ_r . If the capacitance so formed is one third of the original capacitance, find ϵ_r . [3.33 .]

Electric Field

- 12 A parallel-plate capacitor has plates 0.15 mm apart, a plate area of $1\,000 \text{ cm}^2$, and a dielectric with a relative permittivity of 3. Find the electric flux density, the electric field intensity and the voltage between the plates if the capacitor has a charge of $0.5 \mu\text{C}$. [$5 \mu\text{C}/\text{m}^2$; 188.5 kV/m ; 28.3 V .]
- 13 Calculate the dielectric flux between two parallel flat metal plates each 35 cm square with an air-gap of 1.5 mm between them, the potential difference being 3 000 V. A sheet of insulating material 1.5 mm thick is inserted between the plates, and the potential difference is raised to 7 400 V. What is the relative permittivity of this material if the charge is now $32 \mu\text{C}$? [$2.16 \mu\text{C}$; 6.]
- 14 The capacitance of two parallel metal sheets, each 100 cm^2 in area, separated by a dielectric 2 mm thick, is $2 \times 10^{-4} \mu\text{F}$. A potential difference of 20 kV is applied. Find (a) the total dielectric flux in coulombs; (b) the potential gradient in kilovolts per cm; (c) the relative permittivity of the material; (d) the electric flux density. [(a) $4 \mu\text{C}$; (b) 100 kV per cm; (c) 4.5; (d) $400 \mu\text{C}/\text{m}^2$.]
- 15 The electric field strength in a mass of porcelain (relative permittivity 6) in air is 1 000 V per cm. At the inner surface of the porcelain the field makes an angle 45° to the normal and emerges into the air. Find the angle of emergence of the external field, and its magnitude. [9.5° ; 4 300 V per cm.]

II. ELECTRIC FIELDS

Electric Force

- 16 A parallel plate capacitor is immersed in alcohol of relative permittivity 26. The plates are charged to a potential difference of 20 kV, and the distance between them is 1.5 cm. Determine per cm² of plate area the force between the plates and the energy. [0.020 kN; 306 μJ.]
- 17 Find the greatest mechanical force, produced electrostatically, that can be exerted on 1 cm² of conducting material in air. Assume that air becomes conducting at a voltage gradient of 30 kV per cm. [0.003 97 N.]

Voltage Gradient

- 18 Two series-connected, flat-plate capacitors have plate areas of 0.2 m² and 0.04 m², plate separations of 0.5 mm and 0.125 mm, and permittivities of 1 and 6, respectively. Calculate the total voltage across the two capacitors that will produce a gradient of 100 kV per cm between the plates of the first capacitor. [6.04 kV.]
- 19 A steady voltage of 1 500 V is applied across two parallel metal discs of 10 cm radius, and 14 mm apart. Between the discs are three layers of dielectric: $t_1 = 2$ mm, $\epsilon_1 = 3$; $t_2 = 5$ mm, $\epsilon_2 = 4$; $t_3 = 7$ mm, $\epsilon_3 = 6$. Calculate the voltage gradient and the energy stored in each dielectric. [1 625, 1 220, 812 V/cm; 21.9, 41.3, 38.4 μJ.]
- 20 A flat-plate capacitor of 10 cm² superficial area has three dielectrics each 1 mm thick, and of relative permittivity 2, 3 and 5. Find the potential difference between the plates in volts for a charge of 0.01 μC. Draw curves showing the potential across and the potential gradient through the dielectrics. Draw a line on each curve showing the potential and potential gradient for the same charge with air as the dielectric, and state the potential difference required. [1.17 kV; 3.39 kV.]
- 21 Two concentric metal spheres, of radius r and R respectively, are maintained at a potential difference V . What ratio must the radii have for minimum voltage gradient in the intervening medium, for a given value of R ? [$r = \frac{1}{2}R$.]
- 22 Draw to scale equipotentials at every 5 kV, and a curve showing the variation of voltage gradient, between two concentric conductors maintained at a difference of 50 kV. The outer and inner radii of the inner and outer conductors are 1 cm and 10 cm respectively. What is the potential midway between the two conductors? [13.0 kV.]
- 23 A 2-mm diameter wire is co-axial with a porcelain tube ($\epsilon_r = 5$) of inside diameter 6 mm and outside diameter 12 mm.

A steady voltage of 1 000 V is applied between the wire and a metal sheath outside the porcelain. Find the voltage gradients in each of the dielectrics.

[Air : max. 8 150, min. 2 720 V/cm ; porcelain : max. 545, min. 273 V/cm ; air at outside surface of porcelain : 1 365 V/cm.]

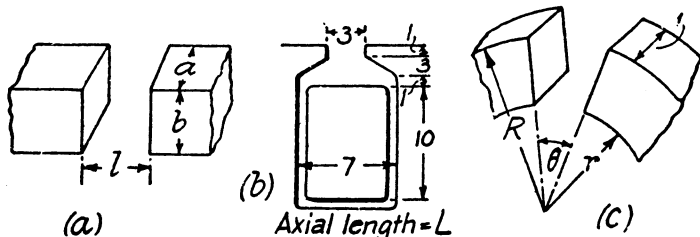
- 24 A 50-pF compressed-gas capacitor for a 100-kV Schering bridge is of the concentric cylinder type. It is desired to limit the maximum stress in the inter-electrode space to 60 kV per cm. Estimate suitable values for the radius and active length of the inner, low-voltage electrode, if the inner radius of the high-voltage electrode is 7 cm. [3·7 cm ; 57·3 cm.]
- 25 The distance between the axes of two parallel conductors of radii 2 cm is 8 cm. If the conductors are maintained at a steady potential difference of 10 kV, draw a curve showing the variation of the voltage gradient along a line joining the axes of the conductors. Determine the ratio of the maximum and minimum values of gradient, and compare this with the similar ratio when the distance between axes of the conductors is reduced to 6 cm. [1·5 ; 1·25.]
- 26 If the breakdown dielectric strength of air be 38 kV per cm, estimate the greatest line voltage that can be used on a 3-phase transmission line with solid smooth conductors each 1 cm in diameter, spaced 2 m apart. [197 kV (peak).]
- 27 A parallel-plate capacitor has a dielectric comprising a sheet of mica 1 mm thick (relative permittivity 5·5; resistivity $10^{16} \Omega\text{-cm}$) and a layer of oil 5 mm thick (2·2; $10^{10} \Omega\text{-cm}$). Find the root-mean-square voltage gradient in each dielectric material when a potential difference of 50 kV at 50 Hz is applied. [Oil : 91·9 kV per cm; mica : 70·4 kV per cm.]
- 28 Two electrodes are so arranged that spark over occurs at a voltage of 3 000 V at normal temperature and pressure. What voltage will have to be applied under working conditions of (a) 75 N/cm² and 300° C, (b) 25 cm mercury and -30° C at a high altitude. Take normal conditions as 15° C and 10 N/cm² or 76 cm mercury. [(a) 12 800 V; (b) 1 185 V.]

CHAPTER III

MAGNETIC FIELDS

Permeance and Permeability

- 1 From the magnetization curves in Figs. 1 and 2 plot curves of relative permeability to a base of flux density (a) for cast steel ; (b) for cast iron.
- 2 An iron ring with a mean circumference of 140 cm and cross-section 12 cm^2 is wound with 500 turns of wire. When the exciting current is 2 A, the flux is found to be 1.2 mWb. What is the relative permeability of the iron ? [1 113.]
- 3 Develop expressions for permeances of the air-paths in the following arrangements. Disregard fringing and assume that the lines of force are rectilinear in (a) and (b), and circular arcs in (c) :—



[(a) ab/l ; (b) $1.55 L$; (c) $(1/\theta) \log_e (R/r)$.]

Magnetic Field

- 4 Two concentrated coils each 20 cm in diameter and wound with 100 turns, are placed coaxially 10 cm apart with their planes parallel. A direct current of 1 A is passed in the same direction through each coil. Draw a curve showing the magnetic field strength at any point on the common axis up to a distance of 80 cm from one coil.
- 5 A glass tube 2.5 cm diameter and 90 cm long is completely and uniformly wound with 900 turns of copper wire 0.9 mm diameter, insulated to 1.0 mm diameter. Find the strength of the magnetic field at the centre of the solenoid thus formed when a potential difference of 4 V is applied to its terminals. Temperature 20° C . [2 000 AT/m.]

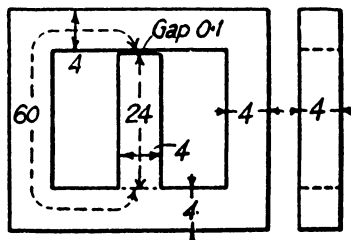
MAGNETIC CIRCUIT

- 6 Calculate the flux density (a) at the surface of, (b) 10 cm away from the centre of, a long straight isolated conductor 0.5 cm in diameter carrying a direct current of 100 A.
 [(a) 8 mWb/m²; (b) 0.2 mWb/m².]
- 7 A long straight wire of diameter 1 cm carries a direct current of 1 000 A. Draw a curve showing magnetic field strength both inside and outside the wire on a radial line extending 10 cm from the axis of the wire.
- 8 A wire is bent into a plane square of 30 cm side, and a current of 250 A is circulated round it. What is the magnetic flux density at the centre of the square? [0.945 mWb/m².]

Magnetic Circuit

- 9 A cast-steel electromagnet has an air-gap of length 2 mm and an iron path of length 30 cm. Find the number of ampere-turns necessary to produce a flux density of 0.8 Wb/m² in the gap. Neglect leakage and fringing. *[1 500.]
- 10 A cast steel ring has a circular cross-section 3 cm in diameter and a mean circumference of 80 cm. The ring is uniformly wound with a coil of 600 turns. (a) Estimate the current required to produce a flux of 0.5 mWb in the ring. (b) If a saw-cut 2 mm wide is made in the ring, find approximately the flux produced by the current found in (a). (c) Find the current value which will give the same flux as in (a). Assume the gap density to be the same as in the iron and neglect fringing.
 *[(a) 0.89 A; (b) 0.184 mWb; (c) 2.78 A.]
- 11 An iron ring of mean length 50 cm has an air gap of 1 mm, and a winding of 200 turns. If the permeability of the iron is 300 when a current of 1 A flows through the coil, find the flux density. [94.2 mWb/m².]

- 12 A 680-turn coil is wound on the central limb of the cast-steel frame shown. A total flux of 1.6 mWb is required in the gap. Find what current is required. Assume that the gap density is uniform and that all lines pass straight across the gap. Dimensions given are in centimetres. *[1.95 A.]



- 13 A ring of cast steel has an external diameter of 24 cm and a square cross-section of 3 cm side. Inside and across the ring an ordinary steel bar $18 \times 3 \times 0.4$ cm is fitted with negligible gap. Calculate the number of ampere-turns to be applied to one-half

III. MAGNETIC FIELDS

of the ring to produce a flux density of 1 Wb/m^2 in the other half. Neglect leakage. [700 AT.]

- 14 A ring of ordinary sheet steel stampings, of mean diameter 25 cm and net cross-sectional area 20 cm^2 , has 2 000 turns. Find the mean rates of change of flux and of flux density when the exciting current is raised from zero to (a) 0.25, (b) 1, (c) 3, (d) 5 A respectively, using the magnetization curves given in Figs. 1 and 2.

$$* \left[\begin{array}{l} (a) 10; (b) 3.08; (c) 1.17; (d) 0.74 \text{ mWb.} \\ (a) 5; (b) 1.54; (c) 0.58; (d) 0.37 \text{ Wb/m}^2. \end{array} \right]$$

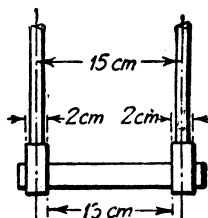
Magnetic Force

- 15 A straight conductor 100 cm long and carrying a direct current of 50 A lies perpendicular to a uniform magnetic field of 1 Wb/m^2 . Find (a) the mechanical force acting on the conductor; (b) the mechanical power in watts to move the conductor against the force at a uniform speed of 5 m per sec; (c) the electromotive force generated in the conductor. Show that the electrical power produced is equal to the mechanical power used in producing motion.

$$[(a) 50 \text{ N}; (b) 250 \text{ W}; (c) 5 \text{ V}.]$$

- 16 Find the force per metre length between two conductors 5 cm apart carrying 1 000 and 5 000 A respectively. [20 N.]

- 17 The sketch represents the contacts of an oil switch. Assuming that the vertical conductors are long compared with the distance between them find approximately the force on the contact bar when the switch carries 10 000 A.



$$[53 \text{ N}.]$$

- 18 A long straight conductor carrying a current of 1 500 A is placed at the centre of a coil whose boundary is two-thirds of the circumference of a circle of radius 12 cm and the closing chord. The coil has 1 500 turns and carries a current of 4 A. Find the couple tending to rotate the coil. [0.15 Nm.]

- 19 A flexible wire 1 m long is formed into a square, the plane of which is perpendicular to a uniform field having a flux density of 1 mWb/m^2 . Find the work done if the wire, under the electromagnetic forces set up when a current of 10 A is passed through it, assumes a circular shape. [$0.171 \times 10^{-3} \text{ J}$.]

LIFTING POWER

INDUCED E.M.F.

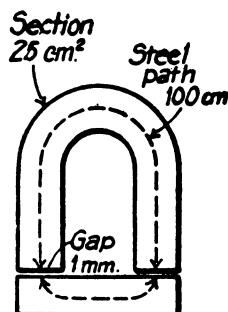
- 20 A circular coil of T turns, radius r m, negligible inductance, and resistance R ohms, is rotated about a diameter in a uniform magnetic field of density B webers/m², at an angular velocity ω radians per sec. The axis of the rotation is perpendicular to the direction of the field. The ends of the coil are joined. Find the work done per revolution. $[B^2 T^2 \omega r^4 \pi^3 / R \text{ J.}]$

Lifting Power

- 21 Determine the force necessary to separate two surfaces with 100 cm² of contact area when the flux density normal to the surfaces is 1 Wb/m². $[8\ 980 \text{ N.}]$

- 22 Calculate the force of attraction on the armature of the cast steel electromagnet shown when the flux density is 1 Wb/m². Find the ampere-turns required to produce this density.

* $[2\ 000 \text{ N; } 2\ 500.]$



- 23 A horse-shoe type relay needs an excitation of 1 800 ampere-turns to raise the armature when the gap is 1.25 mm. Each pole shoe has an area of 2 cm² and the length of iron path is 40 cm. (a) Find the pull on the armature, assuming the iron to be unsaturated. (b) If the gap closes to 0.1 mm, find the force needed to pull the armature away, the excitation remaining constant. $[(a) 129 \text{ N; } (b) 410 \text{ N.}]$

Induced E.M.F.

- 24 A wire of length 50 cm moves at right angles to its length at 40 m per sec in a uniform magnetic field of density 1 Wb/m². Calculate the electromotive force induced in the conductor when the direction of motion is (a) perpendicular to the field, (b) inclined at 30° to the direction of the field.

$[(a) 20 \text{ V; } (b) 10 \text{ V.}]$

- 25 A square coil of 10 cm side and with 100 turns is rotated at a uniform speed of 1 000 rev per min about an axis at right-angles to a uniform magnetic field having a flux density of 0.5 Wb/m². Calculate the instantaneous value of the induced electromotive force when the plane of the coil is (a) at right-angles to the field, (b) at 30° to the field, and (c) in the plane of the field.

$[(a) 0; (b) 45.3 \text{ V; } (c) 52.4 \text{ V.}]$

III. MAGNETIC FIELDS

- 26 A circular coil of 100 turns with a mean diameter of 30 cm is rotated about a vertical axis in the earth's field at 16 rev per sec. Calculate the instantaneous value of the electromotive force in volts induced in the coil when its plane is (a) perpendicular to, (b) inclined at 30° to, (c) coincident with, the magnetic meridian. Take $H = 14.3 \text{ AT/m}$.
 [(a) 0; (b) 0.0111; (c) 0.0128 V.]
- 27 A copper disc 40 cm diameter is rotated at 3 000 rev per min on a horizontal axis perpendicular to and through the centre of the disc, the axis lying in the magnetic meridian. Two brushes make contact with the disc, one at the edge and the other at the centre. If the horizontal component of the earth's field be 0.02 mWb/m^2 , calculate the electromotive force induced between the brushes. [0.12 mV.]
- 28 A meter driving motor consists of a horizontal disc of aluminium, 20 cm in diameter, pivoted on a vertical spindle, and lying in a permanent magnetic field of density 0.8 Wb/m^2 . The current flow is radial from the spindle to the circumference of the disc. The circuit resistance is 0.225Ω , and a potential difference of 2.30 V is required to pass a current of 10 A through the motor. Calculate the rotational speed of the disc and the power lost in friction. [319 rev per min; 0.5 W.]
- 29 A flat coil of 500 turns of fine wire, and of mean area 600 cm^2 , revolves at a speed of 1 600 rev per min in a uniform magnetic field having a flux density of 80 mWb/m^2 . Calculate the peak, average over a half cycle, and root-mean-square values of the electromotive force induced in the coil. [402 V, 256 V, 284 V.]
- 30 A wire, falling horizontally at right angles to a transverse field having a flux density of 1 Wb/m^2 , has its ends joined by a flexible conductor, of which the parts lying within the field remain vertical under all conditions. If the length of the wire is 10 cm and the total weight and resistance of the circuit are 1 g and 1Ω respectively, find (a) the distance traversed, (b) the velocity acquired, after 0.1 sec. [(a) 3.6 cm; (b) 62 cm/sec.]
- 31 A metal ring of resistivity $2.62 \mu\Omega\text{-cm}$, and density 2.7 g/cm^3 , is allowed to fall from rest with the plane of the ring horizontal in such a manner that each element of the ring moves transversely through a radial field of 0.5 Wb/m^2 . At what velocity is the ring falling after 0.001 sec and what is the ultimate velocity attained?
 [0.269 cm per sec; 0.277 cm per sec.]
- 32 The flux Φ linked by a coil of 100 turns varies during the period T of one complete cycle as follows: from time $t = 0$ to $t = \frac{1}{2}T$, $\Phi = \Phi_m (1 - 4t/T)$; and from $t = \frac{1}{2}T$ to $t = T$,